PHOT 504 Fall (2025-2026) **Quiz 1** Date: 28 October 2025
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Student Name:

Grading: Each quiz counts for 20% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

The duration of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Wave functions and expectation values

Consider the wave function of the ground state of the hydrogen atom:

$$\psi_{100}(r, \theta, \phi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a},$$

with Bohr radius $a = 0.0529$ nm.

(a) Prove that ψ_{100} is normalized.
(b) Then calculate the expectation value $\langle r \rangle$. You can express it in units of the Bohr radius a.

Question 2: Bra-kets in finite dimensions

Assume that the Hamiltonian operator $\hat{H}$ of a two-level system is represented by the following matrix:

$$H = \begin{pmatrix} 1 & i\sqrt{3} \\ -i\sqrt{3} & -1 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

(a) Solve the eigenvalue equation $H|\psi\rangle = E|\psi\rangle$ to obtain eigenenergies E_n of the system.
(b) Then calculate the normalized eigenstates $|\psi_n\rangle$.

Question 3: Harmonic oscillator

Consider a particle in a harmonic oscillator is in a superposition state:

$$|\psi\rangle = \frac{4}{5}|\psi_1\rangle - \frac{3}{5}|\psi_0\rangle$$

(a) Derive an expression for $\hat{x}|\psi\rangle$ using $\hat{x}$ expressed in ladder operators $\hat{a}_+$ and $\hat{a}_-$.
(b) What is the expectation value of the position $\langle \hat{x} \rangle = \langle \psi | \hat{x} | \psi \rangle$? Use the ladder operators as in (a) to simplify the calculation.

Hints: the position operator $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(\hat{a}_+ + \hat{a}_-)$, eigenstates are orthonormal, and acting with ladder operators on eigenstates:

$$\hat{a}_+|\psi_n\rangle = \sqrt{n+1}\,|\psi_{n+1}\rangle, \qquad \hat{a}_-|\psi_n\rangle = \sqrt{n}\,|\psi_{n-1}\rangle$$