PHOT 451: Microscale optical system design
LECTURE 12

Michaël Barbier, Fall semester (2025-2026)

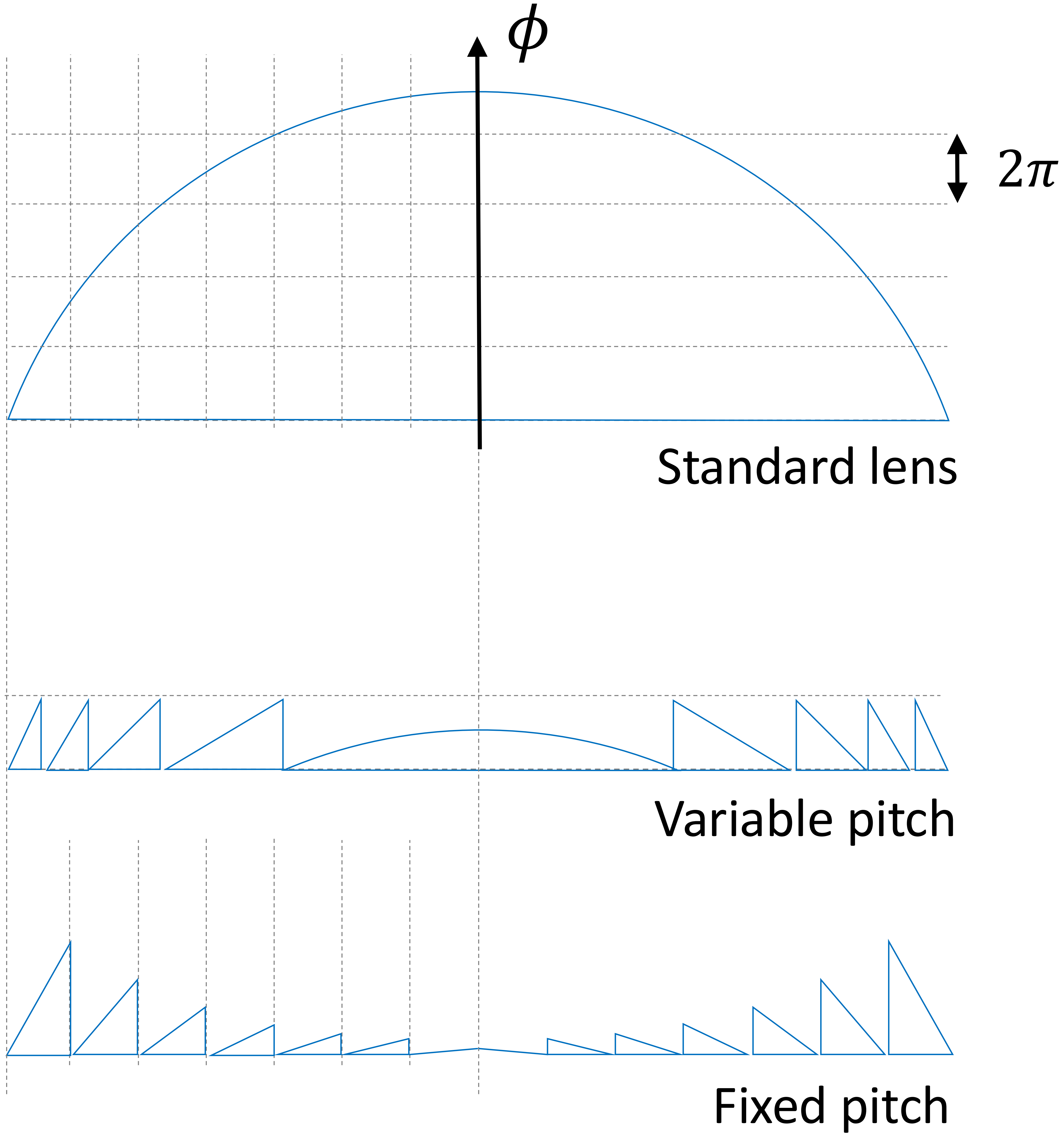
OVERVIEW OF THE COURSE

week	Topic
Week 1	Introduction to micro-scale optical components
Week 2	Light propagation in free space
Week 3	Geometric optics and raycasting
Week 4	Diffraction limit & Abberations
Week 5	**Quiz** + Beam propagation
Week 6	Refractive optical elements Microlenses
Week 7	Blazed Fresnel lenses
Week 8	Digital lenses
Week 9	Diffractive optical elements
Week 10	**Quiz** + Wave guides and beam propagation
Week 11	Wave mixing
Week 12	Gratings, periodic structures
Week 13	photonic crystals
Week 14	Whole optical system optimization

Blazed Fresnel lenses

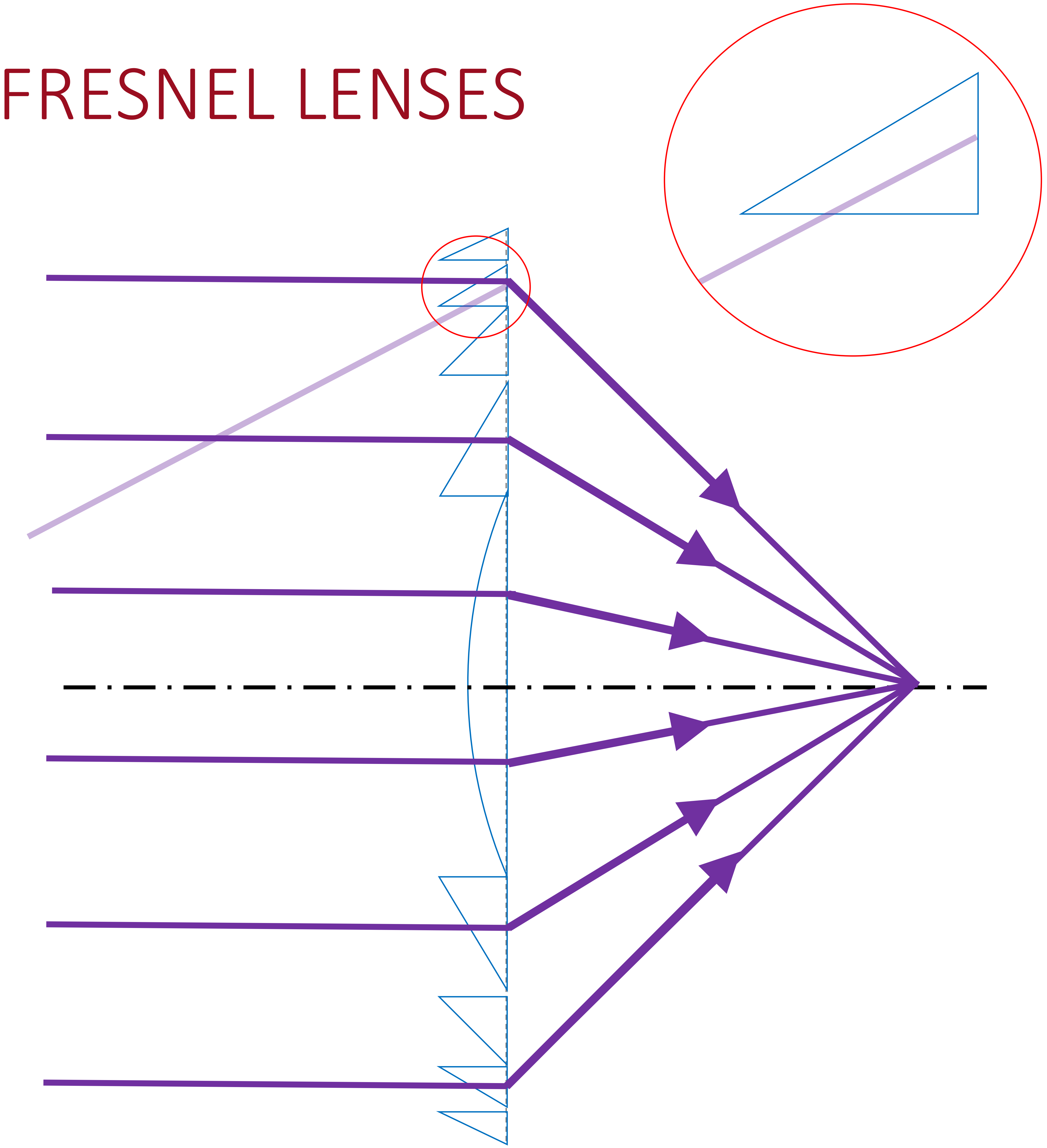
KINOFORM/BLAZED FRESNEL LENSES

- Standard lens:
  - large spherical aberration
  - Uses more material
  - Aberrations increase with curvature of lens
- Kinoform Fresnel lens:
  - Continuous surface
  - Optimize flatness
  - Optimal: max. phase $\phi = 2\pi$
- Different designs:
  - Constant height / phase or
  - Constant width / pitch



KINOFORM/BLAZED FRESNEL LENSES

- Larger diameter of lens possible than with standard lens
- Aspherical shape can be approximated
- Tooth side of the Fresnel lens
  - Incorrect phase difference if rays cross the “side” of a tooth
  - Use the focal point farthest away, rays as parallel as possible

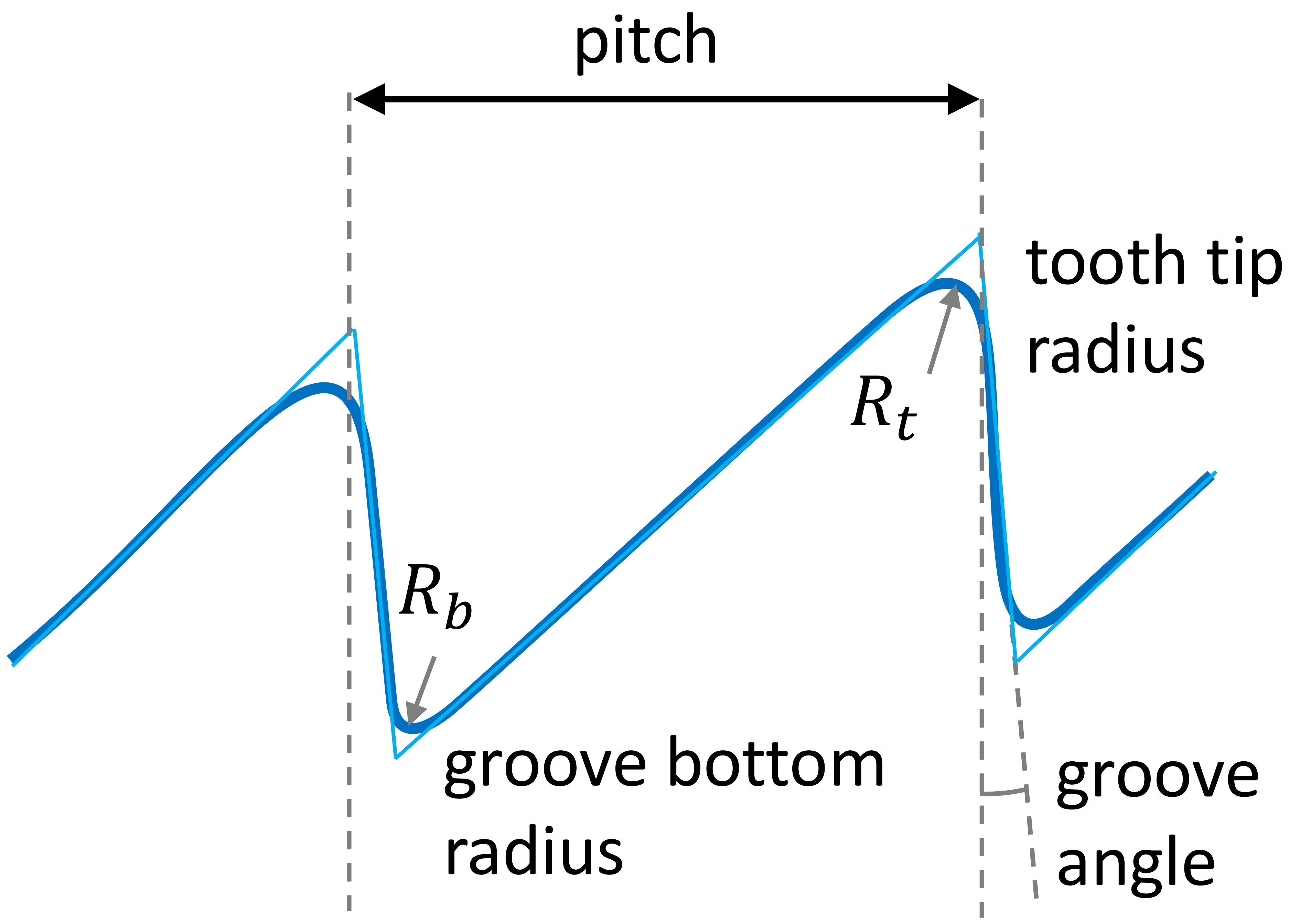


BLAZED FRESNEL LENSES

- Fabrication difficult
  - Diamond turning
  - 3D laser writing

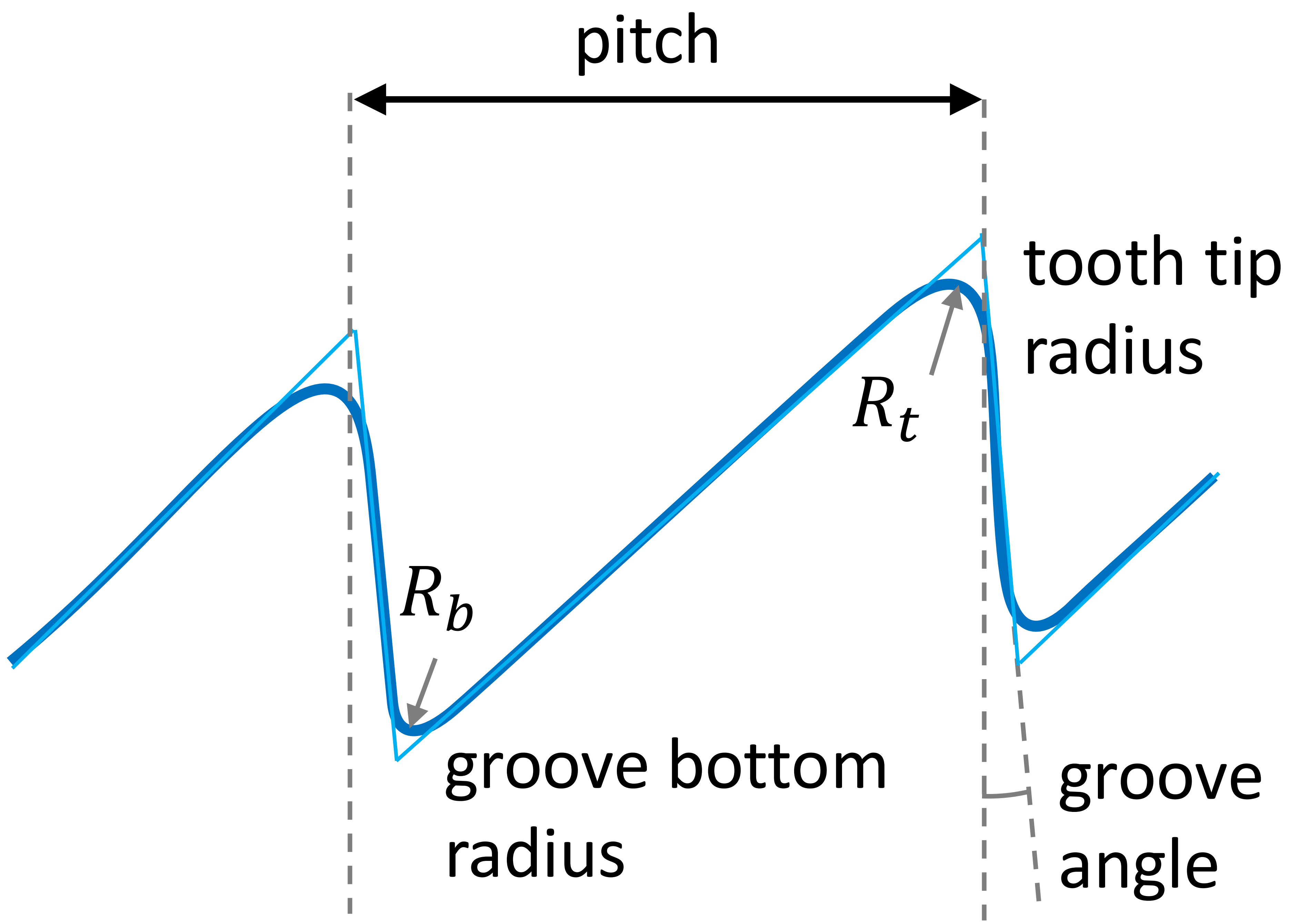
- Fabrication errors of grooves
  - Rounding of groove tip and bottom
  - Non-flat groove slope

- Consequence for Fresnel lens
  - **Dead zones** where scattering of rays takes place
  - **Background glare** and **loss of light**
  - **Chromatic aberration**



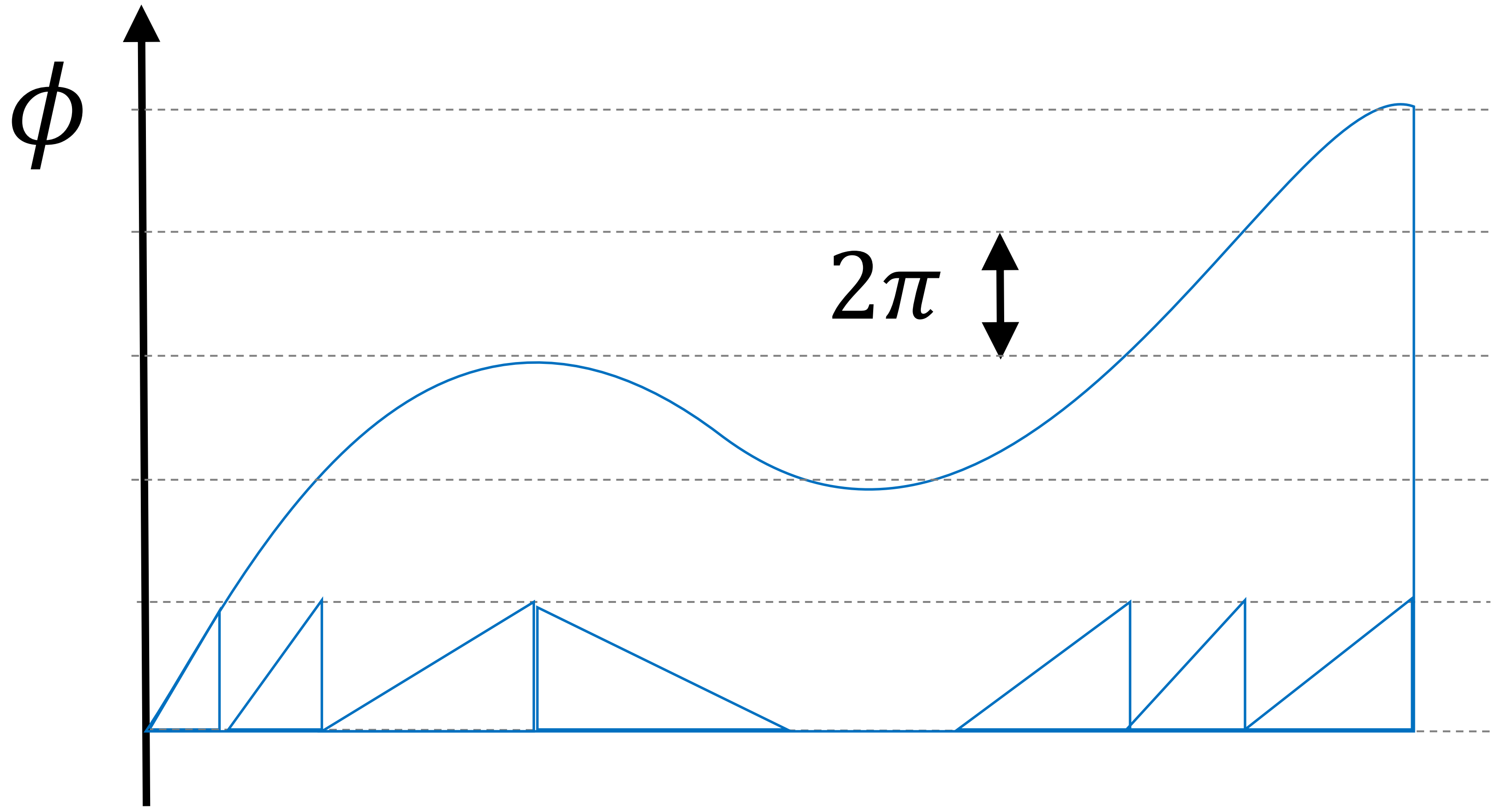
BLAZED FRESNEL LENSES

- Avoiding too much impact of Dead zones and loss of light
  - **Reduce number of Fresnel zones**: trade-off between geometrical aberration and fabrication defects
  - **Special designs** of grooves, e.g. a groove angle
  - Combine more Fresnel lenses

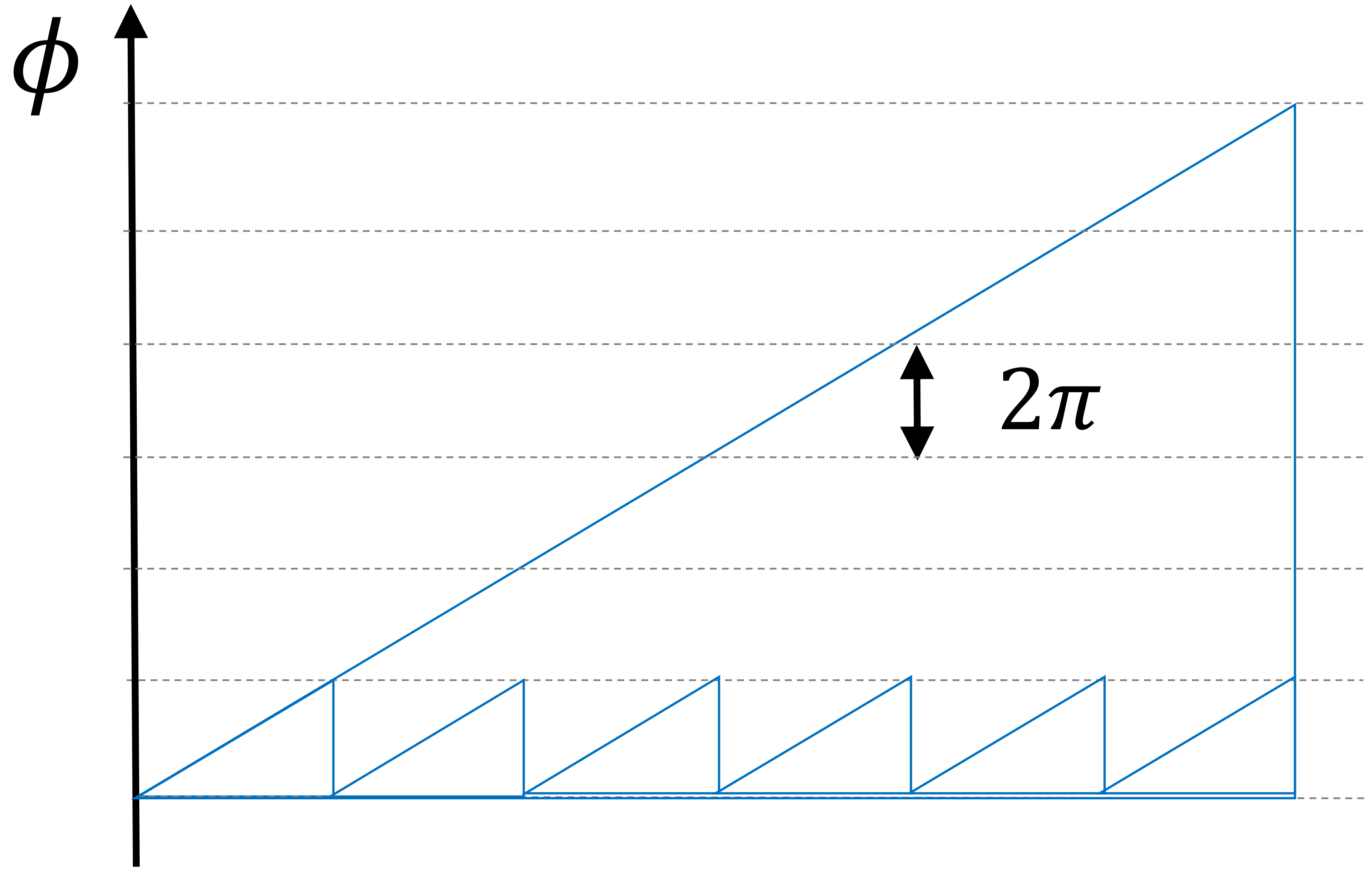


BLAZED PRISMS AND ASPHERICAL SURFACES

- General surfaces can be constructed with same method
  - Convex, concave lenses
  - Prisms
  - Aspherical lenses



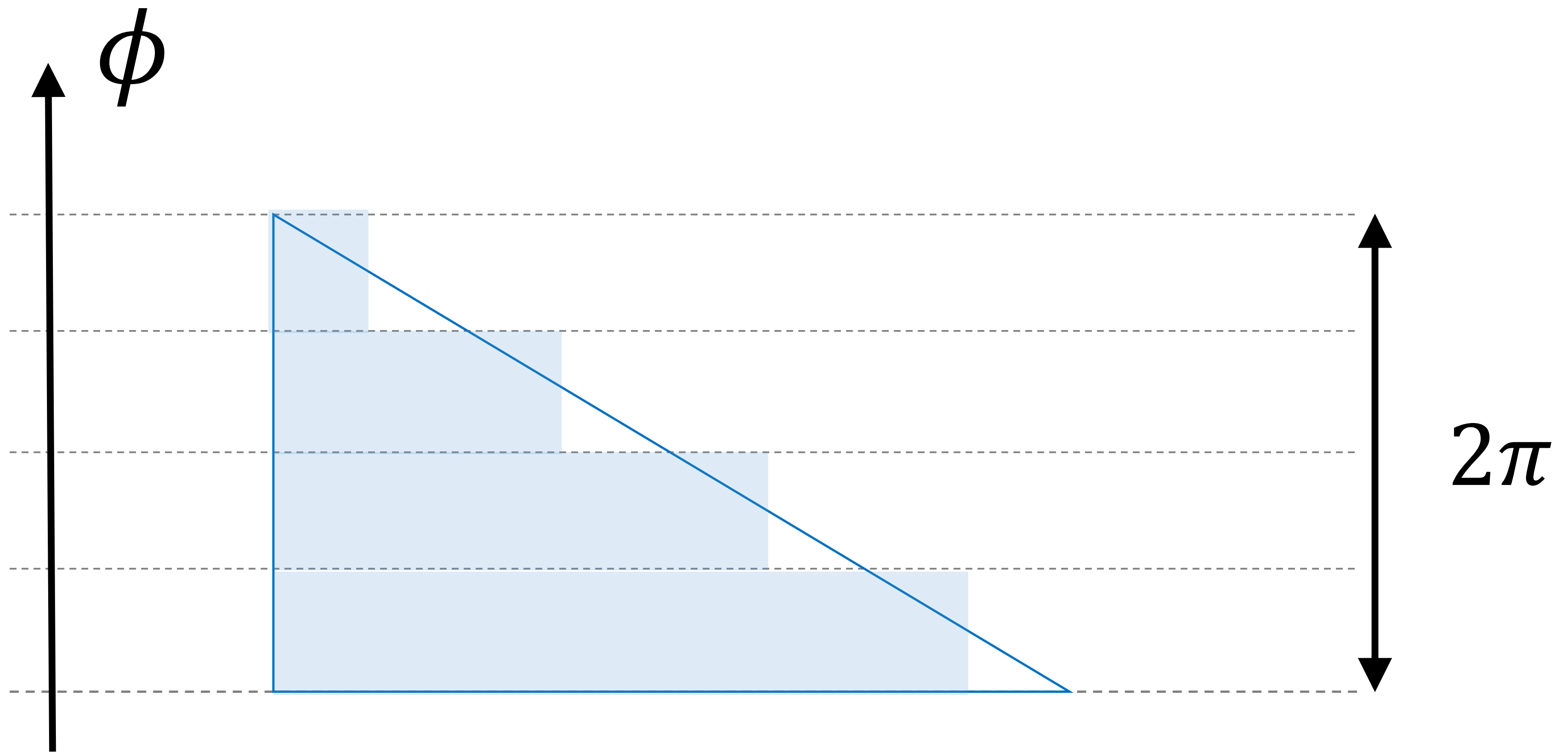
- Linear approximation often sufficient: Easier fabrication



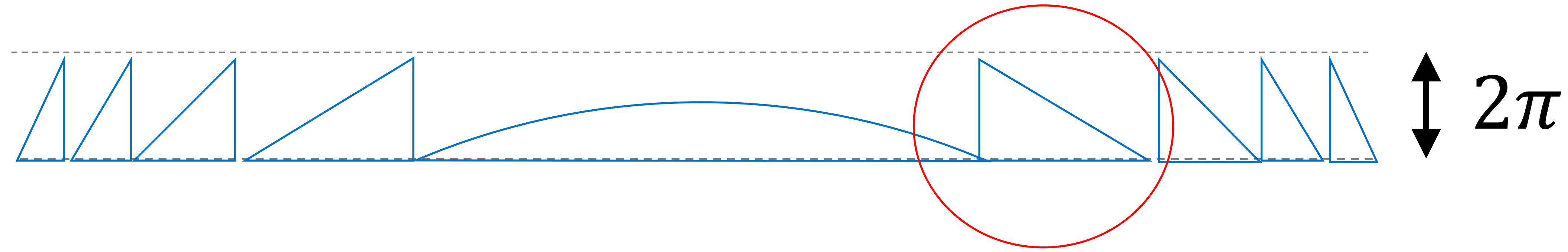
Discrete and Digital Lenses

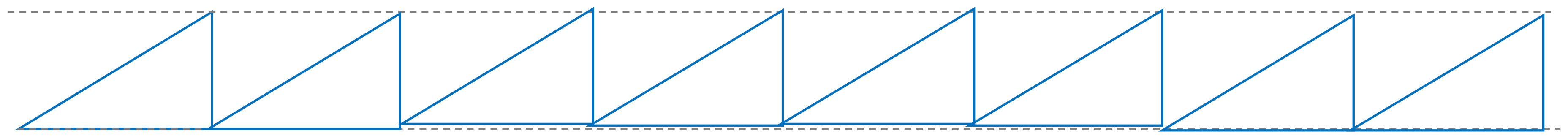
DISCRETE LENSES AND PRISMS

- Multi-level approach
  - N - levels
  - Ideal for lithographic fabrication
  - More levels approximate the exact shape better
  - All not too steep slopes and shapes can be approximated



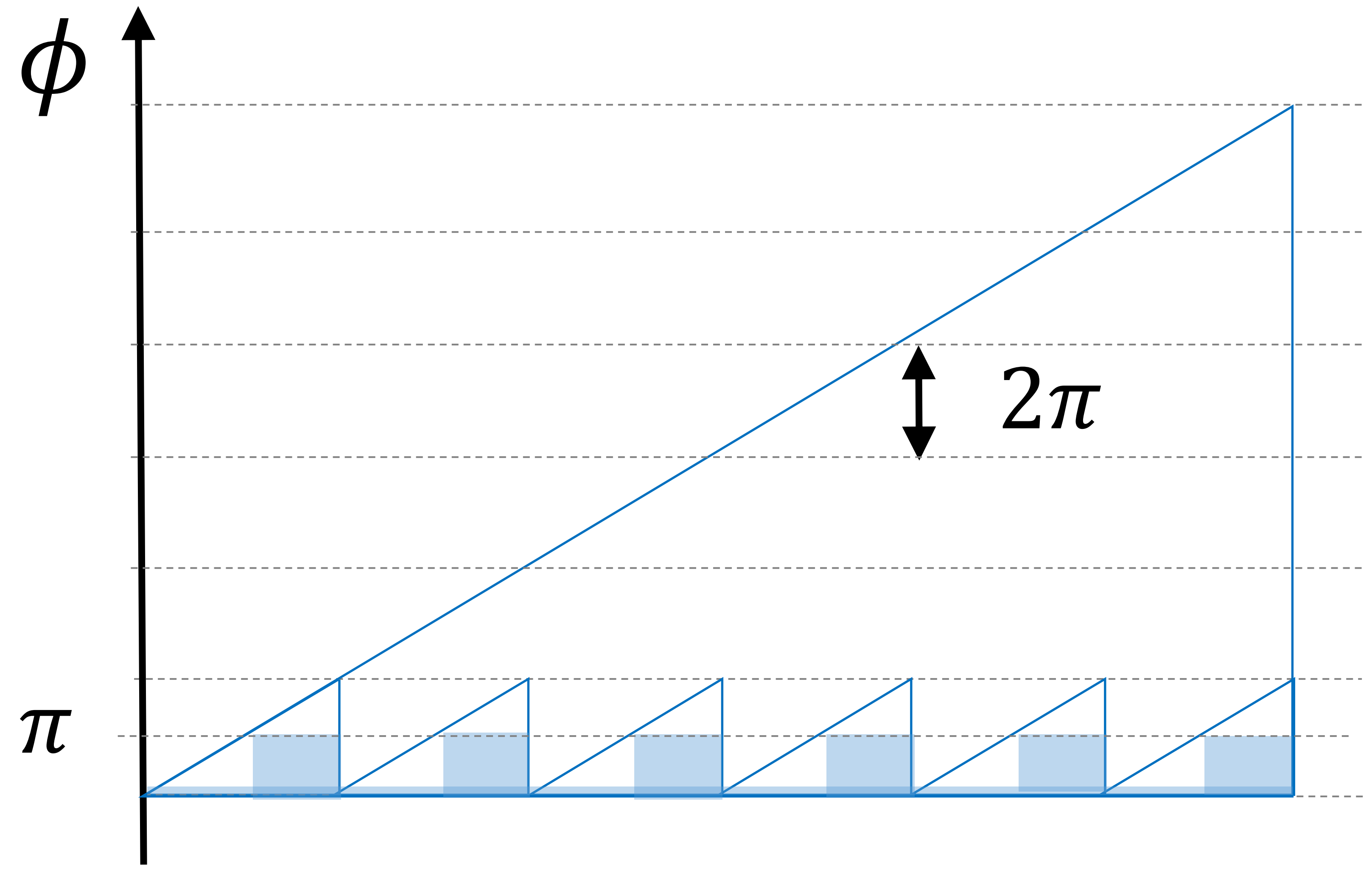
- Aberrations due to
  - Discrete phase jumps
  - Scattering at level steps

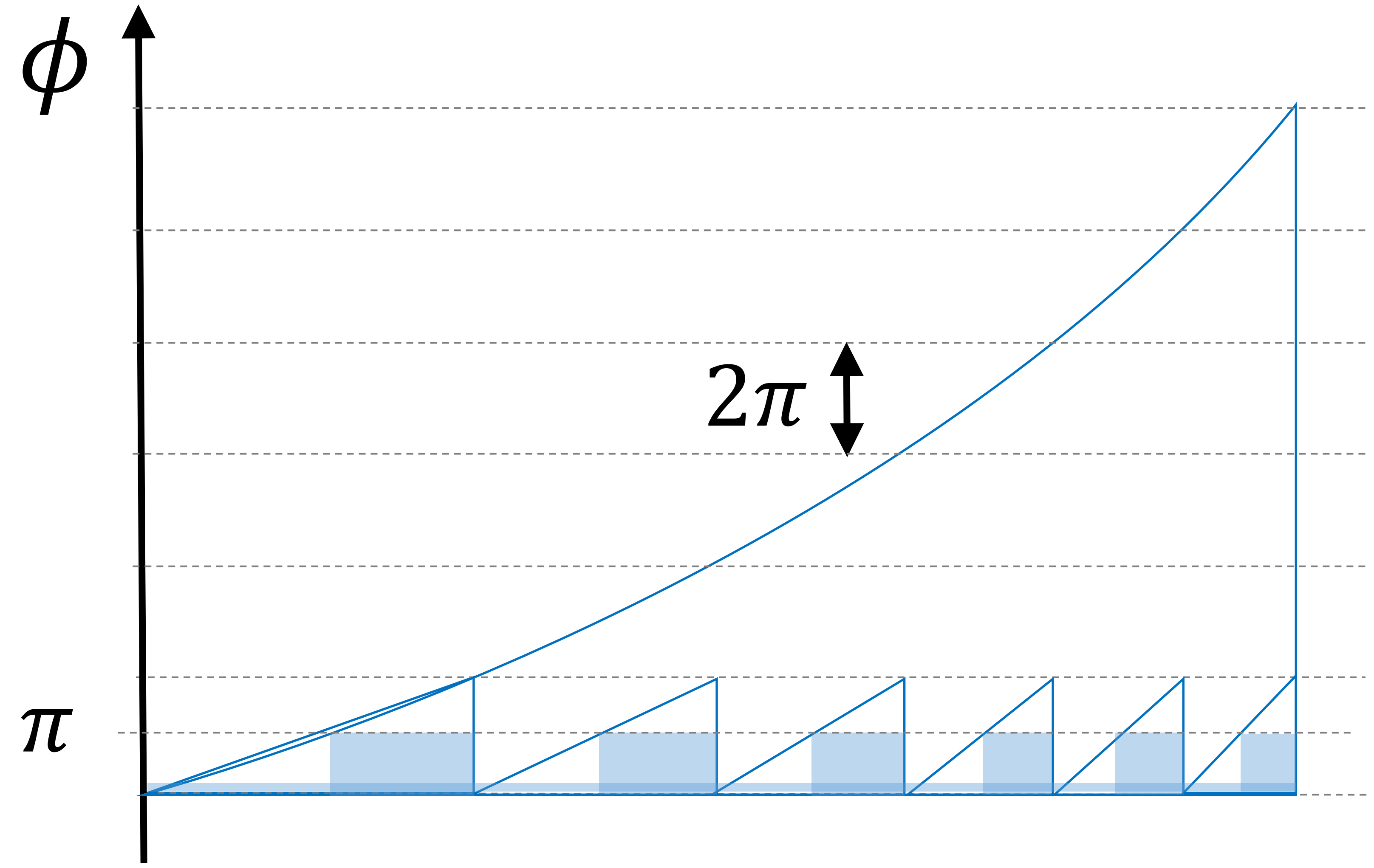




BINARY PRISMS / LENSES

- Binary phase
  - Two levels: 0 or π
- Larger slopes – smaller phase elements needed
- Phase elements
  - Ratio of thickness over width
  - Size changes for lenses
  - Constant for prisms





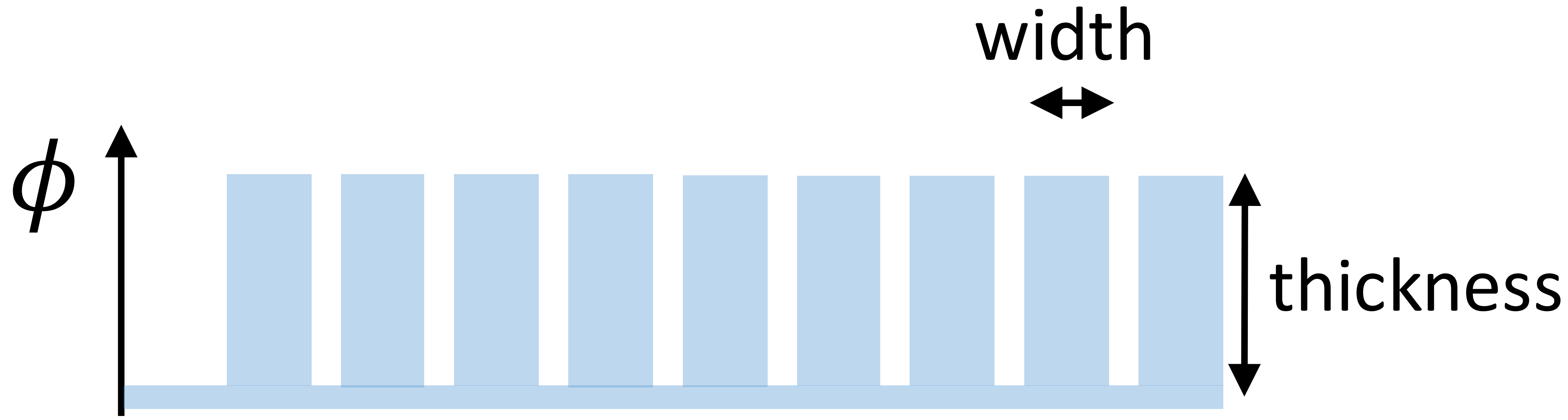
Diffractive elements

THIN & THICK PHASE ELEMENTS

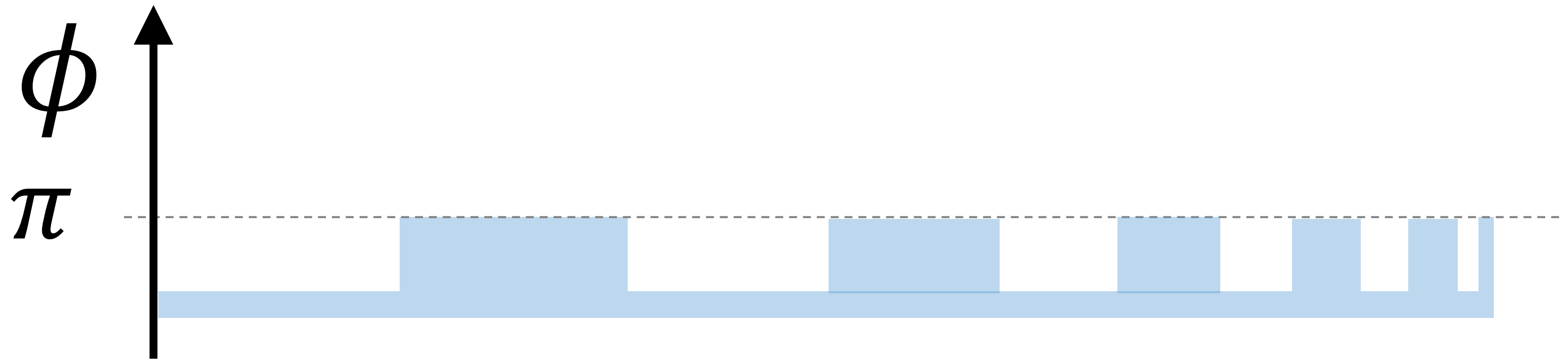
- Ratio of thickness over width
- Thickness for π phase difference depends on:
  - Refractive indices n_2, n_1,
  - Required "slope",
  - Wavelength (chromatic aberration)
- Condition for thin elements:

$$\frac{Thickness}{width} \ll \frac{n_1}{\sqrt{n_2^2 - n_1^2}}$$

Grating with **thin** phase elements



Grating with **thick** phase elements



Lens: **thick** and **thin** phase elements

THIN & THICK PHASE ELEMENTS

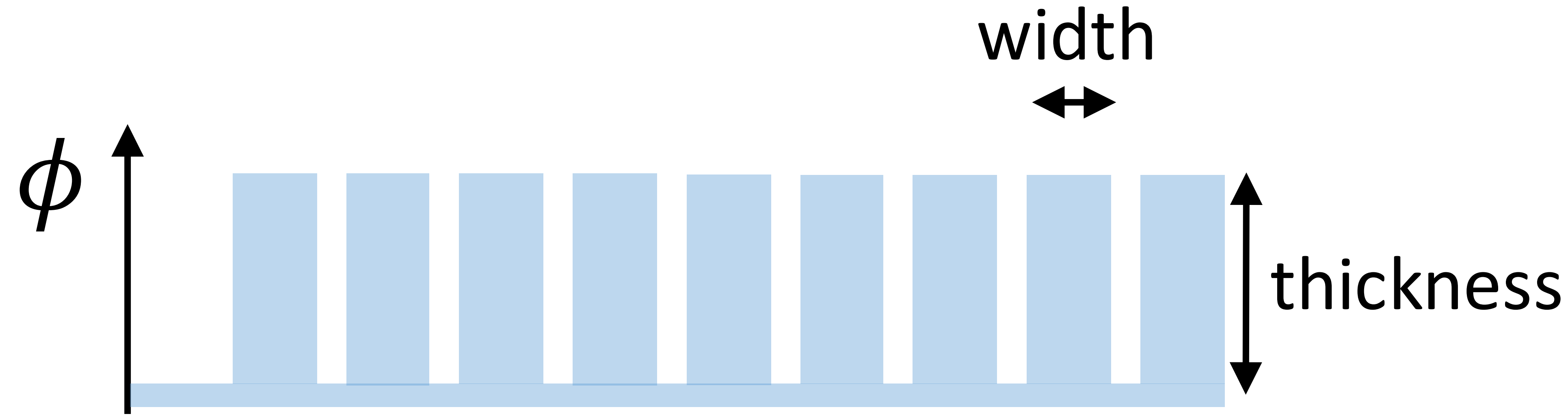
- Kirchoff approximation for thin elements: scattering happen in single plane
- 2D transmission function of element:

$$u_0(x, y) = a(x, y)\, e^{i\varphi(x,y)}$$

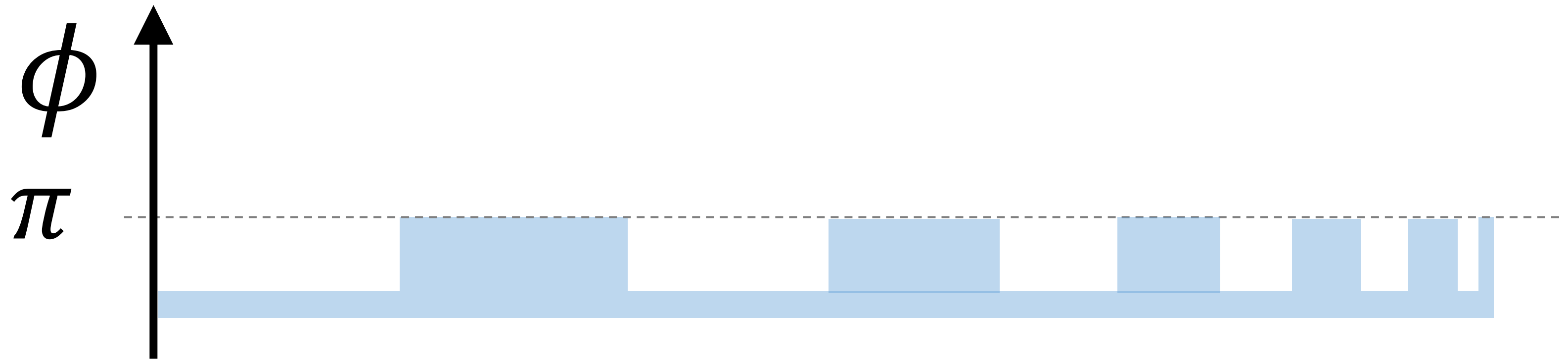
Amplitude: $a(x, y)$

Phase: $e^{i\varphi(x,y)}$

Grating with **thin** phase elements



Grating with **thick** phase elements

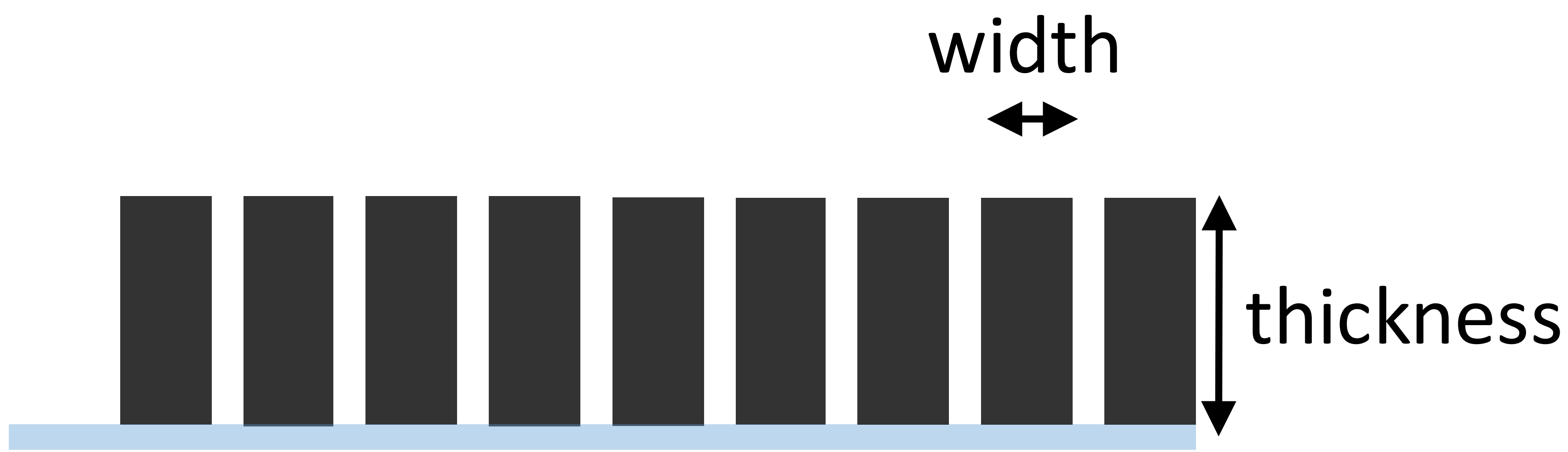


Lens: **thick** and **thin** phase elements

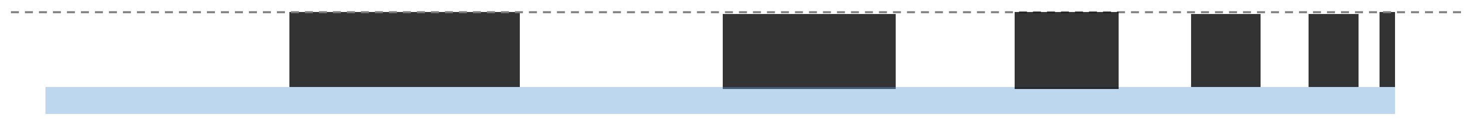
AMPLITUDE VERSUS PHASE

- Changing amplitude i.e. transparency of elements instead of phase
- Binary elements: opaque
- Amplitude elements similar to phase but
  - Higher loss of light
  - Often simpler fabrication
  - Diffraction-based

Grating with **thin** amplitude elements



Grating with **thick** amplitude elements

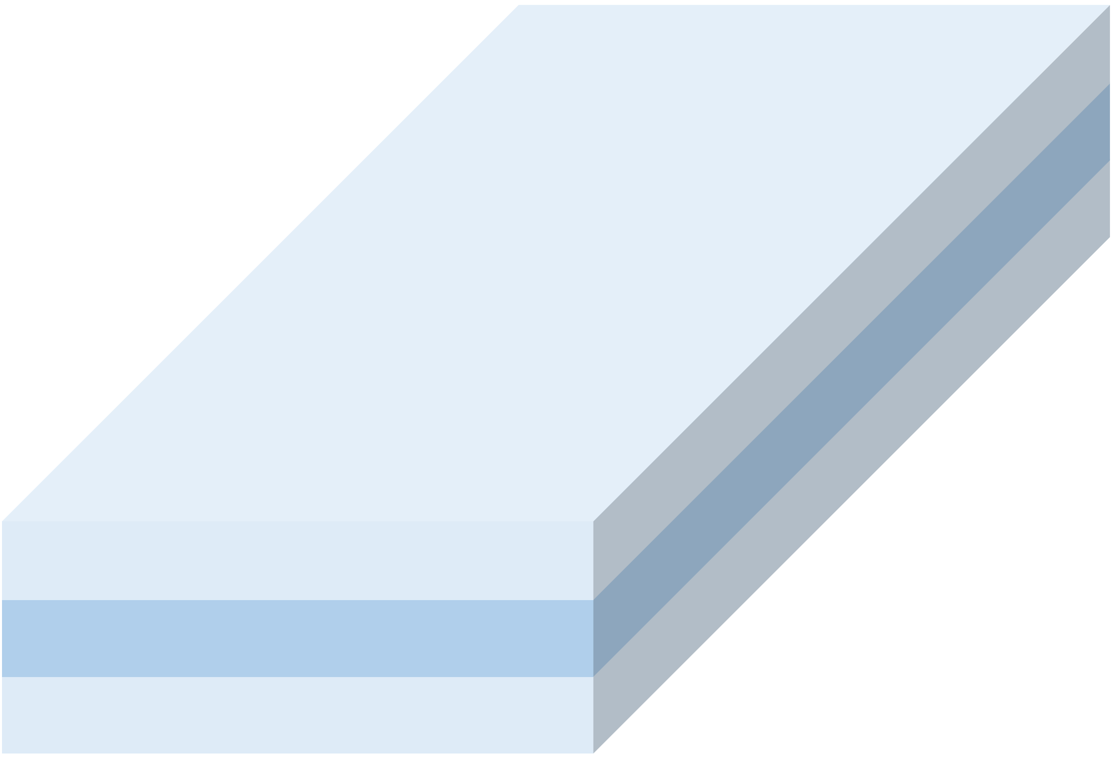


Lens: **thick** and **thin** amplitude elements

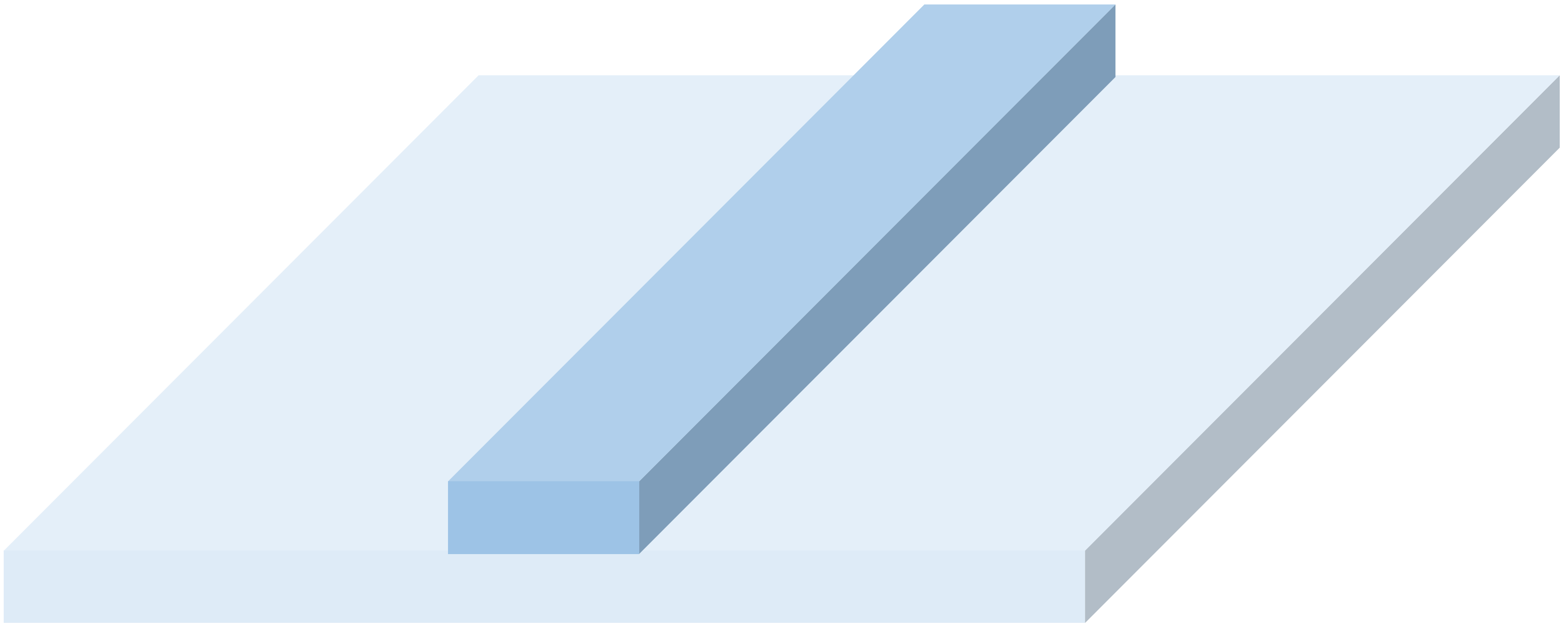
Waveguides & Fibers

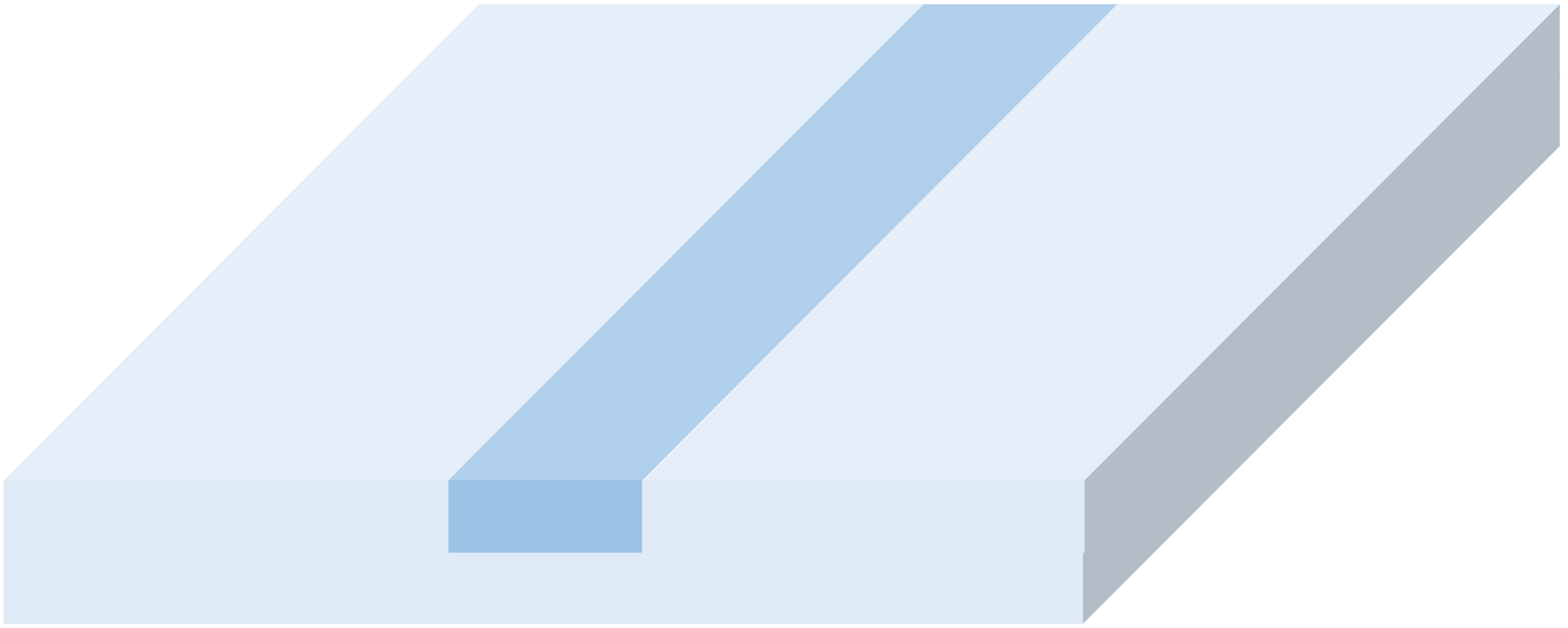
SLABS, WAVEGUIDES AND FIBERS

2D slabs

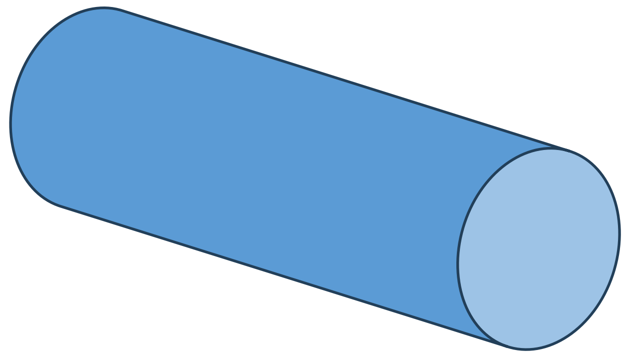


waveguide

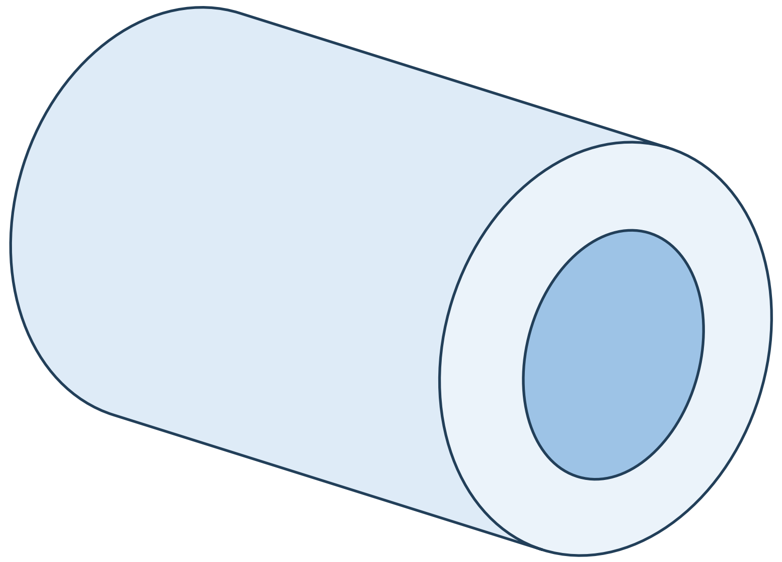




Fiber core



Fiber with cladding



DIELECTRIC WAVEGUIDES: GEOMETRIC DESCRIPTION

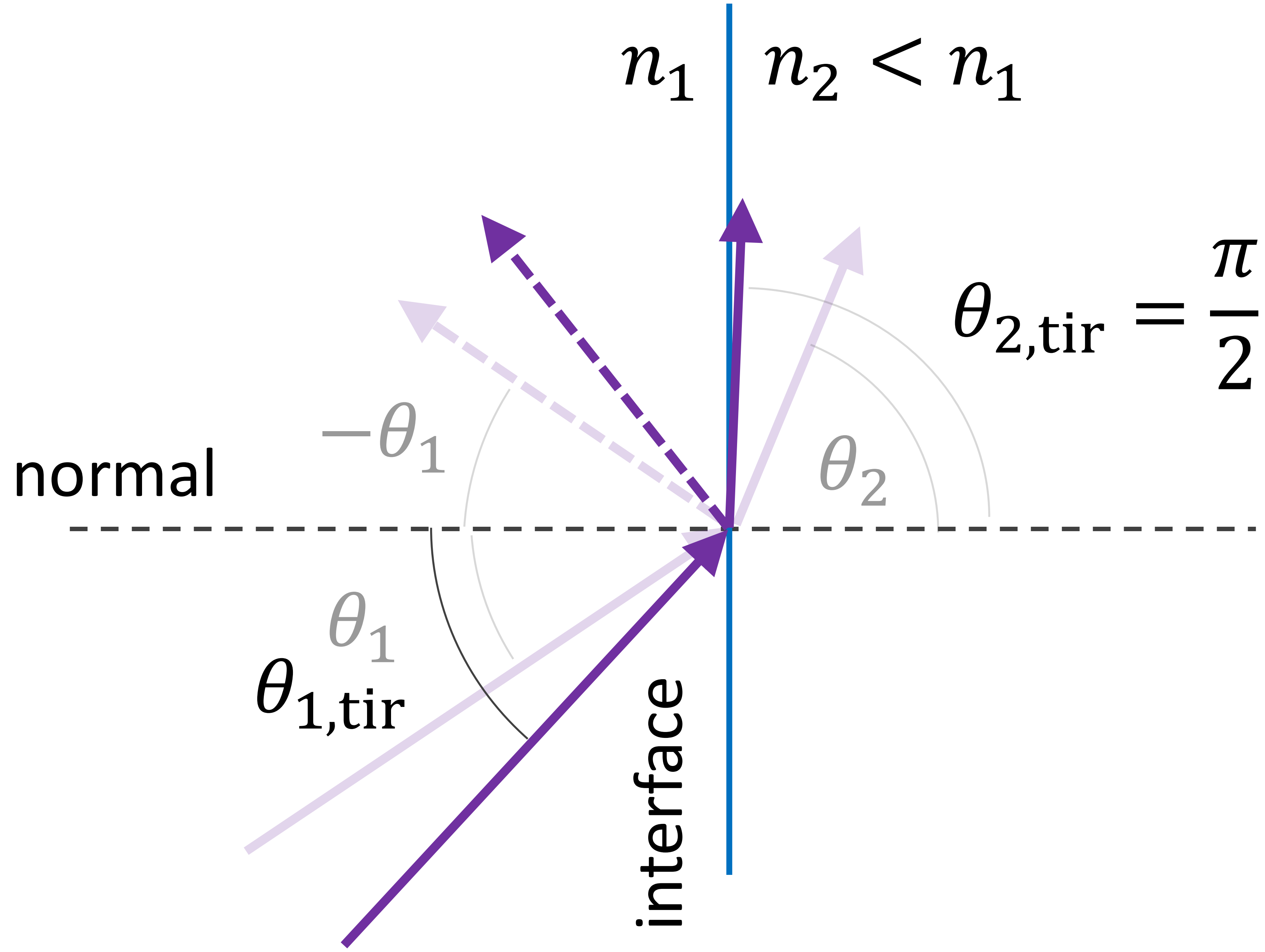
- Principle: Total internal reflection

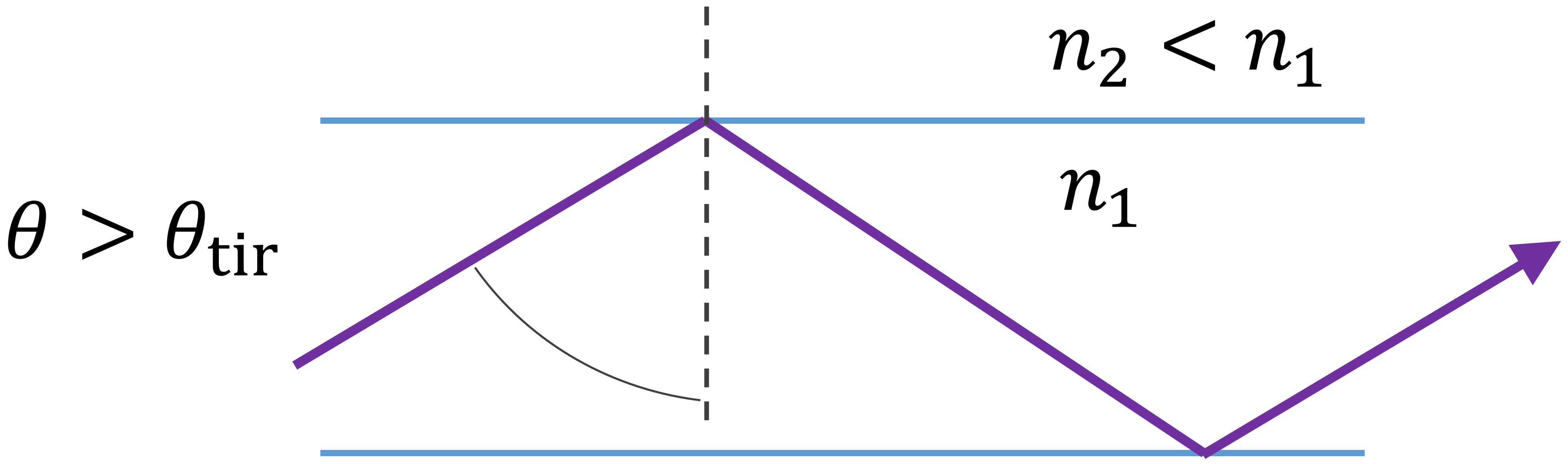
- Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- No solution beyond $\theta_2 = \frac{\pi}{2} = 90°$

$$\Rightarrow \quad \sin\theta_{1,\mathrm{tir}} = \frac{n_2}{n_1} < 1$$

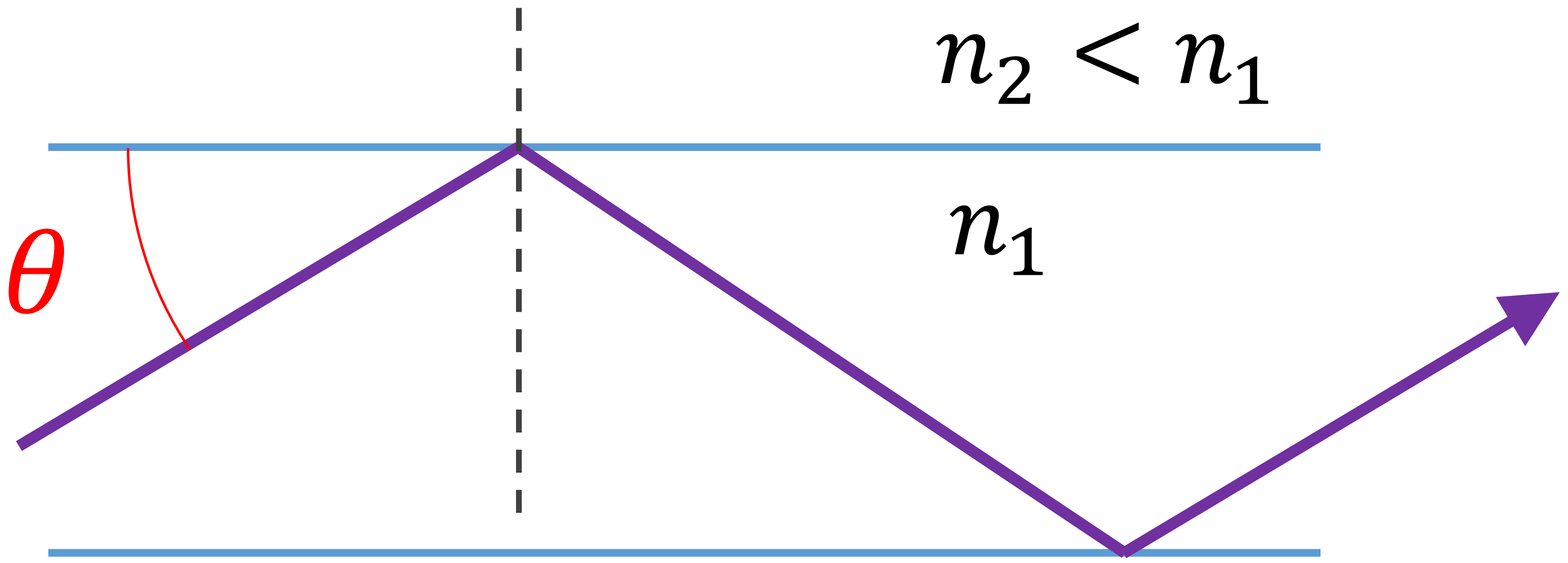




(SCALAR) WAVES: EIGENMODES FOR PERFECT MIRRORS

(From now θ angle with interface)

- Raytracing insufficient
- Interference is expected

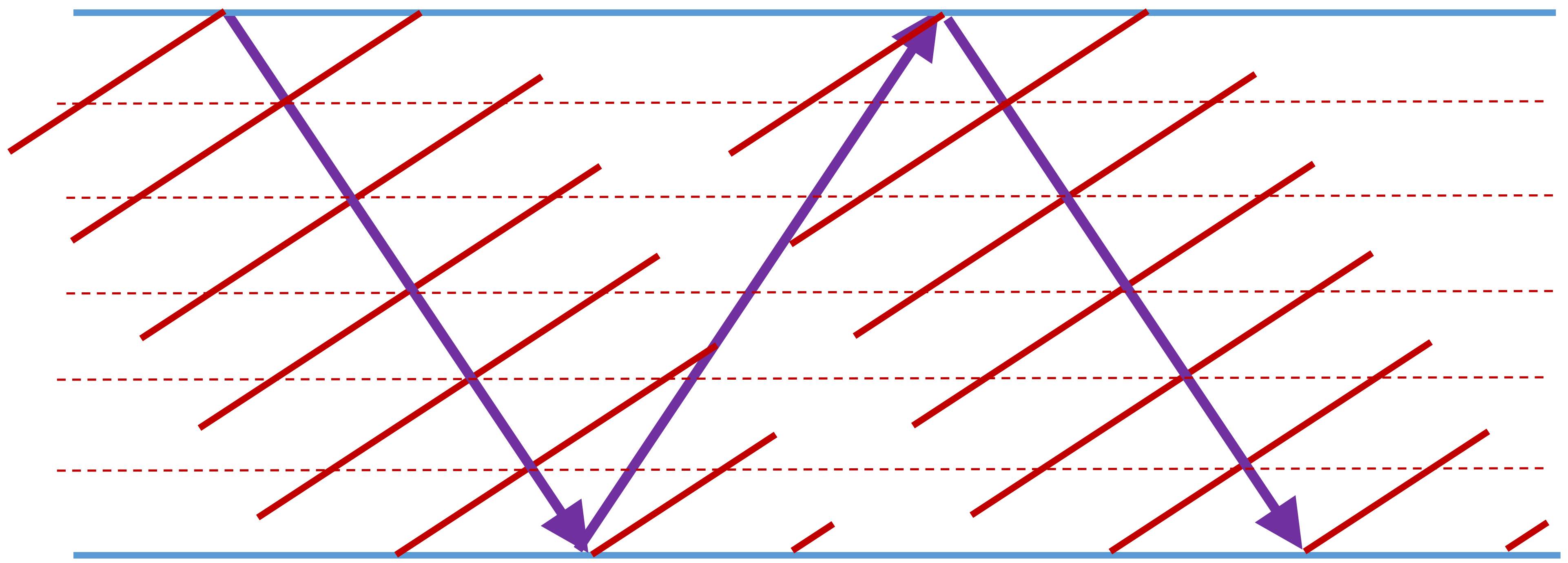


- Require: Constructive interference

Discrete set of angles

Normal direction: Standing waves

Along interface: Propagation



(SCALAR) WAVES: EIGENMODES FOR PERFECT MIRRORS

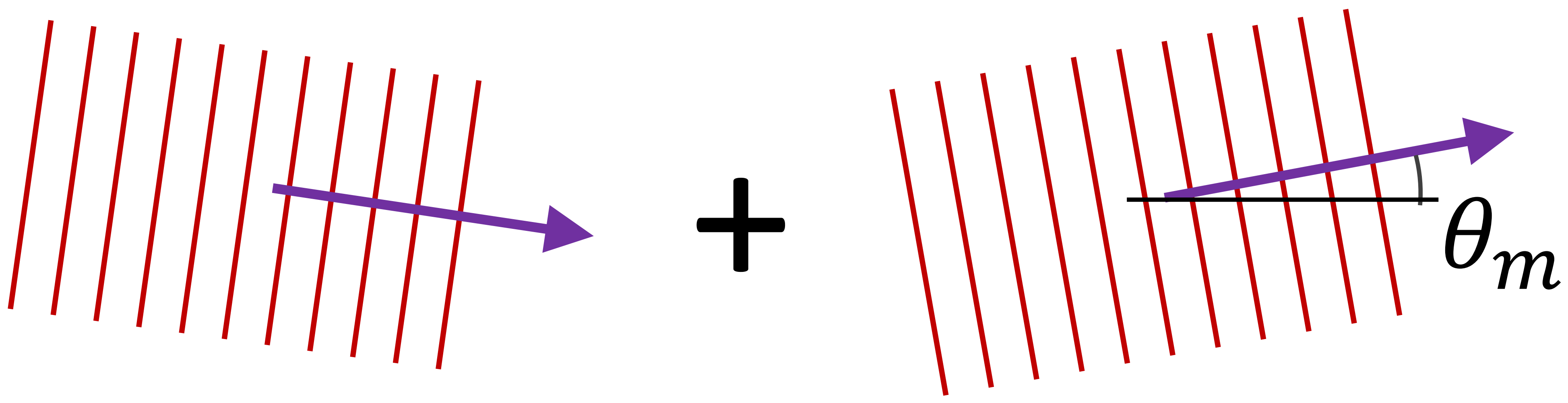
- Require: Constructive interference

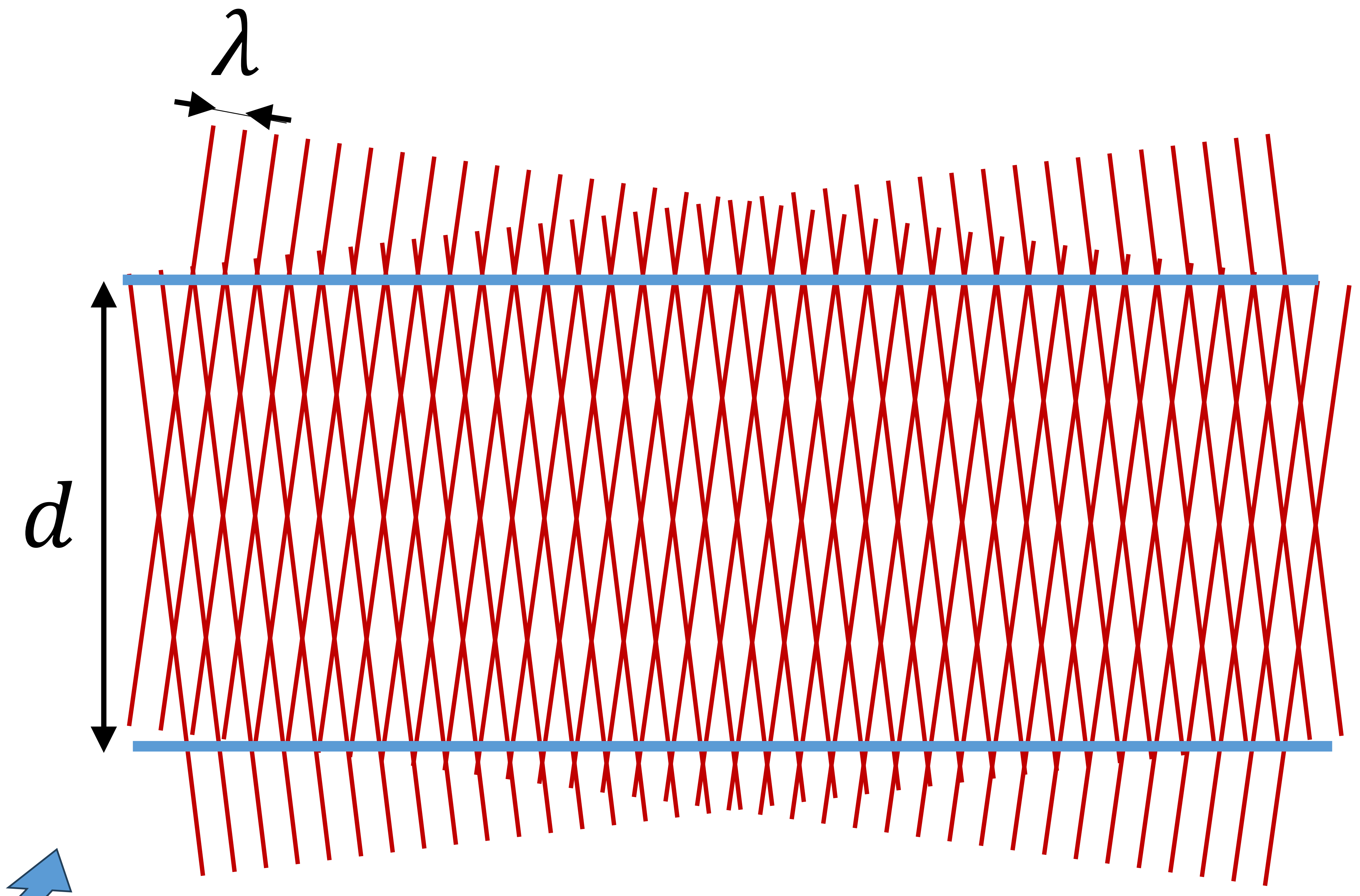
  Discrete set of angles

- Condition for angles of mode m

$$\sin\theta_m = m\,\frac{\lambda}{2d}, \qquad m = 1, 2, \dots$$

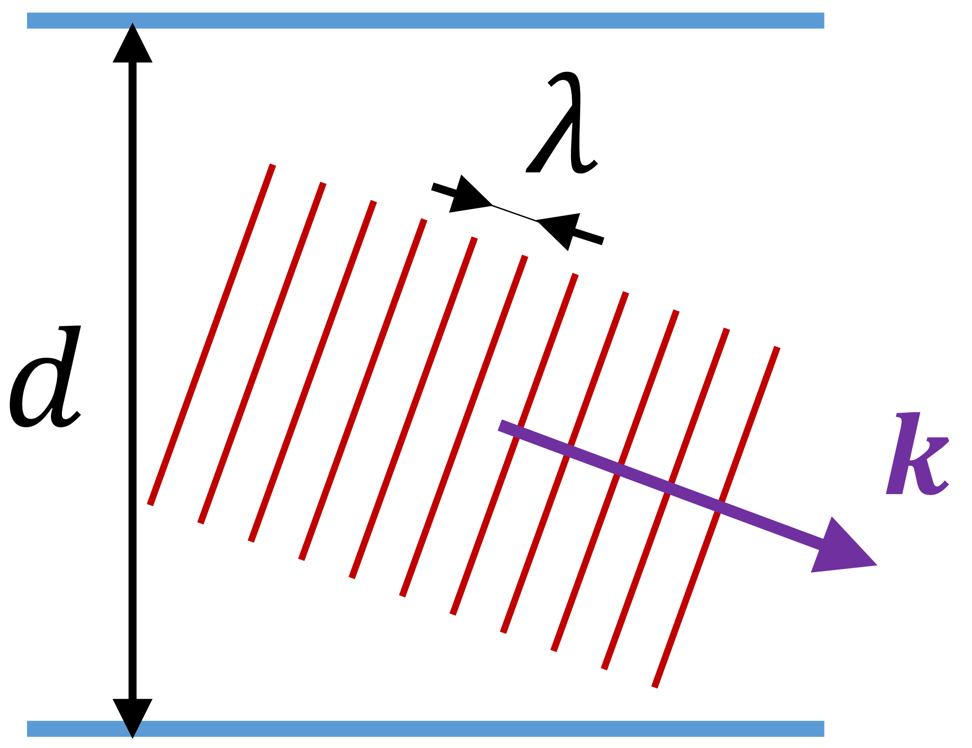
- Linear combination of two plane waves
- Visualization by Moire patterns independent of z





(SCALAR) WAVES: EIGENMODES FOR PERFECT MIRRORS

$$\sin\theta_m = m\,\frac{\lambda}{2d}, \qquad m = 1, 2, \ldots$$



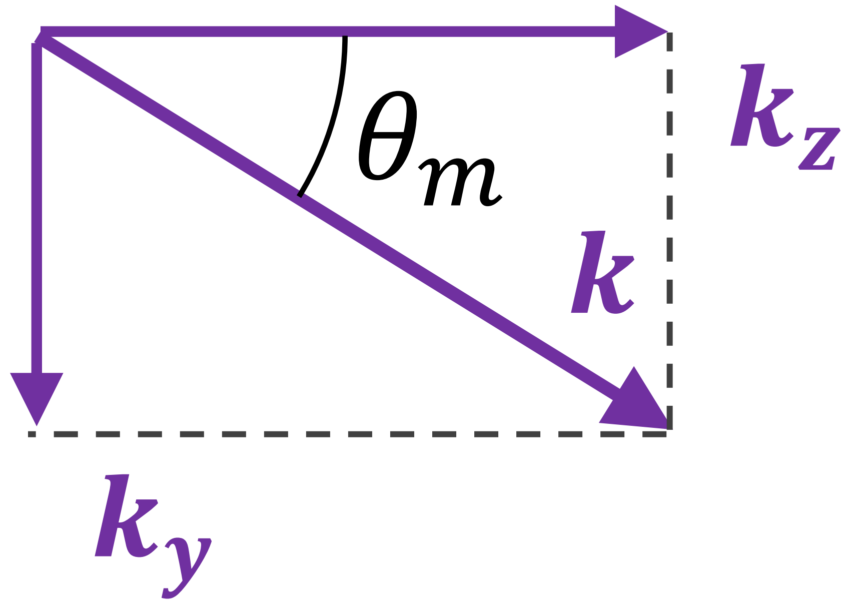
- Number of modes: $m_{\max} < \frac{2d}{\lambda}$

➡ Single mode $2\mathrm{d} \gtrsim \lambda$

- Wave vector

$$\beta_m^2 \equiv k_z^2 = k^2 - m^2\,\frac{\pi^2}{d^2}$$

$$k_y = m\,\frac{\pi}{d}$$



- Higher mode ➡ smaller β_m
  larger k_y

(SCALAR) WAVES: EIGENMODES FOR PERFECT MIRRORS

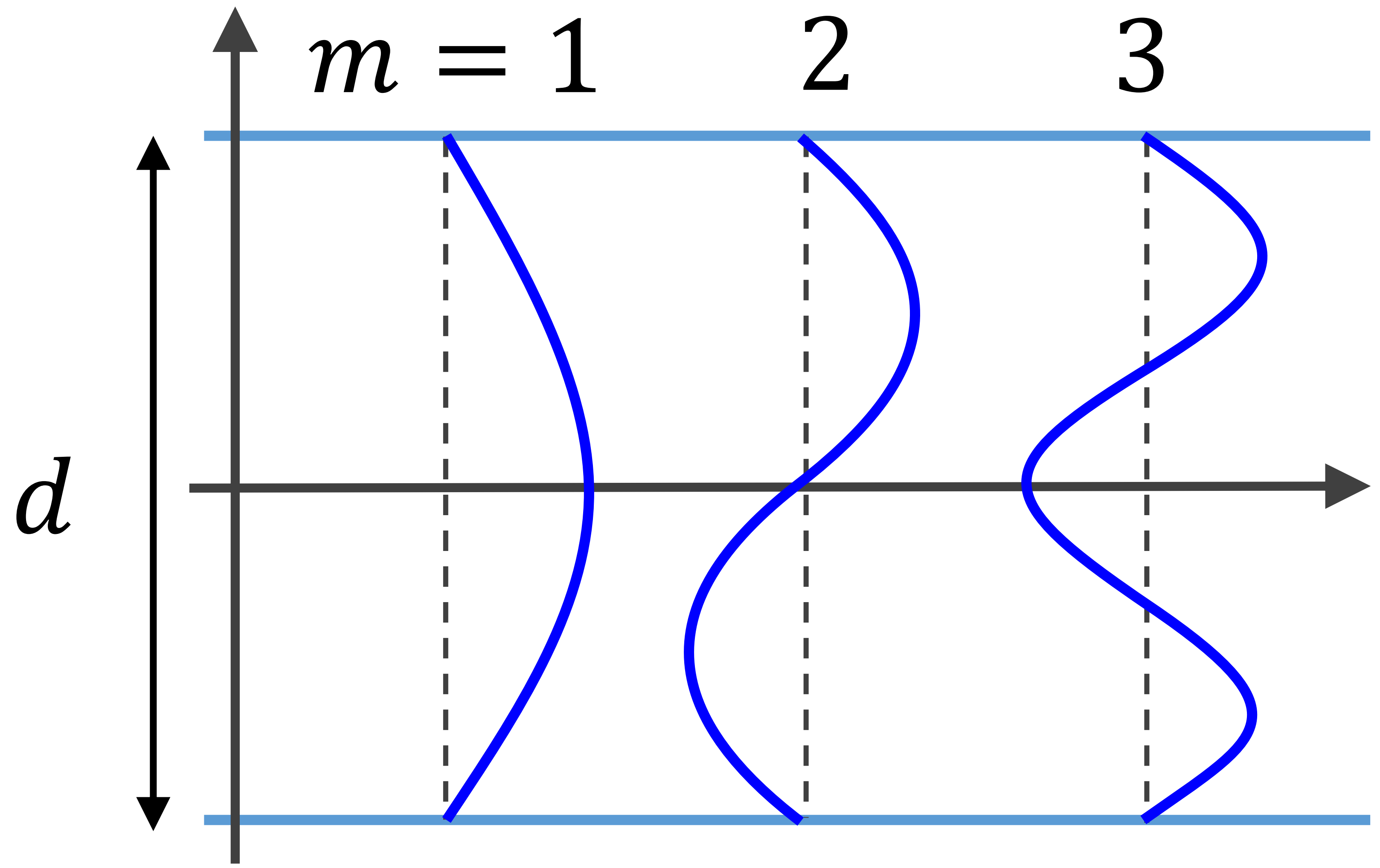
- Assume the walls are perfect mirrors

- Solutions are standing waves

symmetric

anti-symmetric

$$\begin{cases} \sqrt{2/d}\ \cos\left(\frac{m\pi y}{d}\right)\ e^{-i\beta_m z},\ \ m = 1, 3, \ldots \\ \\ \sqrt{2/d}\ \sin\left(\frac{m\pi y}{d}\right)\ e^{-i\beta_m z},\ \ m = 2, 4, \ldots \end{cases}$$



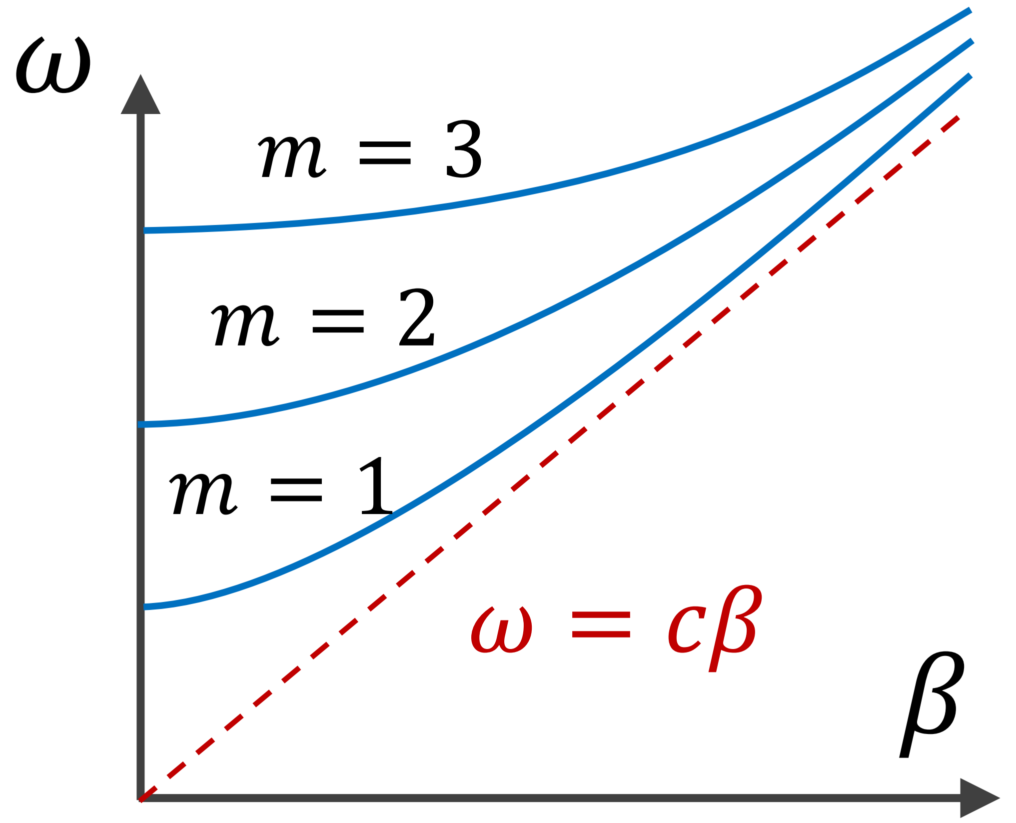
(SCALAR) WAVES: EIGENMODES FOR PERFECT MIRRORS

- (Minimal) Cut-off wavelength $\lambda_c = 2d$
- Cut-off angular frequency $\omega_c = \frac{2\pi c}{\lambda_c} = \frac{\pi c}{d}$
- Dispersion relation

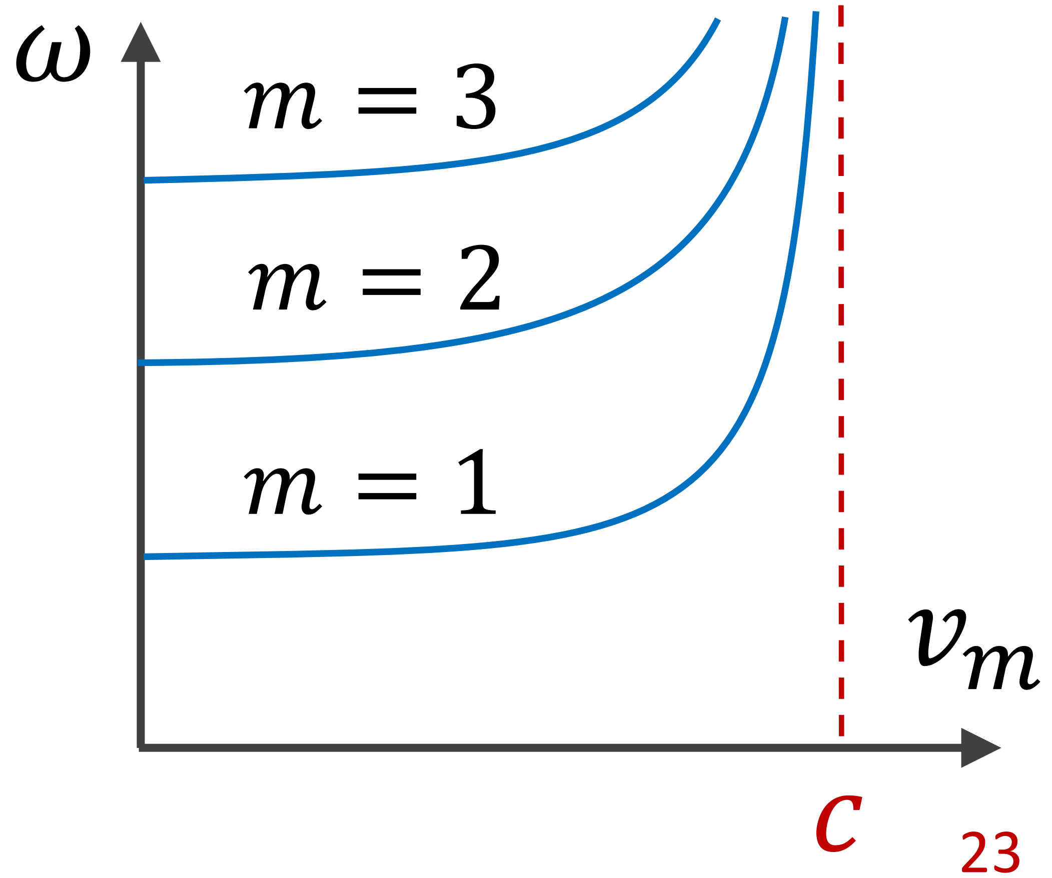
$$\beta_m = \frac{\omega}{c}\sqrt{1 - m^2\frac{\omega_c^2}{\omega^2}} \Leftarrow \begin{cases} \beta_m^2 = k^2 - m^2\frac{\pi^2}{d^2}, \\ \mathrm{k} = \frac{\omega}{c} \end{cases}$$

- Group velocity $v_m = c\sqrt{1 - m^2\frac{\omega_c^2}{\omega^2}}$

Dispersion relation



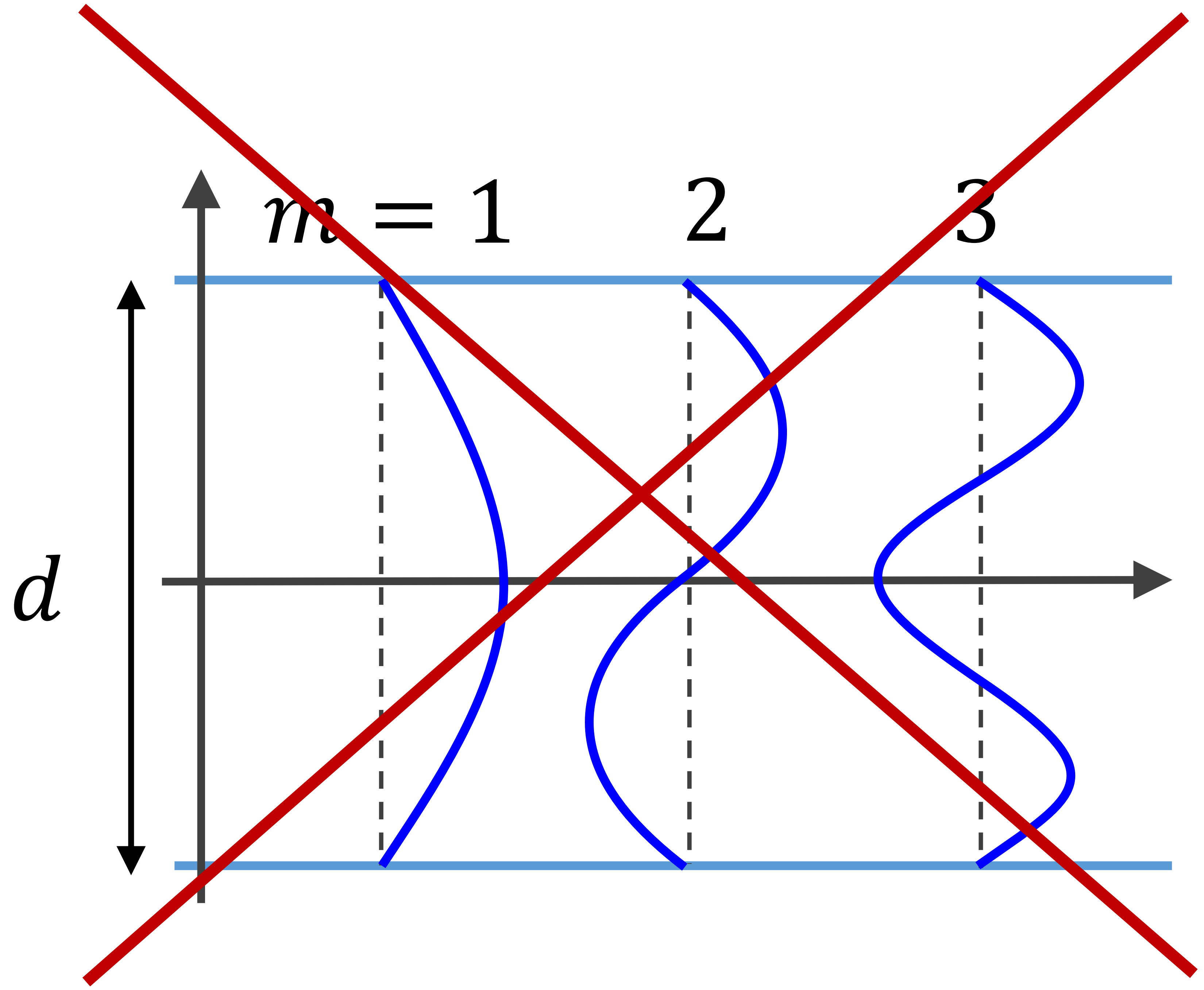
Group velocity



SCALAR WAVES VERSUS ELECTROMAGNETIC WAVES

- Two problems:
  - Walls are not always perfect mirrors
  - Scalar wave ignore electric and magnetic interaction but polarization matter for interaction with walls

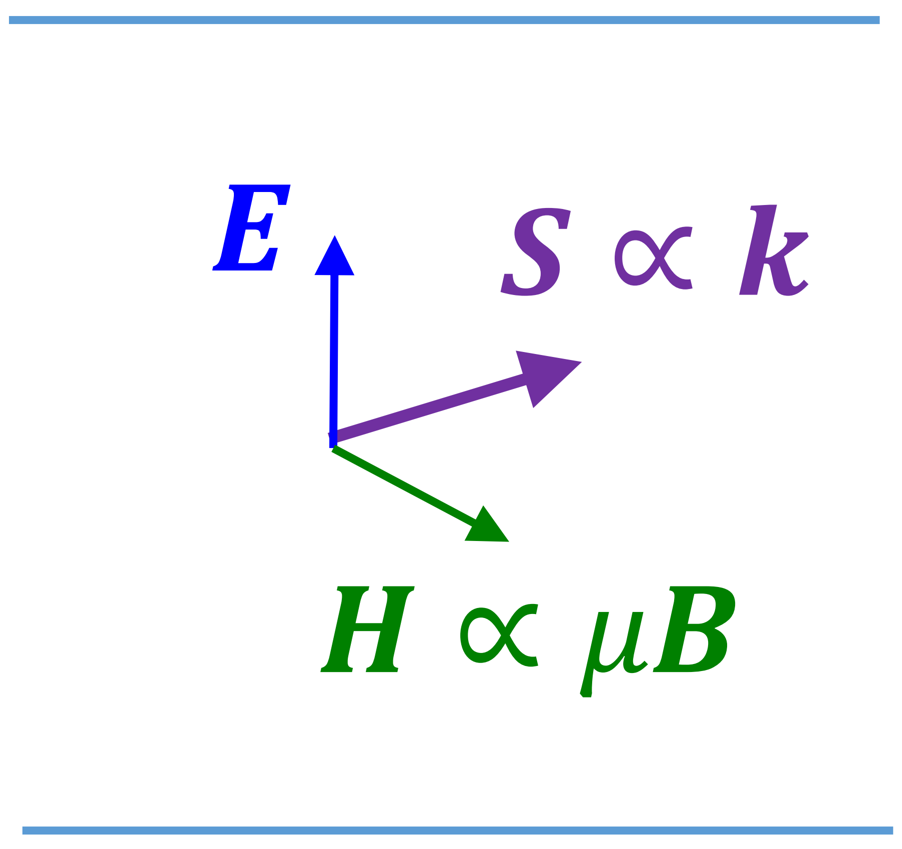
- We have to consider:
  - Difference TE and TM polarization
  - Partial penetration of wall
  - Eigenmodes will slightly change

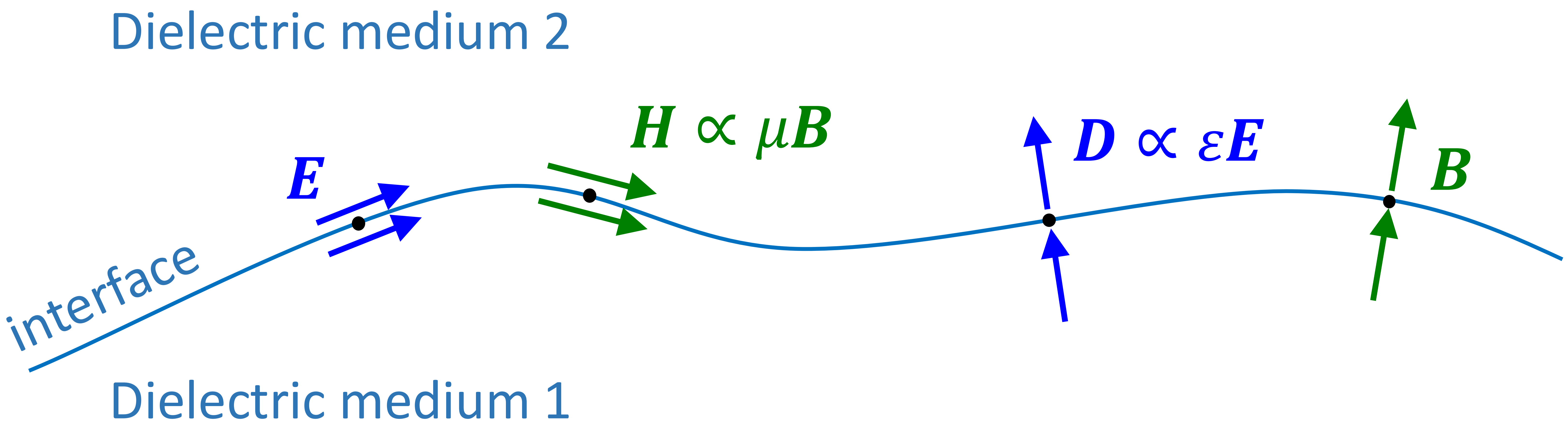


BUT: Most previous formulas are approximately correct

ELECTROMAGNETIC WAVES: POLARIZATION & INTERFACES

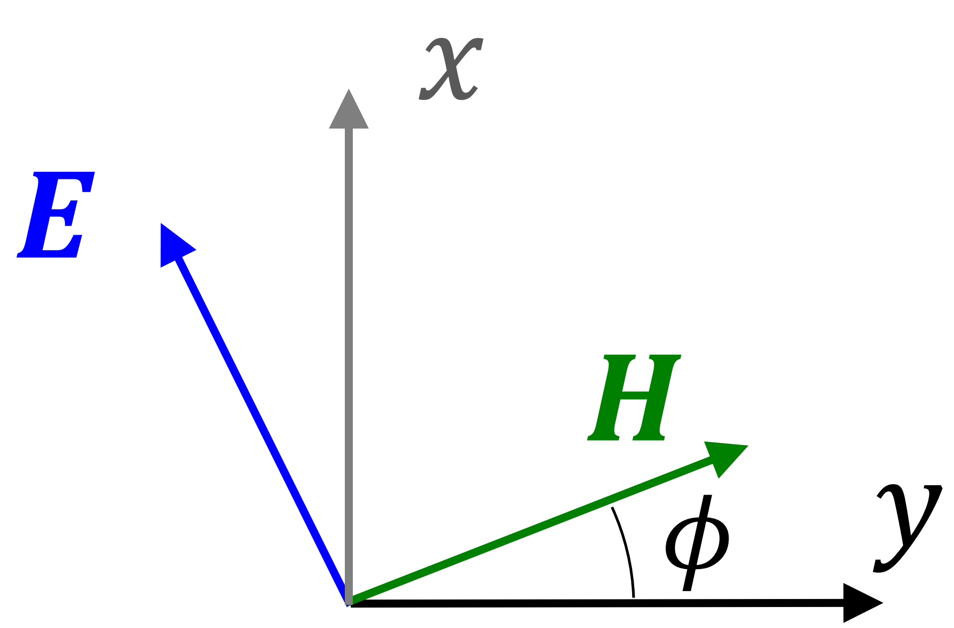
- Polarization matters for interfaces of a waveguide
- In a **homogeneous medium**:
  - Tangential components: $\boldsymbol{E}$ and $\boldsymbol{H} \propto \mu\boldsymbol{B}$ continuous
  - Normal components: $\boldsymbol{B}$ and $\boldsymbol{D} \propto \varepsilon\boldsymbol{E}$ continuous
- If medium also isotropic then $\boldsymbol{H} \parallel \boldsymbol{B}$ and $\boldsymbol{D} \parallel \boldsymbol{E}$





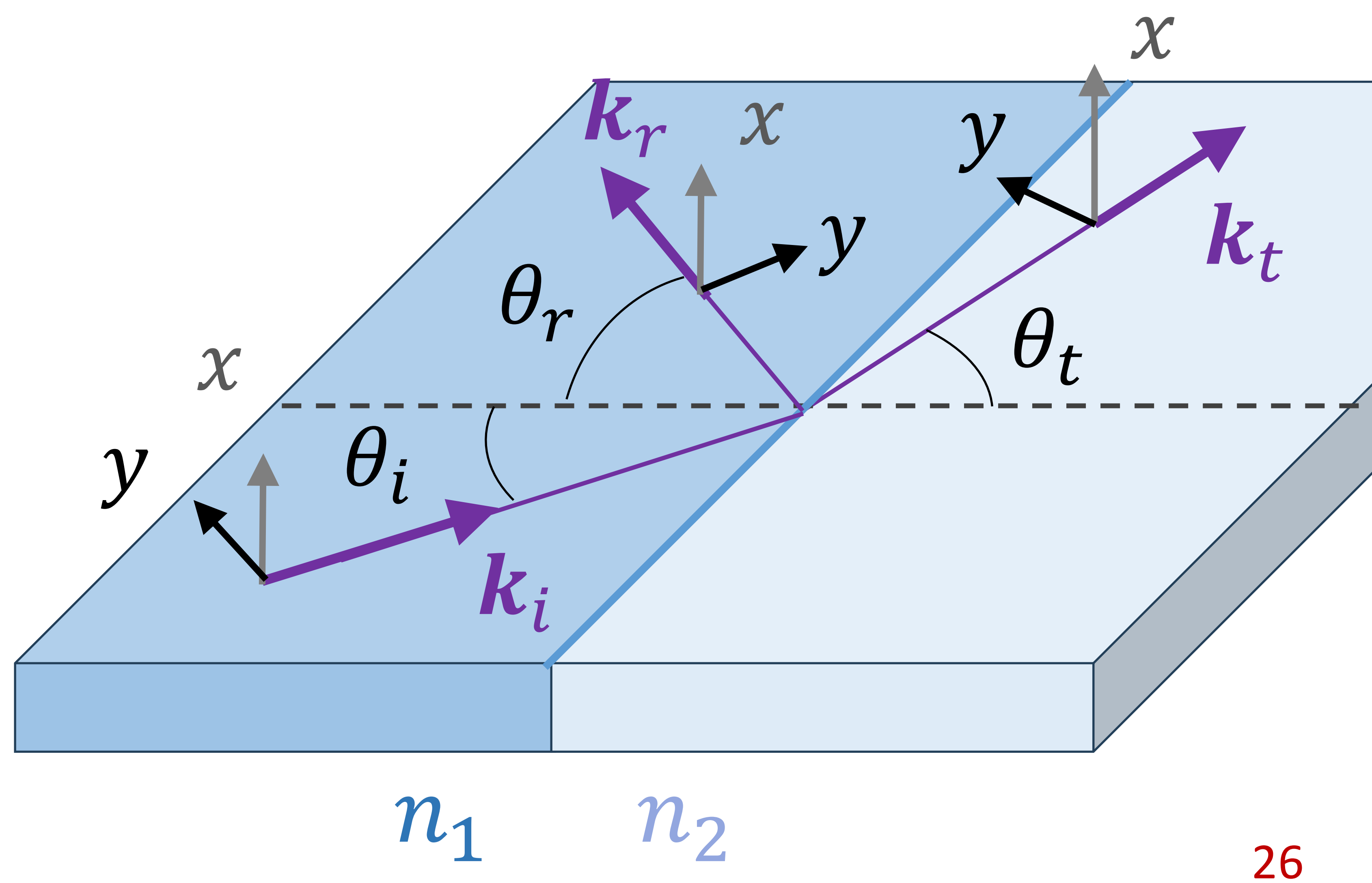
EXTENDING SNELL'S LAW: FRESNEL EQUATIONS

- Assume linear polarization
- Assume x-polarization and y-polarization
- Superposition ➡ any polarization angle ϕ
- Fresnel equations: Reflection and Transmission coefficients for x and y polarized part



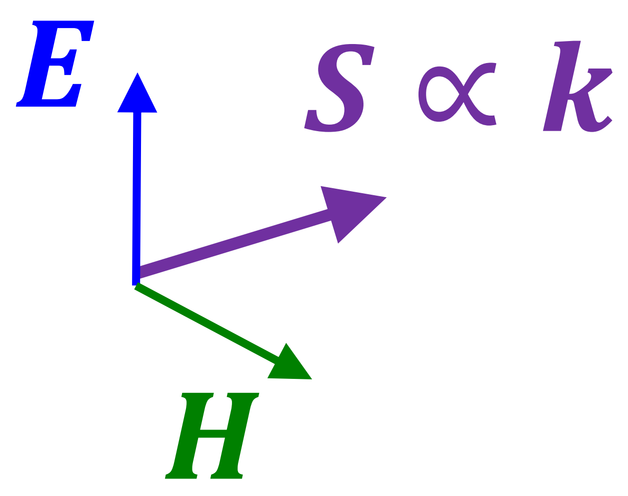
$$r_x = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}, \qquad t_x = 1 + r_x$$

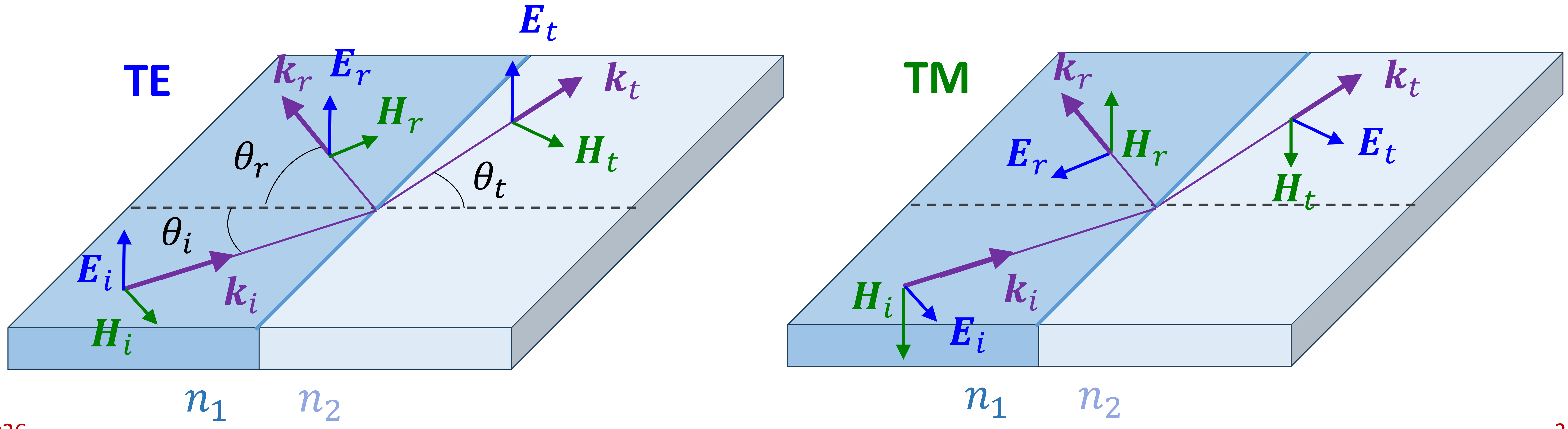
$$r_y = \frac{n_1 \sec\theta_i - n_2 \sec\theta_t}{n_1 \sec\theta_i + n_2 \sec\theta_t}, \qquad t_y = 1 + \frac{\cos\theta_i}{\cos\theta_t} r_y$$



POLARIZATION-DEPENDENCY: TE AND TM POLARIZATION

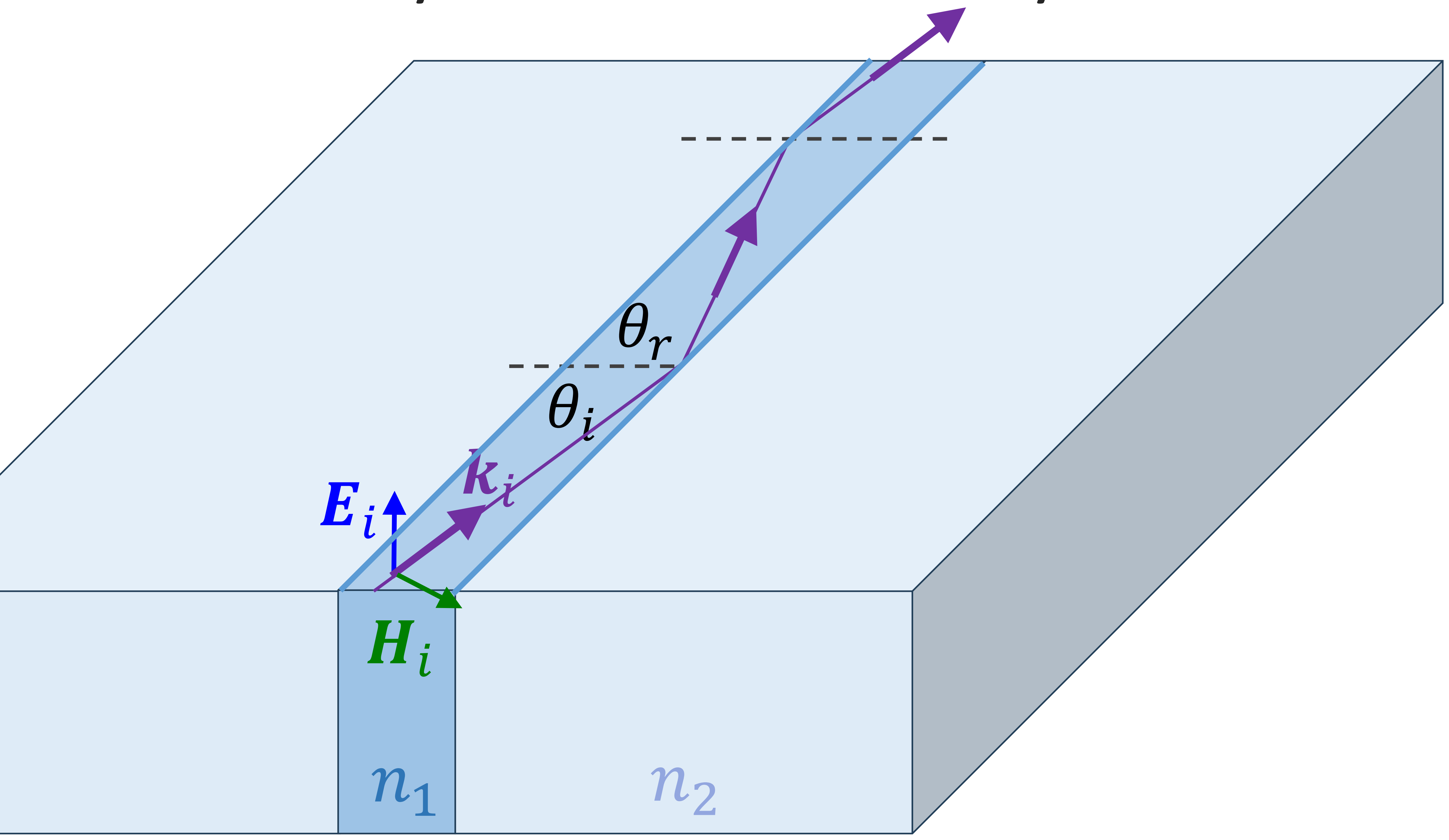
- (Linear) Polarization direction:
  - **TE**: Transverse Electric (S - polarization)
    E-field orthogonal to interface
  - **TM**: Transverse Magnetic (P - polarization.)
    H-field orthogonal to interface

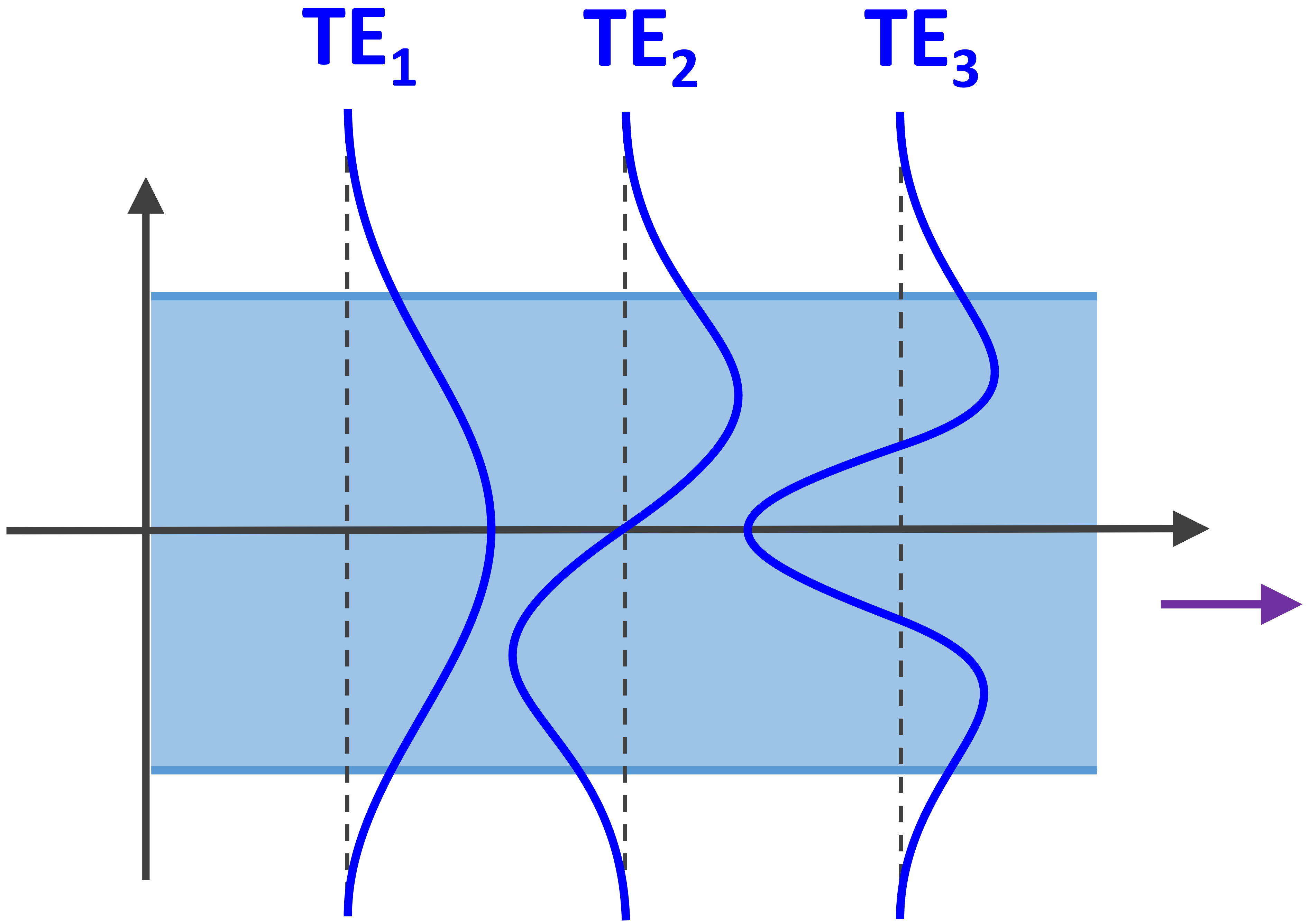




EIGENMODES: TE

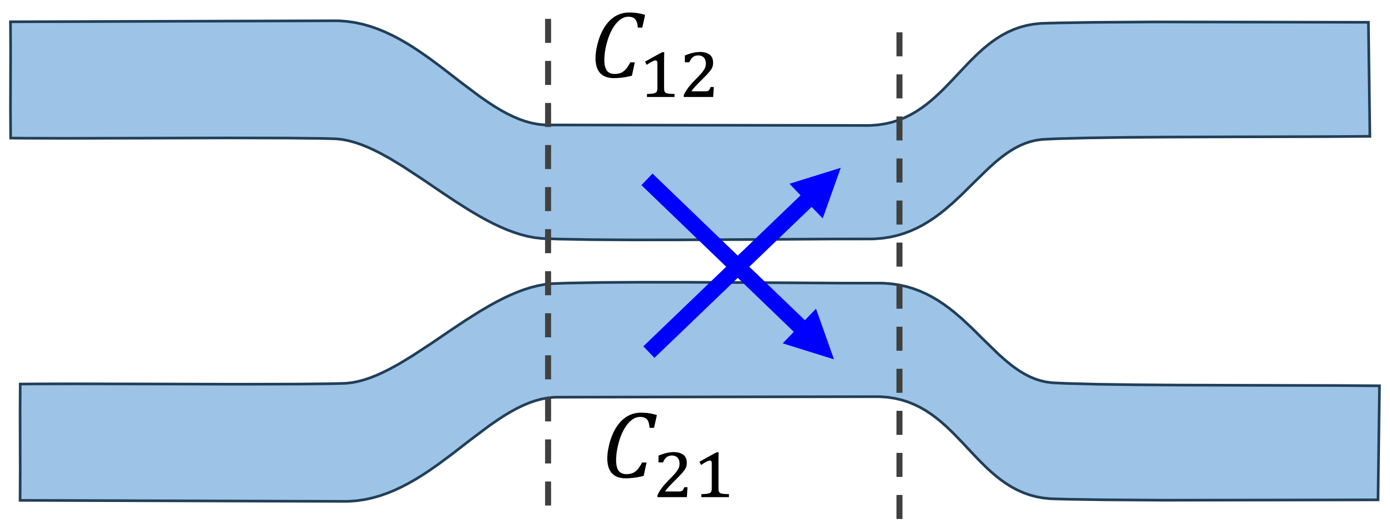
- Evanescent E and H-field outside waveguide
- Spatial fields similar to perfect mirror walls case
  - Multiple modes, slightly broadened because well penetration
  - Symmetric and anti-symmetric modes



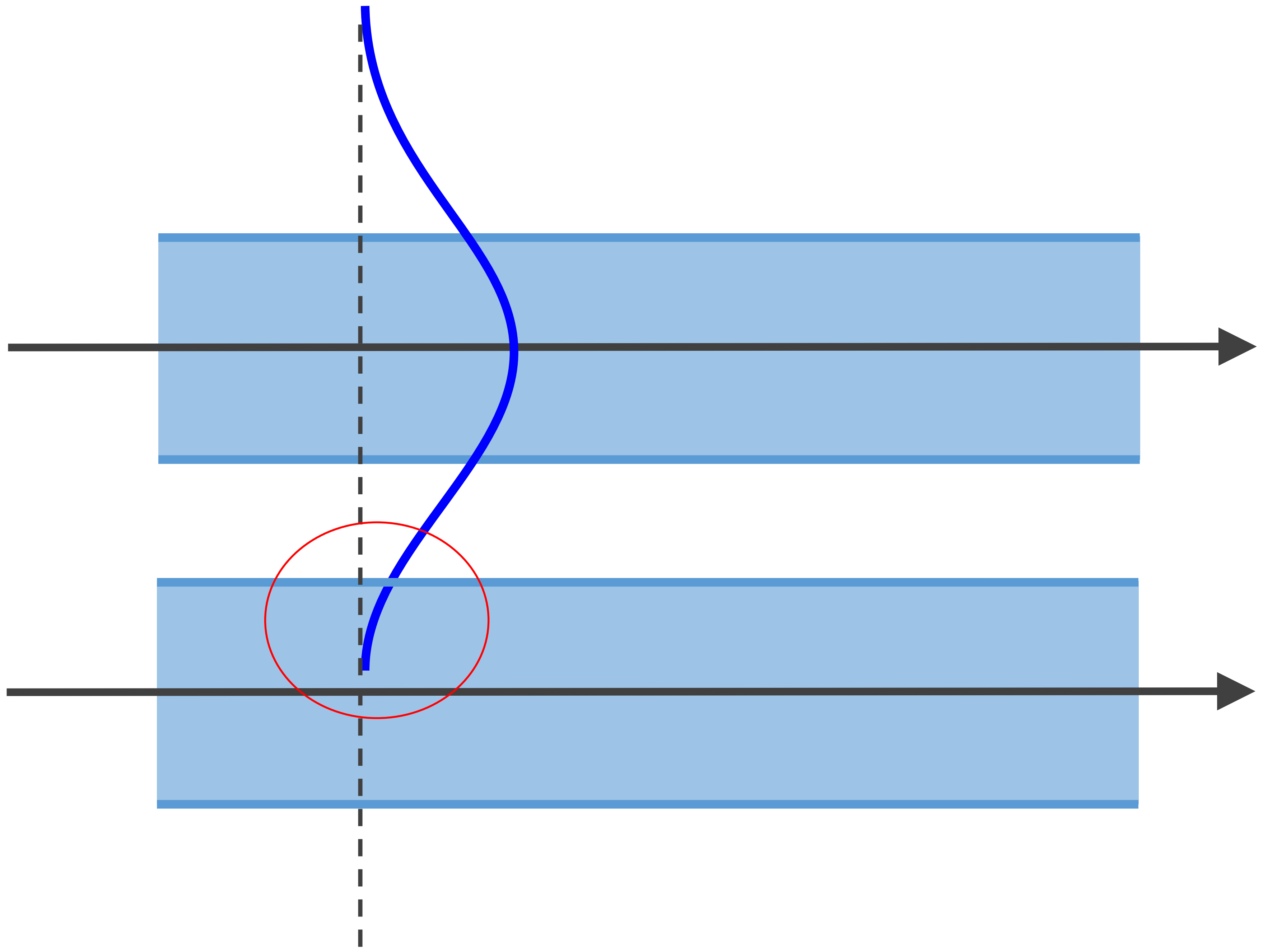


WAVEGUIDE MODE COUPLING

- Directional couplers:
  - Waveguides in close proximity
  - Over a specific coupling length L
  - Separation distance
  - Coupling constants C_{12} and C_{21}

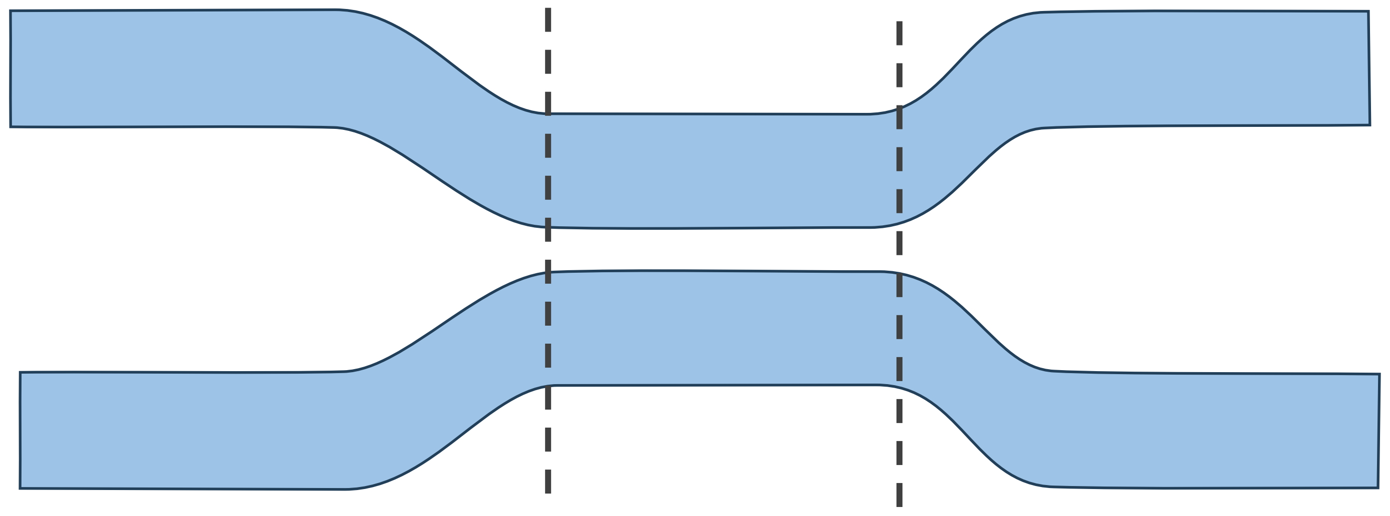


- Evanescent part of the eigenmodes of one waveguide overlaps with the other waveguide
- Optical power can be transferred



WAVEGUIDE MODE COUPLING

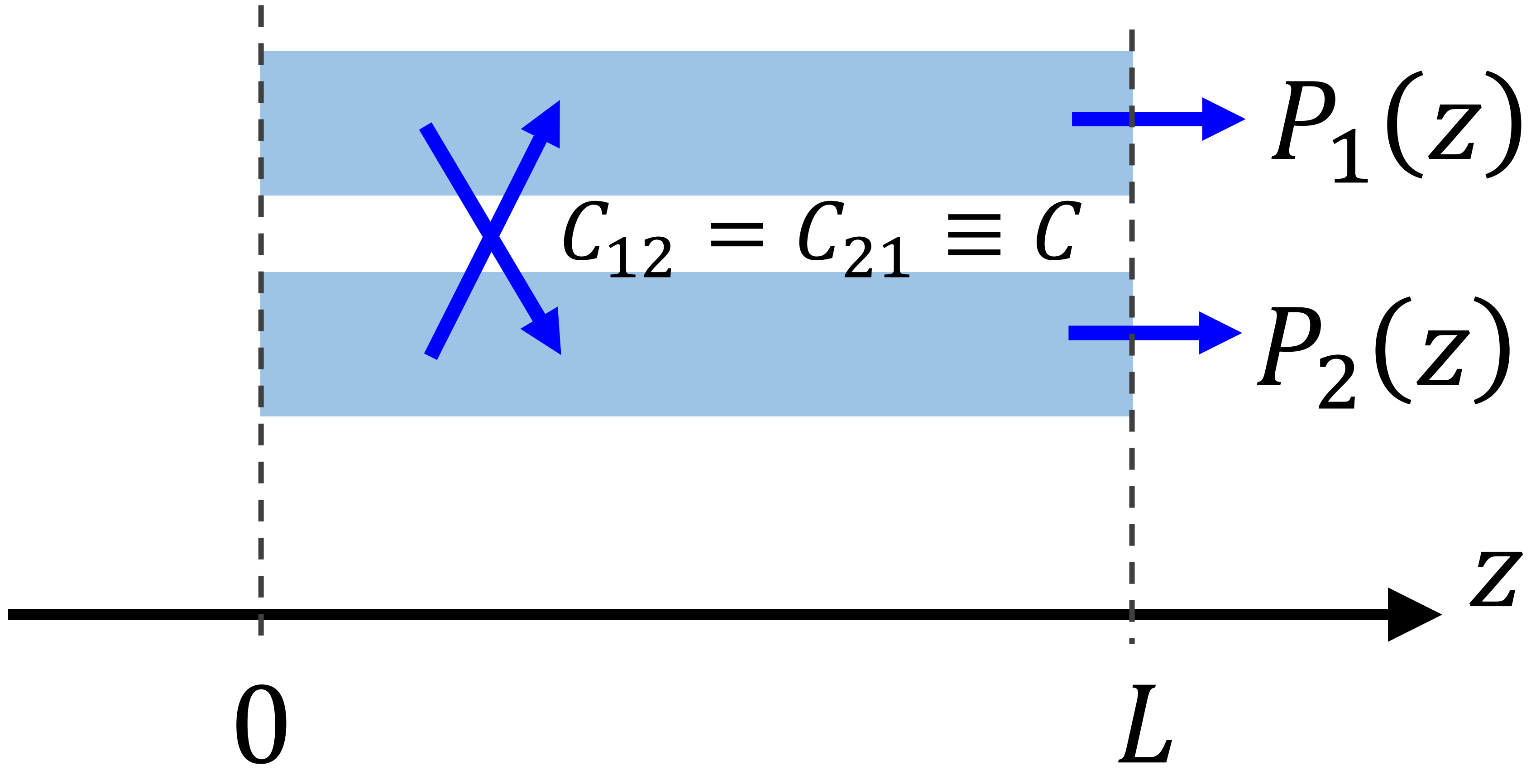
- Transfer of optical power for 2 identical waveguides
  - Oscillates between 0 and 1, according to the coupling length
  - 3dB coupler will split power 50/50



$$P_1(z) = P_1(0)\cos^2(Cz)$$
$$P_2(z) = P_1(0)\sin^2(Cz)$$

With C is the coupling coefficient

- Full power transfer: $L = \pi/2C$
- Split power 50/50: $L = \pi/4C$



DIELECTRIC (GLASS) FIBERS

- Similar concept as waveguides but with cylindrical symmetry
  - Larger core diameter: higher eigenmodes allowed
  - Larger ratios of core-cladding refractive indices n_1/n_2 give stronger localized waves
  - Directional couplers also possible: Evanescent part of the eigenmodes of one fiber can overlap with another fiber if fibers fused together
- Fibers less appropriate for photonic circuits but better for long distances

Fiber with cladding