PHOT 504 Fall (2025-2026) **Quiz 2** Date: 2 December 2025
Michaël Barbier Student No:
Student Name:

Grading: Each quiz counts for 20% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

The duration of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Spin expectation values

An electron is in spin state: $\chi = \frac{1}{\sqrt{5}} \begin{pmatrix} i \\ -2 \end{pmatrix}$

(a) Calculate the expectation value $\langle S_y \rangle = \langle \chi | \hat{S}_y | \chi \rangle$ where $\hat{S}_y$ is represented by the matrix $S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

(b) Then calculate the standard deviation $\sigma_{S_y} = \sqrt{\langle S_y^2 \rangle - \langle S_y \rangle^2}$.

Question 2: Fermions in a Harmonic Oscillator

Assume two noninteracting fermions in a 1D harmonic oscillator potential (ignore spin). One particle is in state ψ_0 and the other in ψ_1. Exchange forces will adjust the expectation value for the distance between particles:

$$\langle \Delta x^2 \rangle = \langle x^2 \rangle_0 + \langle x^2 \rangle_1 - 2\langle x \rangle_0 \langle x \rangle_1 + 2|\langle x \rangle_{01}|^2 = \frac{\hbar}{2m\omega} + \frac{3\hbar}{2m\omega} + 0 + 2|\langle x \rangle_{01}|^2$$

where we filled in the expectation values $\langle x^2 \rangle_n = \langle \psi_n | x^2 | \psi_n \rangle = (n + 1/2)\frac{\hbar}{m\omega}$ and are only left with the unknown overlap integral: $\langle x \rangle_{01} = \int_{-\infty}^{+\infty} x \psi_0 \psi_1 dx$.

(a) Explain why the third term is equal to zero.

(b) Calculate the last term of the exchange forces: $2|\langle x \rangle_{01}|^2$.

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} = \alpha\, e^{-\beta^2 x^2/2} \qquad \text{with } \alpha = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}, \quad \beta = \sqrt{\frac{m\omega}{\hbar}}$$

$$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{2m\omega}{\hbar}}\, x\, e^{-\frac{m\omega}{2\hbar}x^2} = \alpha \sqrt{2}\, \beta x\, e^{-\beta^2 x^2/2}$$

Question 3: Symmetry and transformations

Consider a particle in an infinite well: $V(x) = 0$ for $x \in [-a, a]$ and infinite otherwise.

(a) Sketch the first three eigenstates $\psi_1(x)$, $\psi_2(x)$, and $\psi_3(x)$, before and after applying the parity operator ($\Pi : x \longrightarrow -x$). Which are symmetric/anti-symmetric under parity transformation?

(b) Assume the particle has wave function: $\psi(x) = \frac{1}{5}(3\psi_1(x) + 4\psi_2(x))$. What is the expectation value of the parity operator: $\langle \Pi \rangle = \langle \psi | \Pi | \psi \rangle$?