PHOT 451: Microscale optical system design
LECTURE 08

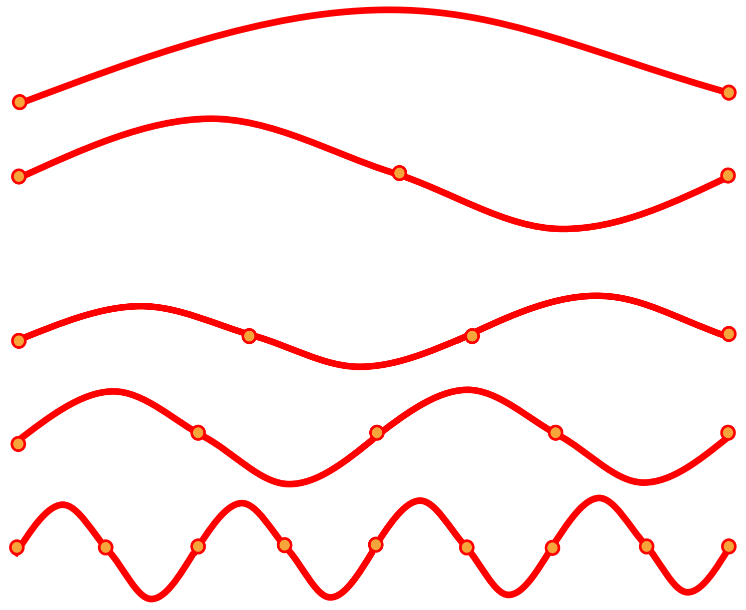
Michaël Barbier, Fall semester (2025-2026)

OVERVIEW OF THE COURSE

week	Topic
Week 1	Introduction to micro-scale optical components
Week 2	Light propagation in free space
Week 3	Geometric optics and raycasting
Week 4	Diffraction limit & Abberations
Week 5	**Quiz** + Beam propagation
Week 6	Refractive optical elements Microlenses
Week 7	Blazed Fresnel lenses
Week 8	Digital lenses
Week 9	Diffractive optical elements
Week 10	**Quiz** + Wave guides and beam propagation
Week 11	Wave mixing
Week 12	Gratings, periodic structures
Week 13	photonic crystals
Week 14	Whole optical system optimization

Light as information

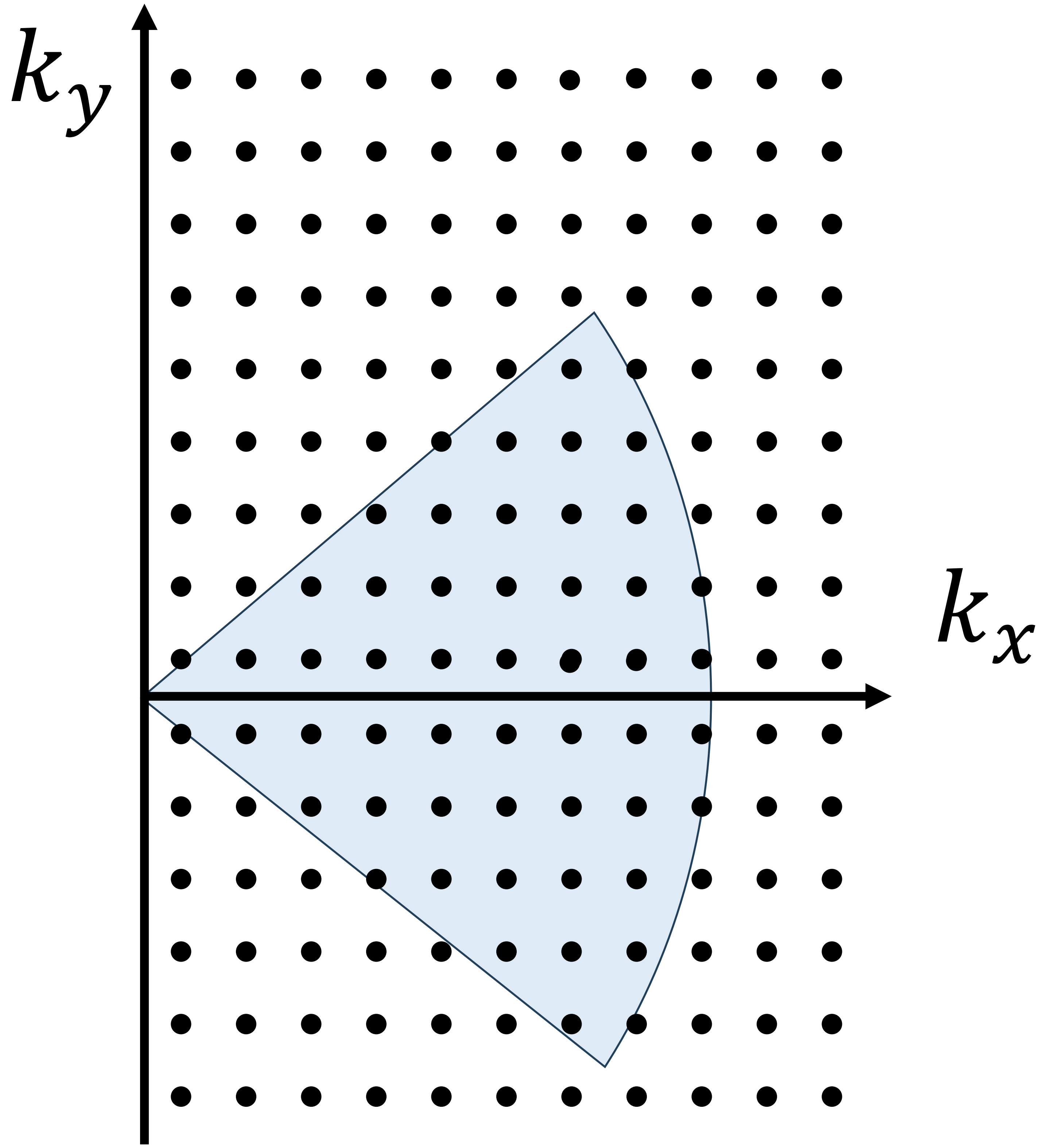
LIGHT AS INFORMATION CARRIER



- Frequency range:
  - Frequency range carries more information
  - Resolution of imaged objects

$$\sigma_{k_x}^2 = \frac{\int_{-\infty}^{+\infty} (k_x - k_0)^2 \, |\tilde{u}(k_x)|^2 \, dk_x}{\int_{-\infty}^{+\infty} |\tilde{u}(k_x)|^2 \, dk_x}$$

$$k_0 = \frac{\int_{-\infty}^{+\infty} k_x |\tilde{u}(k_x)|^2 \, dk_x}{\int_{-\infty}^{+\infty} |\tilde{u}(k_x)|^2 \, dk_x}$$



LIGHT AS INFORMATION CARRIER

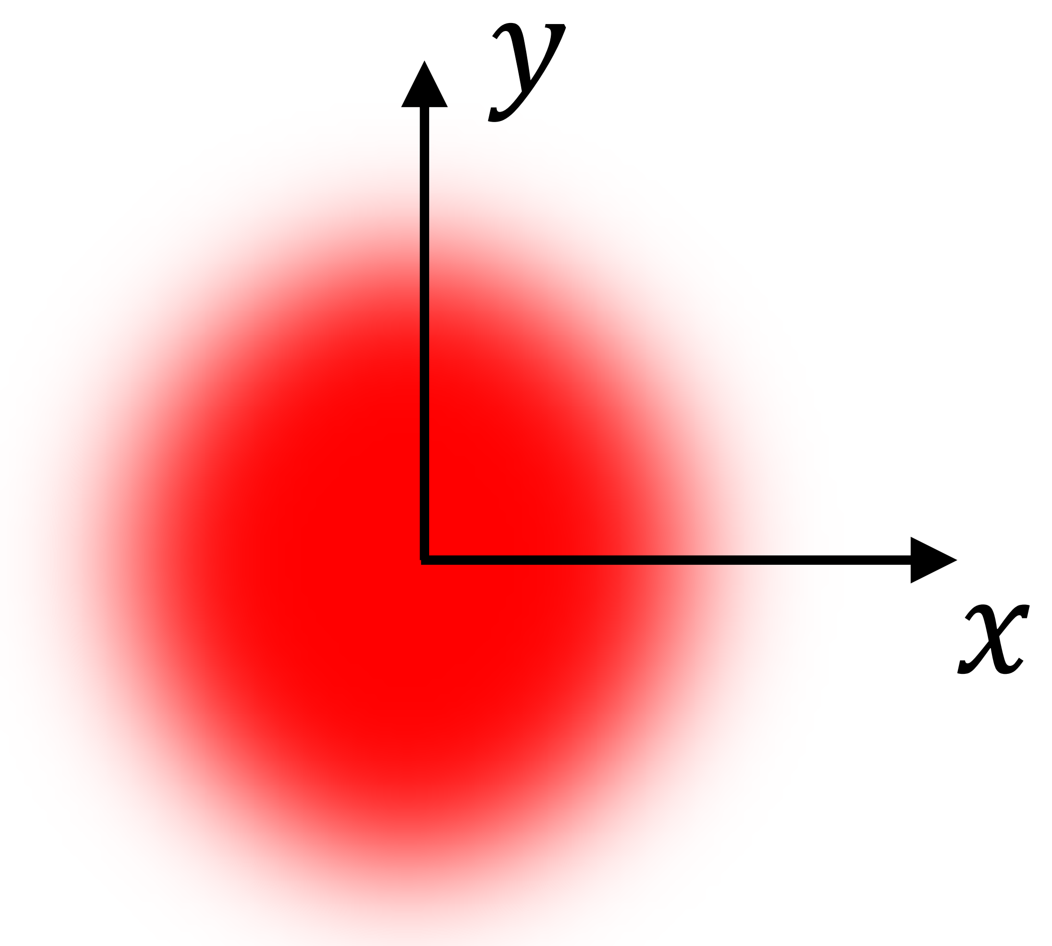
- Complex amplitude optical field $u(x)$

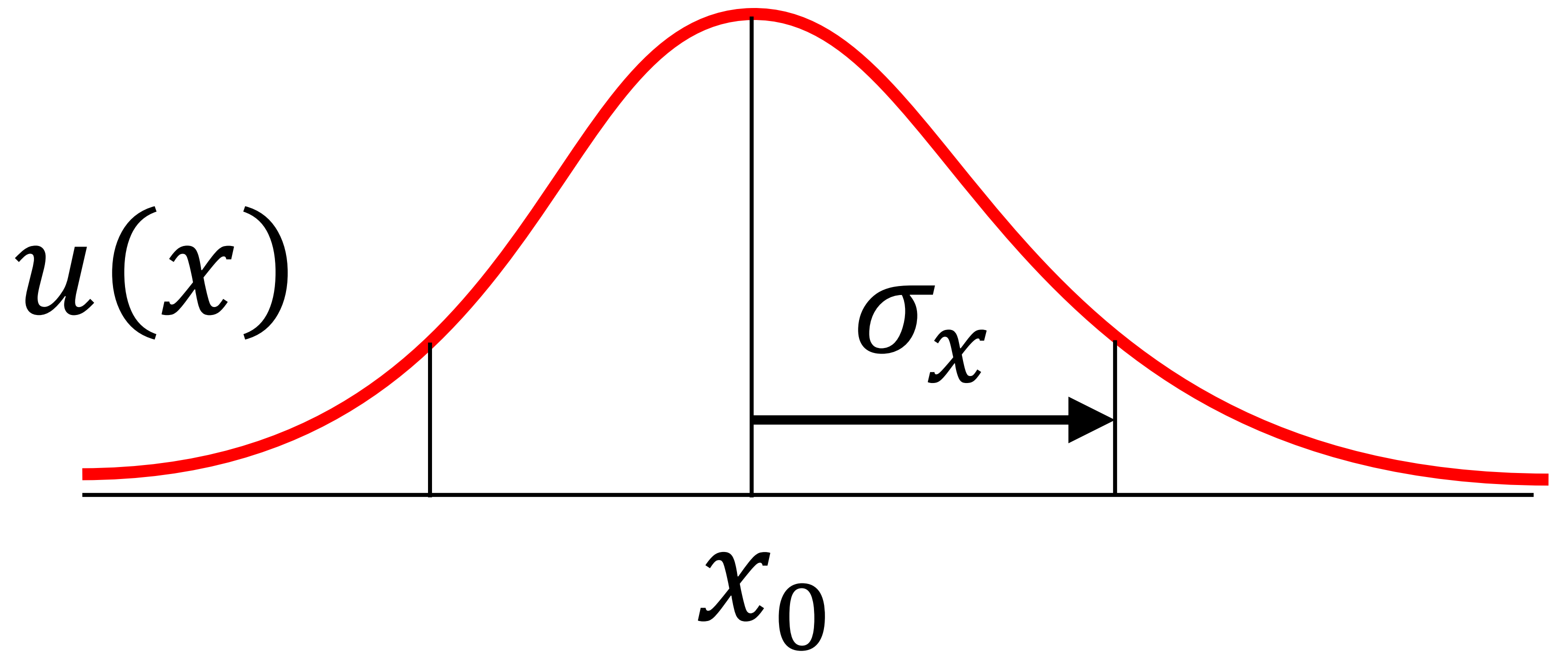
- Size of the object:

$$\sigma_x^2 = \frac{\int_{-\infty}^{+\infty} (x - x_0)^2 \, |u(x)|^2 \, dx}{\int_{-\infty}^{+\infty} |u(x)|^2 \, dx}$$

$$x_0 = \frac{\int_{-\infty}^{+\infty} x \, |u(x)|^2 \, dx}{\int_{-\infty}^{+\infty} |u(x)|^2 \, dx}$$

Example: Gaussian beam





LIGHT AS INFORMATION CARRIER

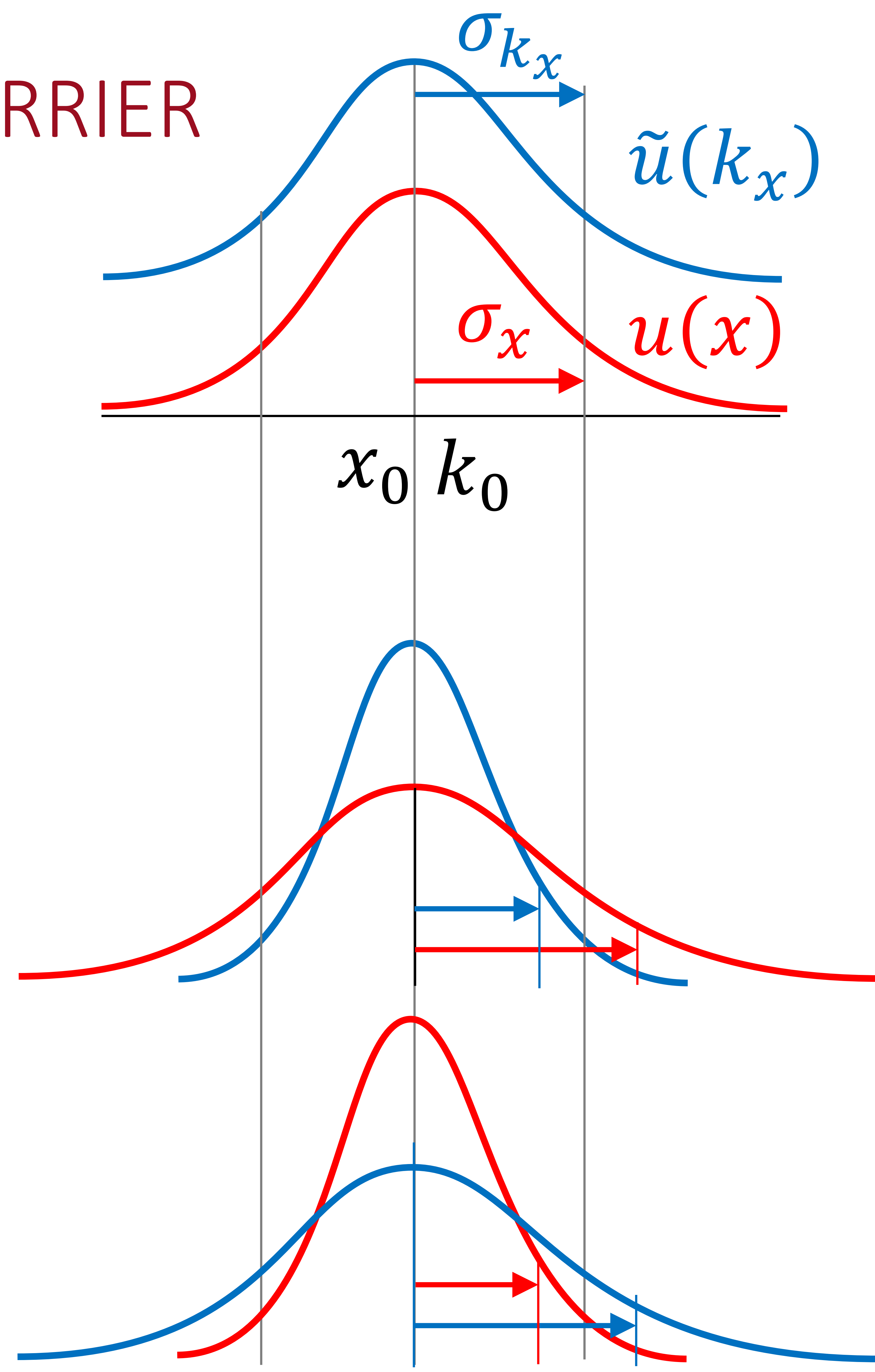
- Space Bandwidth Product (SBP)

$$\mathrm{SBP}_{1\mathrm{D}} = \sigma_x \, \sigma_{k_x}$$

- Minimum SBP for Gaussian $u(x) = e^{-\frac{x^2}{w^2}}$:

$$\mathrm{SBP}_{1\mathrm{D}} = \sigma_x \, \sigma_{k_x} \geq \frac{1}{2}$$

- Similar to Heisenberg uncertainty principle in quantum mechanics



LIGHT AS INFORMATION CARRIER

- Quality factor and SBP

$$M^2 = \mathrm{SBP}_{1\mathrm{D}} = \sigma_x \ \sigma_{k_x} \geq \frac{1}{2}$$

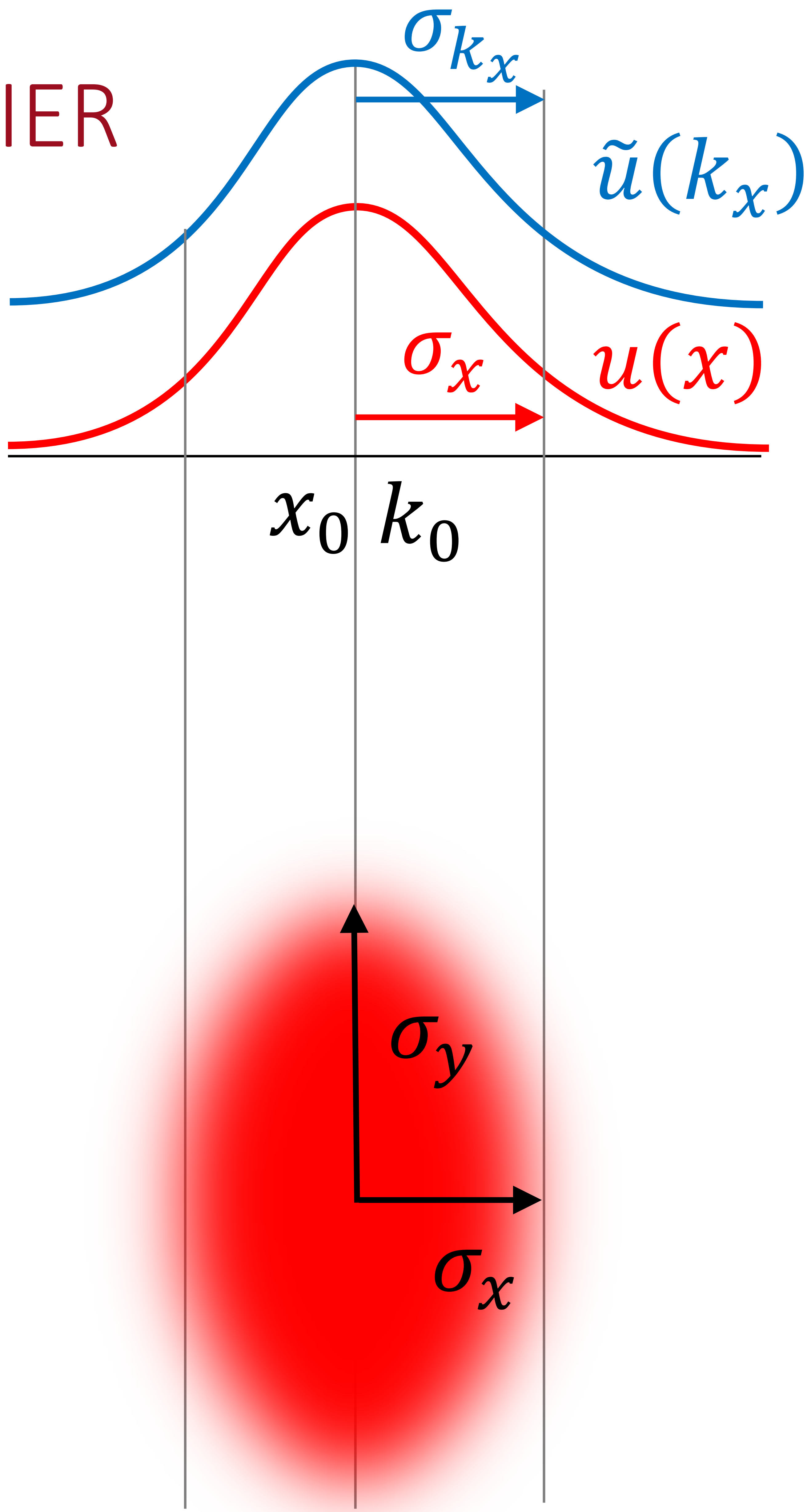
- In two dimensions:

$$\mathrm{SBP}_{2\mathrm{D}} = M^4 = \sigma_r^2 \sigma_k^2$$

- Quality **astigmatic** Gaussian beam

$$\mathrm{SBP}_{2\mathrm{D}} = M_x^2 M_y^2 = \sigma_x \sigma_{k_x} \cdot \sigma_y \sigma_{k_y}$$

$$= \frac{\left(w_x^2 + w_y^2\right)^2}{2 w_x w_y} \geq 1$$



Diffraction limit and resolution

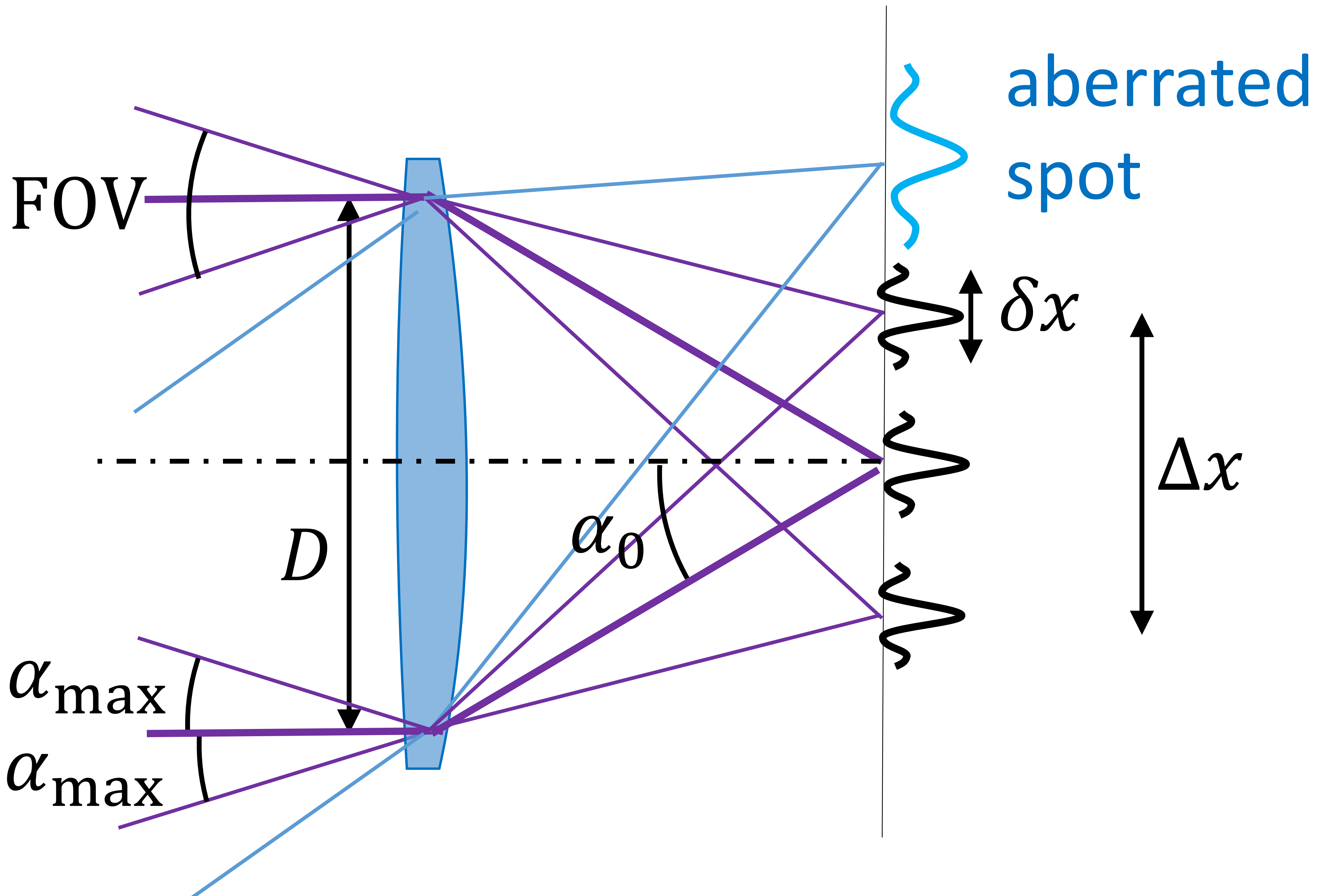
NUMERICAL APERTURE

- Numerical aperture NA

$$\mathrm{NA} = \frac{D}{2f}$$

- Small aberrations:

$$\mathrm{FOV} = 2\alpha_{\max}$$



NUMERICAL APERTURE

- Numerical aperture NA

$$\mathrm{NA} = \frac{D}{2f}$$

- Small aberrations:

$$\mathrm{FOV} = 2\alpha_{\max}$$

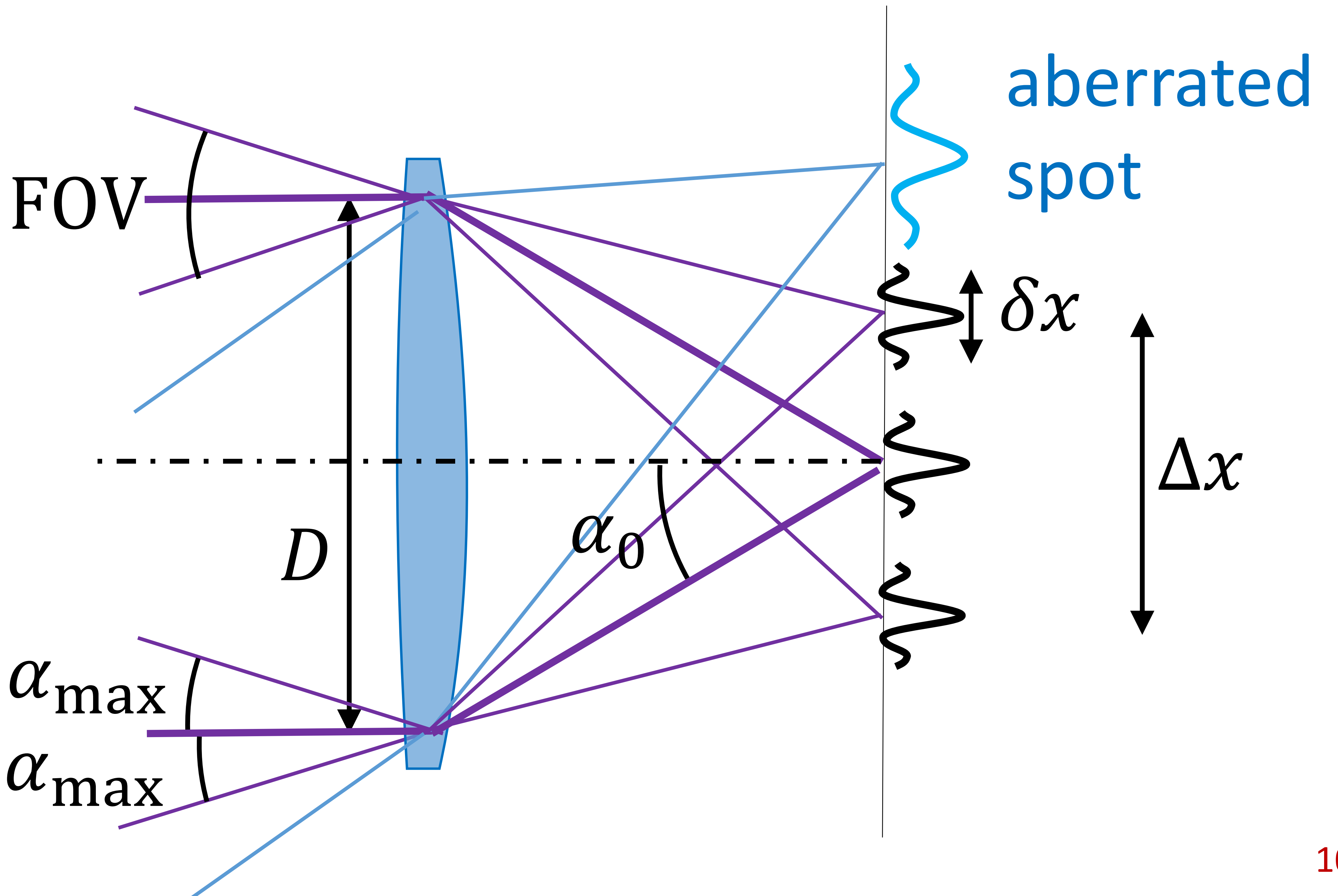
- Space bandwidth product

$$\mathrm{SBP}_{1\mathrm{D}} = \frac{\Delta x}{\delta x}$$

$$\approx 1.22\frac{f}{\lambda}\,\mathrm{FOV}\cdot\mathrm{NA}$$

$$\Delta x = 2f\,\tan(\alpha_{\max}) \approx f\cdot\mathrm{FOV}$$

$$\delta x \approx 2.44\frac{\lambda f}{D} = 1.22\frac{\lambda}{\mathrm{NA}} = 1.22\lambda\,\tan\alpha_0$$



Downscaling of an optical system

DOWNSCALING OPTICAL SYSTEMS

- Assume scaling parameter $\mathcal{M}$
- Option 1: **constant f/# scaling**
  - Focal length $f \rightarrow \mathcal{M}f$
    Lens diameter $D \rightarrow \mathcal{M}D$

Scale both lens diameter D and surface radii R_1 and R_2

$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

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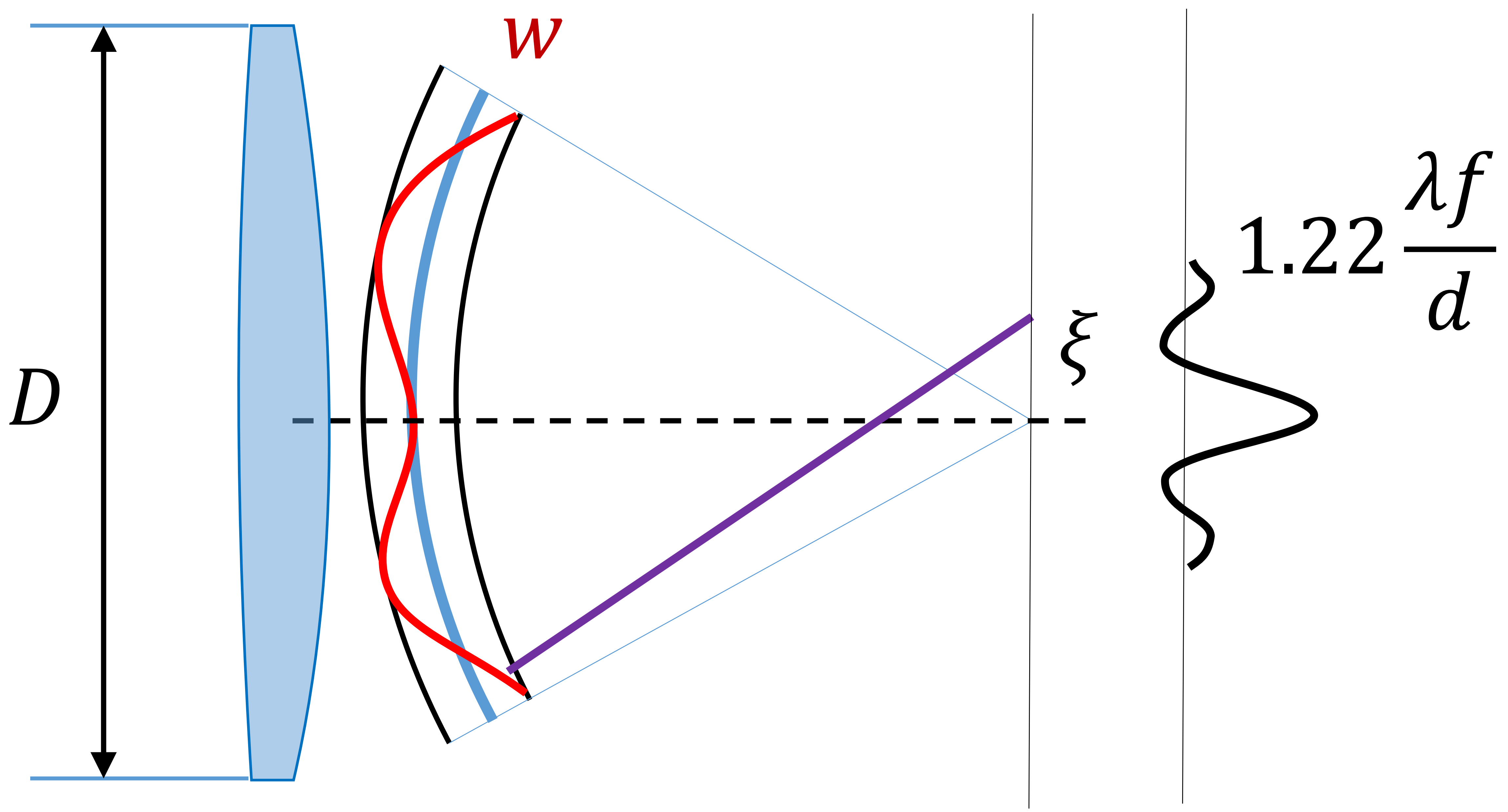
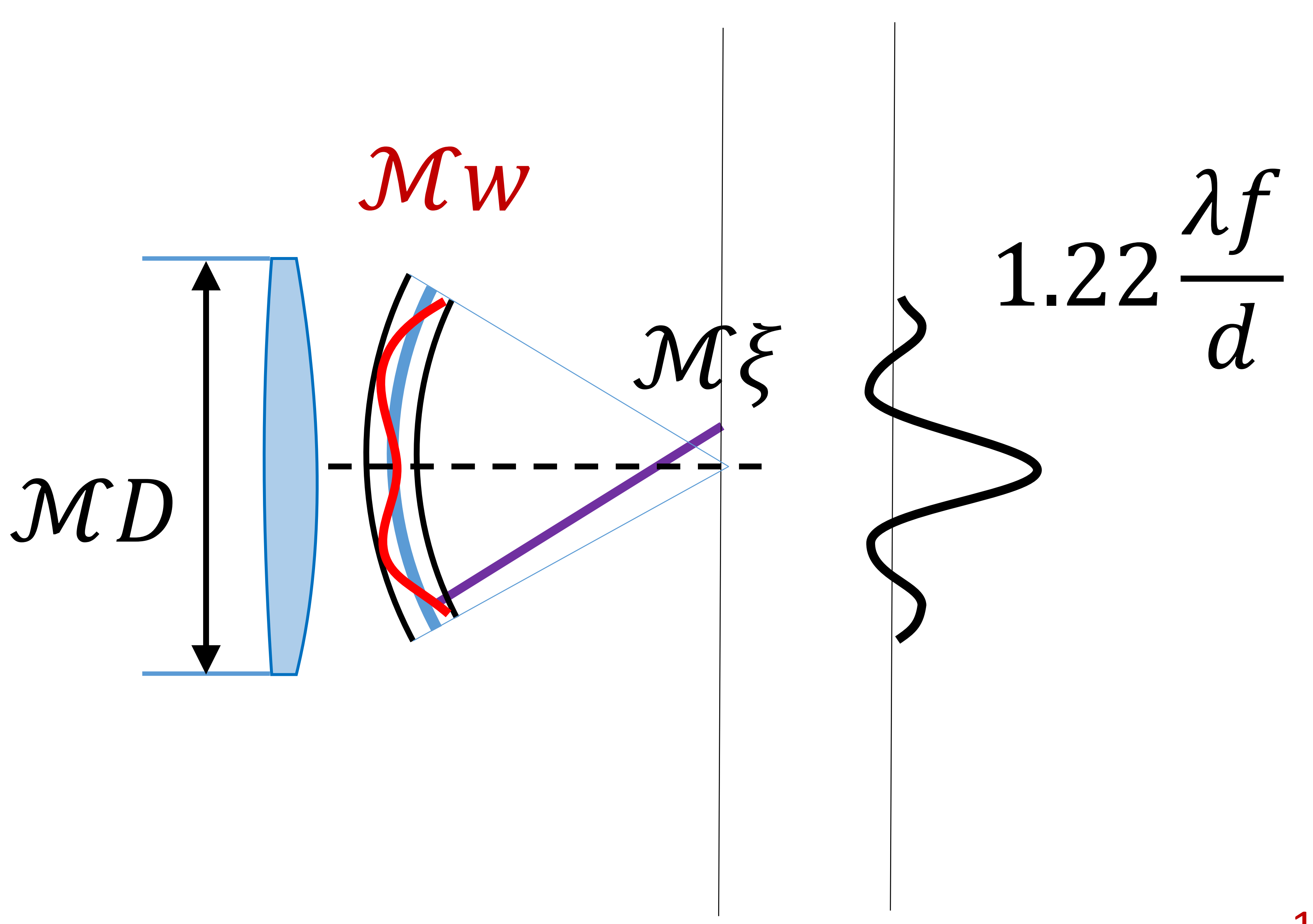




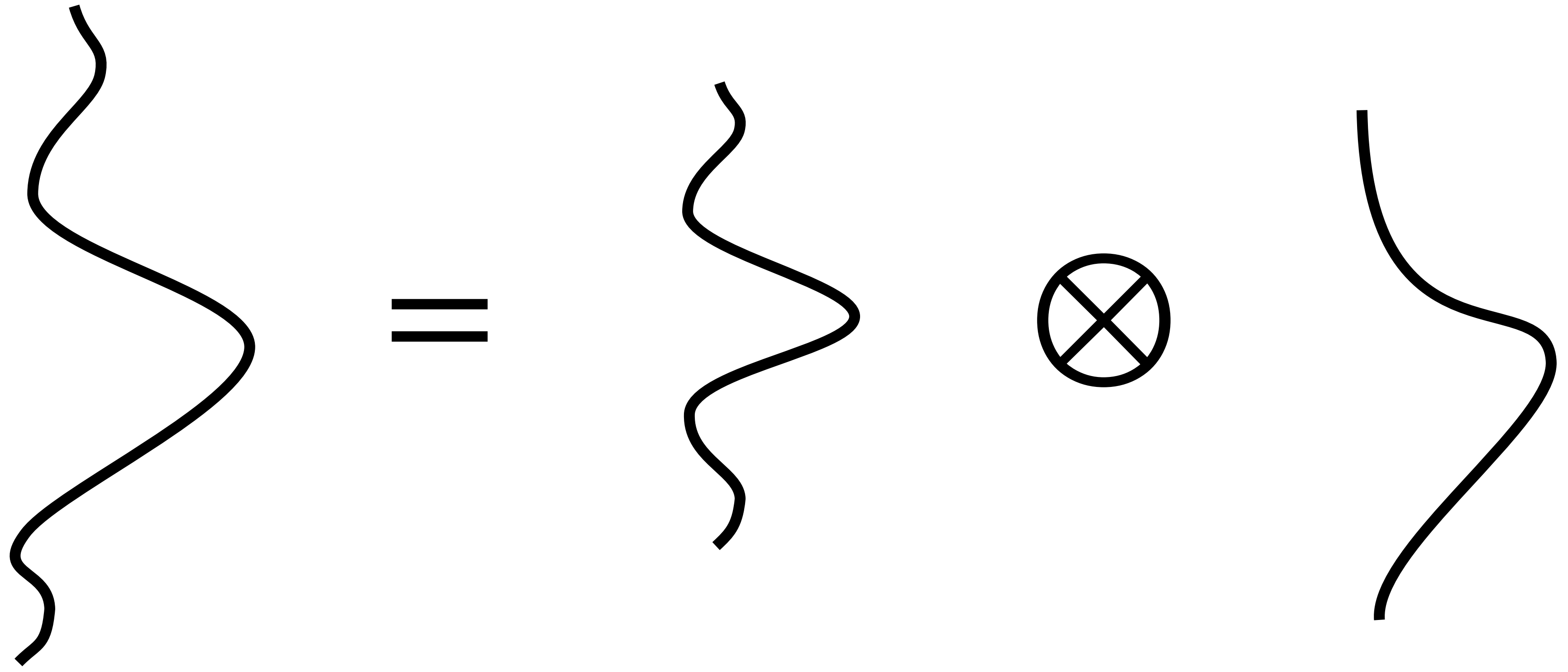
DOWNSCALING OPTICAL SYSTEMS

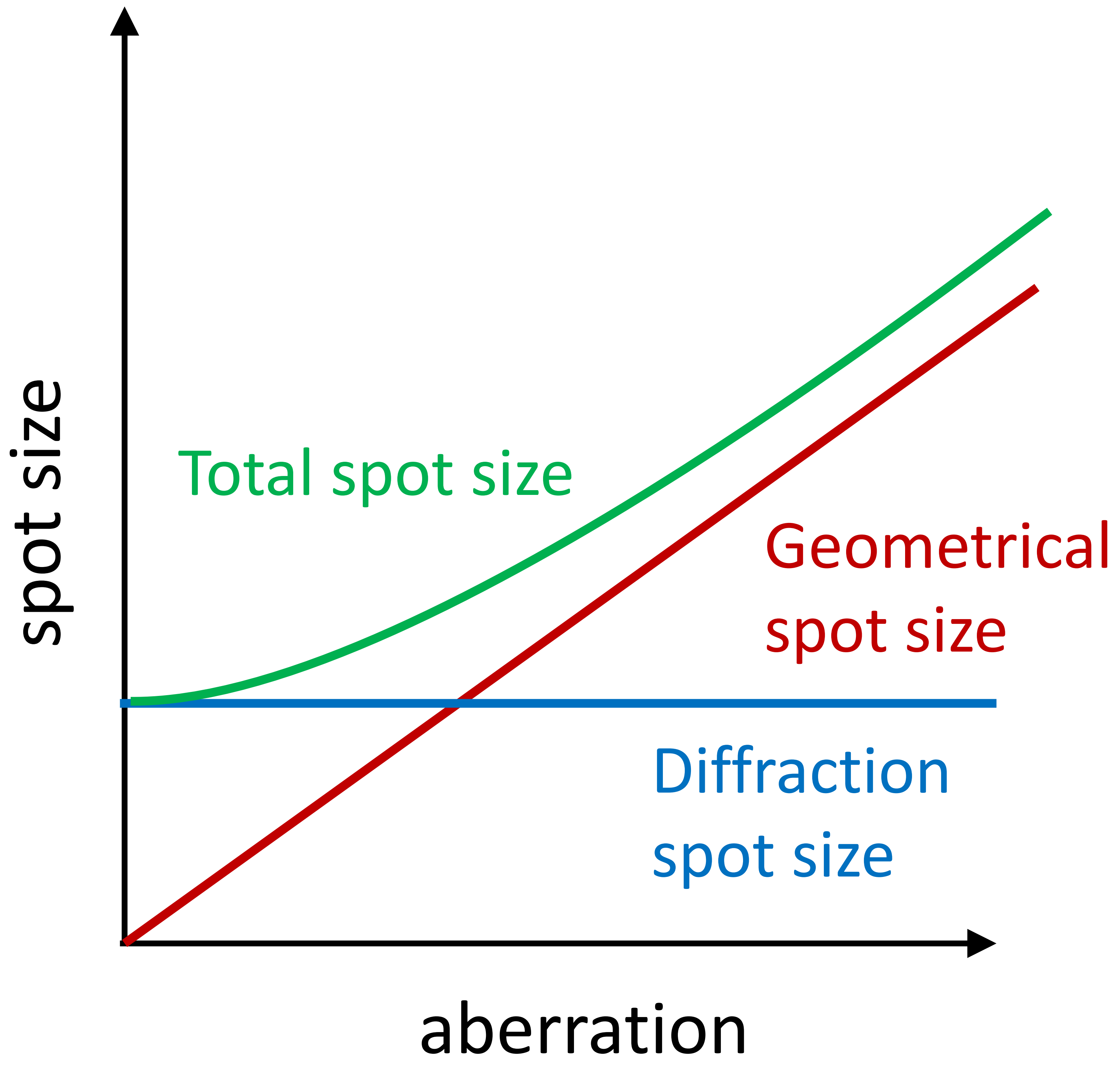
- **Constant f/# scaling**

  Focal length $f \rightarrow \mathcal{M}f$

  Lens diameter $D \rightarrow \mathcal{M}D$

$$\sigma_{\mathrm{tot}}^2 = \sigma_{\mathrm{psf}}^2 + \sigma_{\mathrm{aberration}}^2$$





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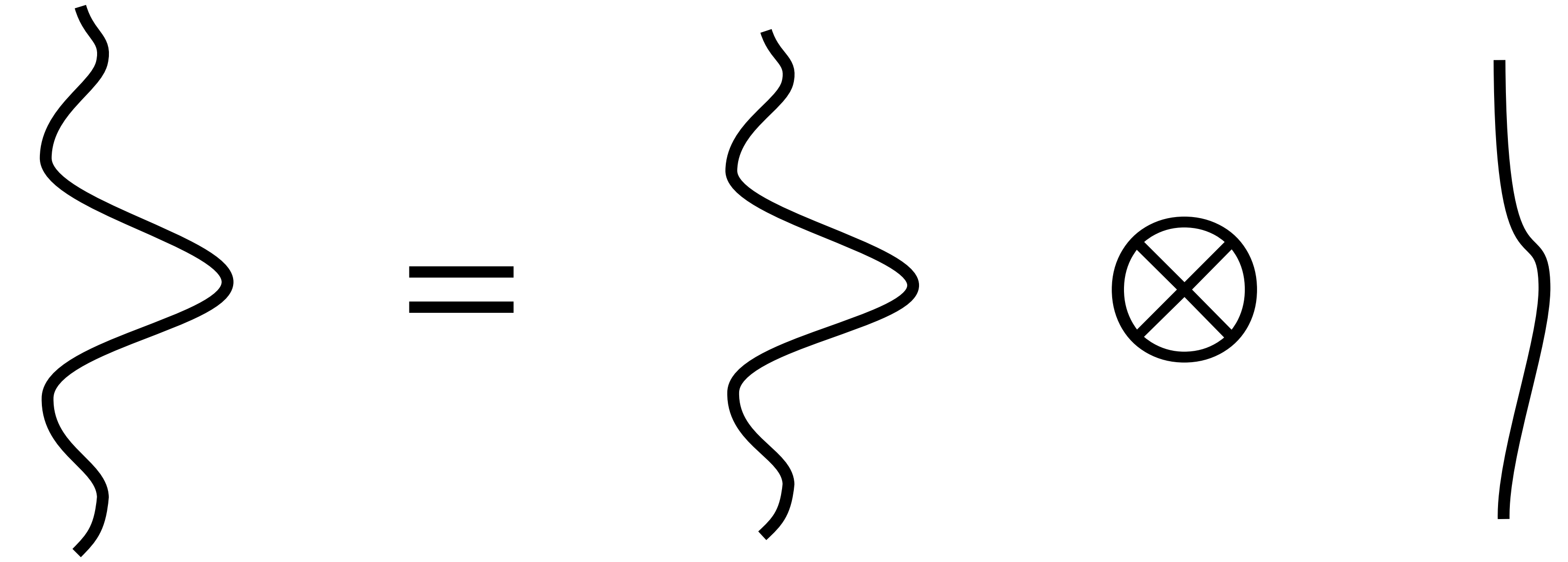
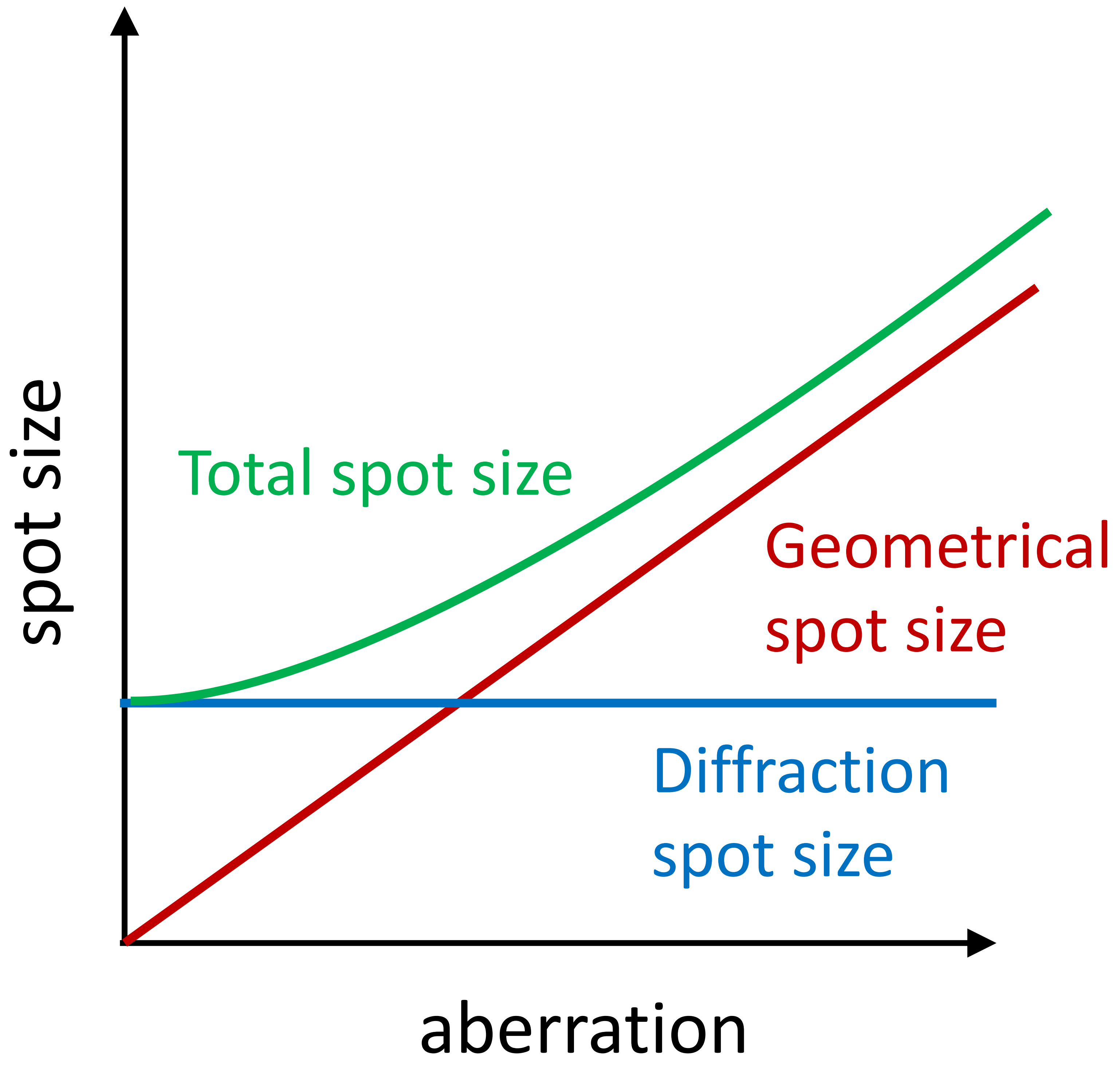




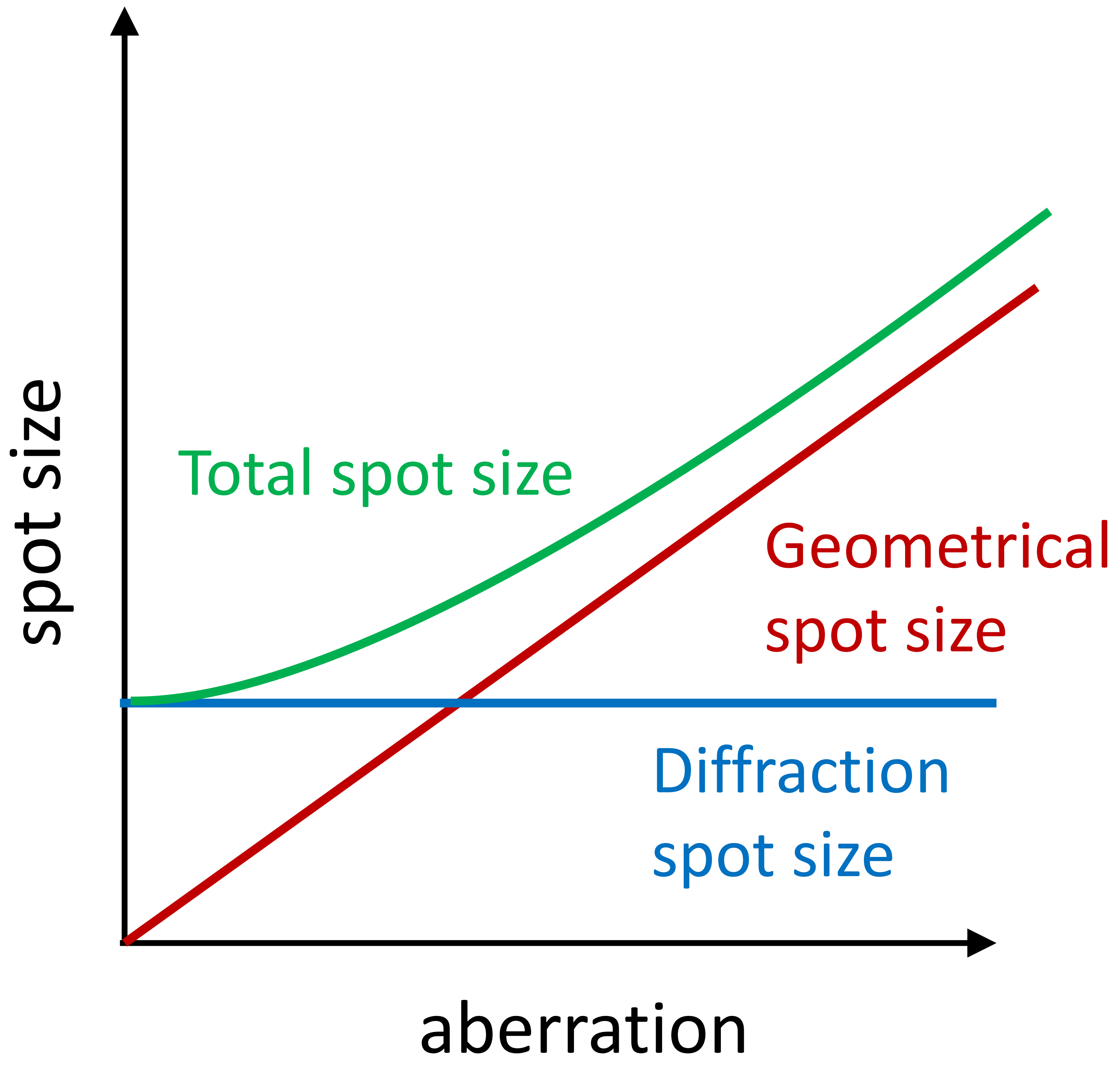
DOWNSCALING OPTICAL SYSTEMS

- **Constant f/# scaling**
  Focal length $f \to \mathcal{M}f$
  Lens diameter $D \to \mathcal{M}D$

- Scaling of resolution?

$$\mathrm{SBP}_{1\mathrm{D}} = \frac{\Delta x}{\delta x} \approx 1.22\frac{f}{\lambda}\ \mathrm{FOV}\cdot\mathrm{NA}$$

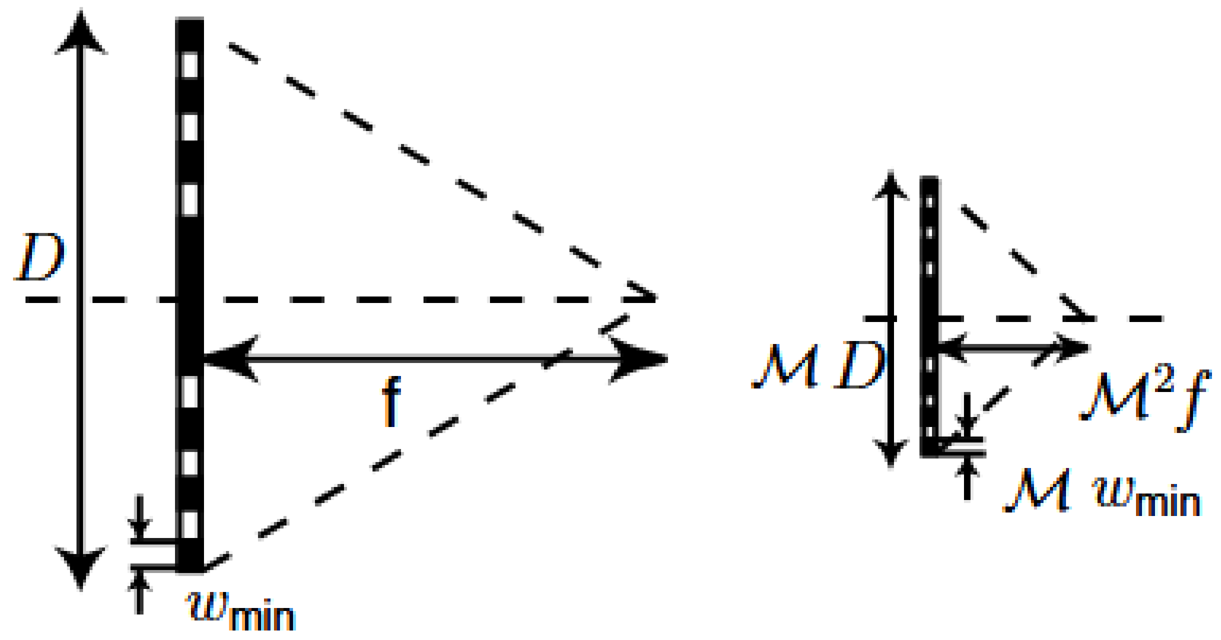
$$\mathrm{SBP} = \frac{\Delta x \Delta y}{\delta x \delta y} \to \frac{\mathcal{M}^2 \Delta x \Delta y}{\delta x \delta y + \mathcal{M}^2 \sigma^2_{\mathrm{aberration}}} \approx \frac{\mathcal{M}^2 \Delta x \Delta y}{\delta x \delta y}$$



DOWNSCALING OPTICAL SYSTEMS

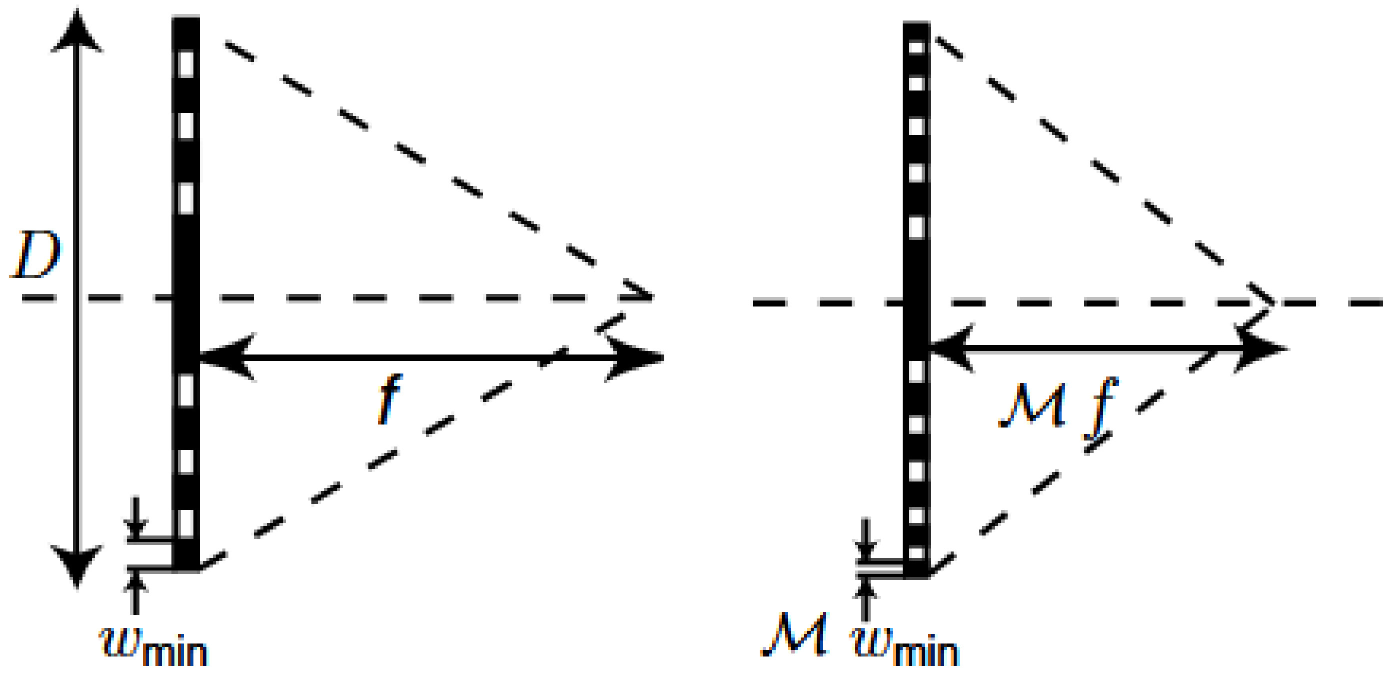
- **Photographic scaling**

Focal length	$f \rightarrow \mathcal{M}^2 f$
Lens diameter	$D \rightarrow \mathcal{M} D$
Minimum feature size	$w_{\mathrm{min}} \rightarrow \mathcal{M} w_{\mathrm{min}}$



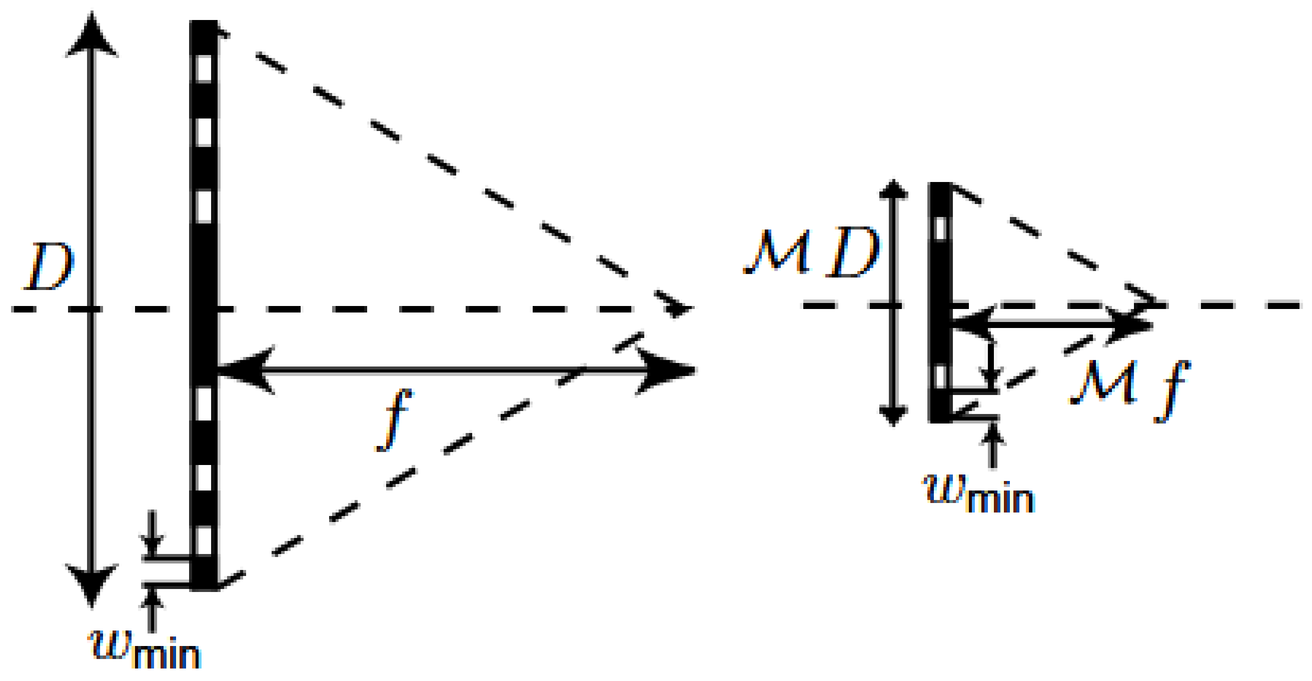
- **Microscopic scaling**

Focal length	$f \rightarrow \mathcal{M} f$	
Lens diameter	$D \rightarrow D$	*(constant)*
Minimum feature size	$w_{\mathrm{min}} \rightarrow \mathcal{M} w_{\mathrm{min}}$	



- **Constant f/# scaling**

Focal length	$f \rightarrow \mathcal{M} f$	
Lens diameter	$D \rightarrow \mathcal{M} D$	
Minimum feature size	$w_{\mathrm{min}} \rightarrow w_{\mathrm{min}}$	*(constant)*



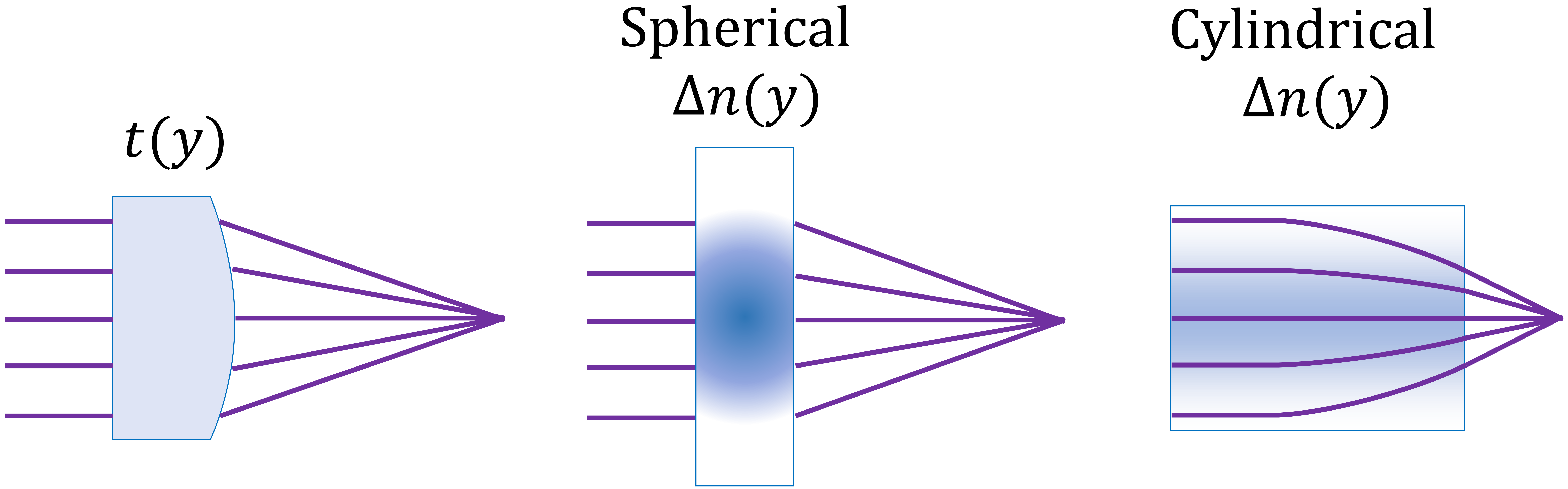
Refractive optics & microlenses

REFRACTIVE ELEMENTS

- Continuous phase difference
- Refractive index Δn and local thickness t

Optical path length $\varphi(y) = \frac{2\pi}{\lambda} \Delta n(y)\, t(y)$

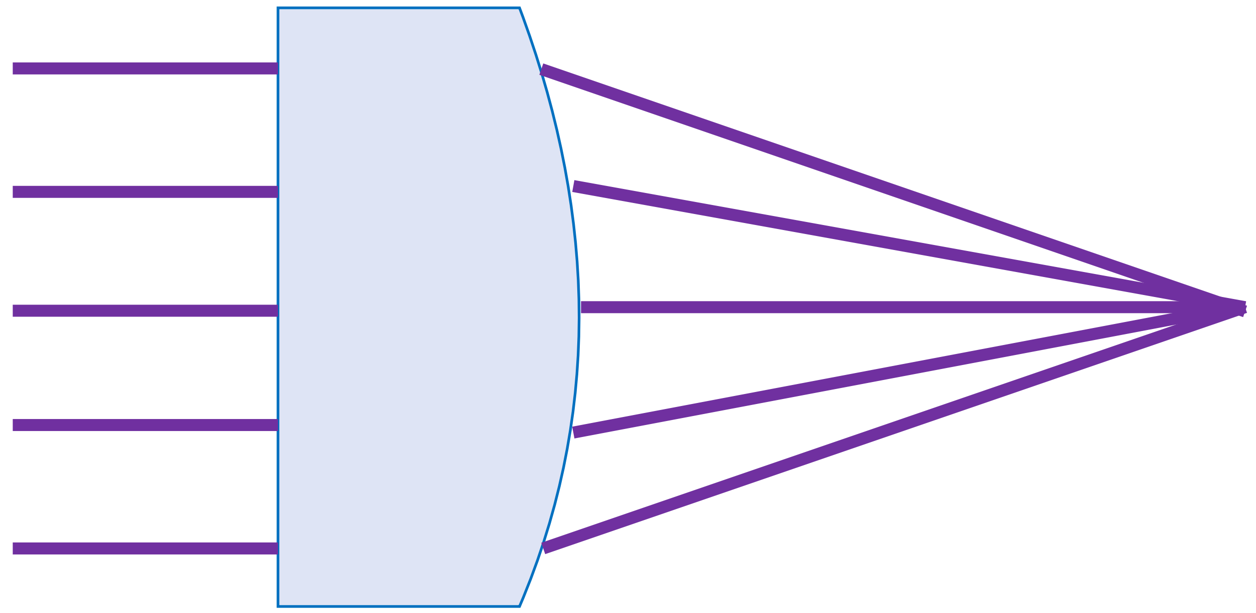
- Same as Snell's law



REFRACTIVE ELEMENTS: MICROLENS

- Fabrication of micro-surfaces:
  - Diamond turning
  - Small diamond tip radii:
  - Shape limited by tip size
    (> microns)

$t(y)$

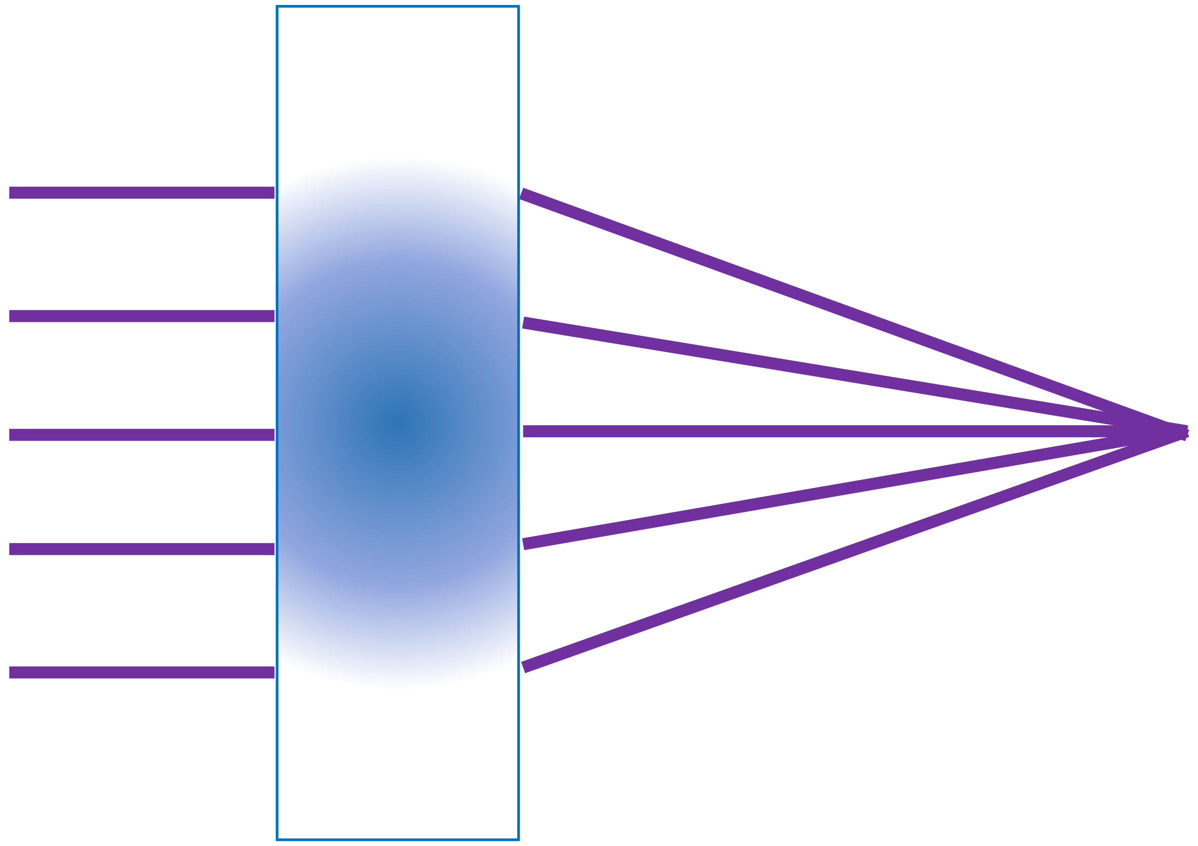


Picture from https://www.precitech.com

REFRACTIVE ELEMENTS: GRIN PLANAR LENS

- Diffusion of positive ions (Ag^{+}) on planar substrate at "single point"
- Concentration of ions in time: $C(y, t)$
- Diffusion of ions often nonlinear (similar to "aspheric" lens)
- Refractive index difference:

$$\Delta n(y) \propto C(y, t)$$



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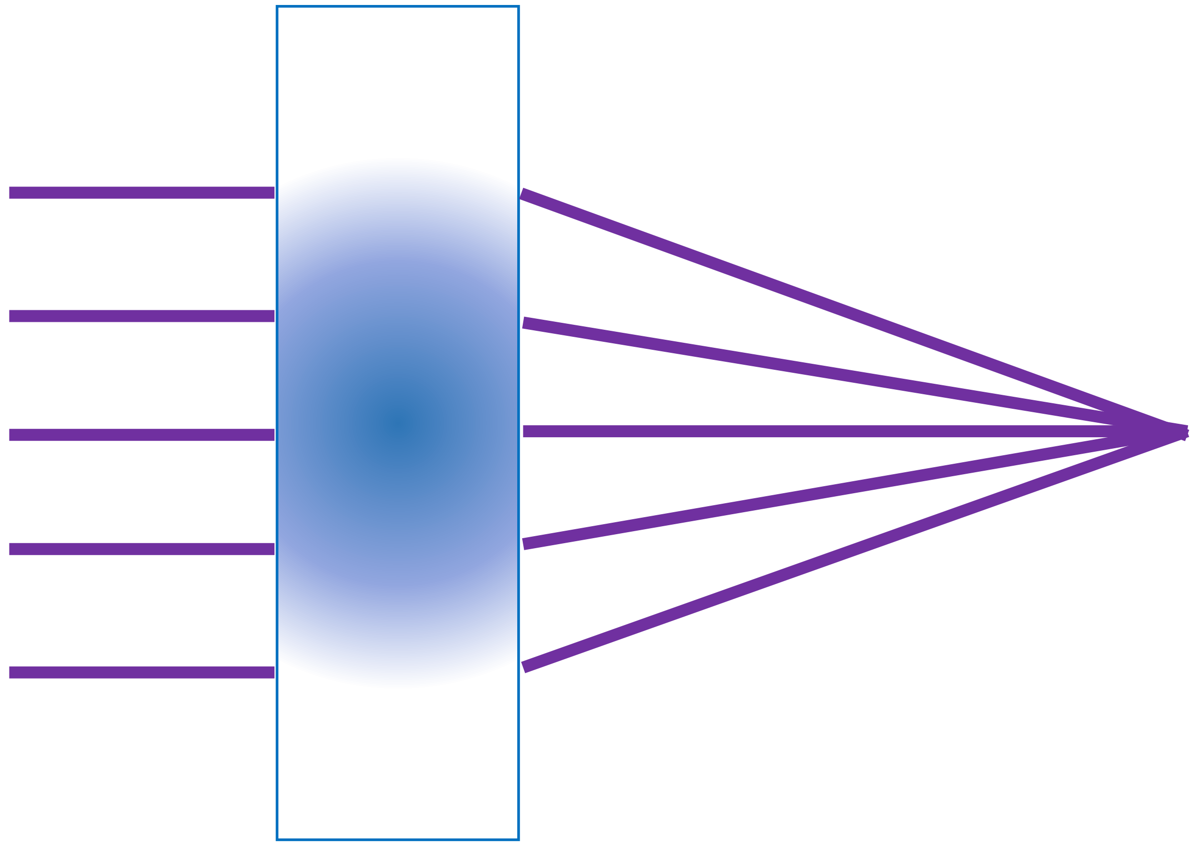
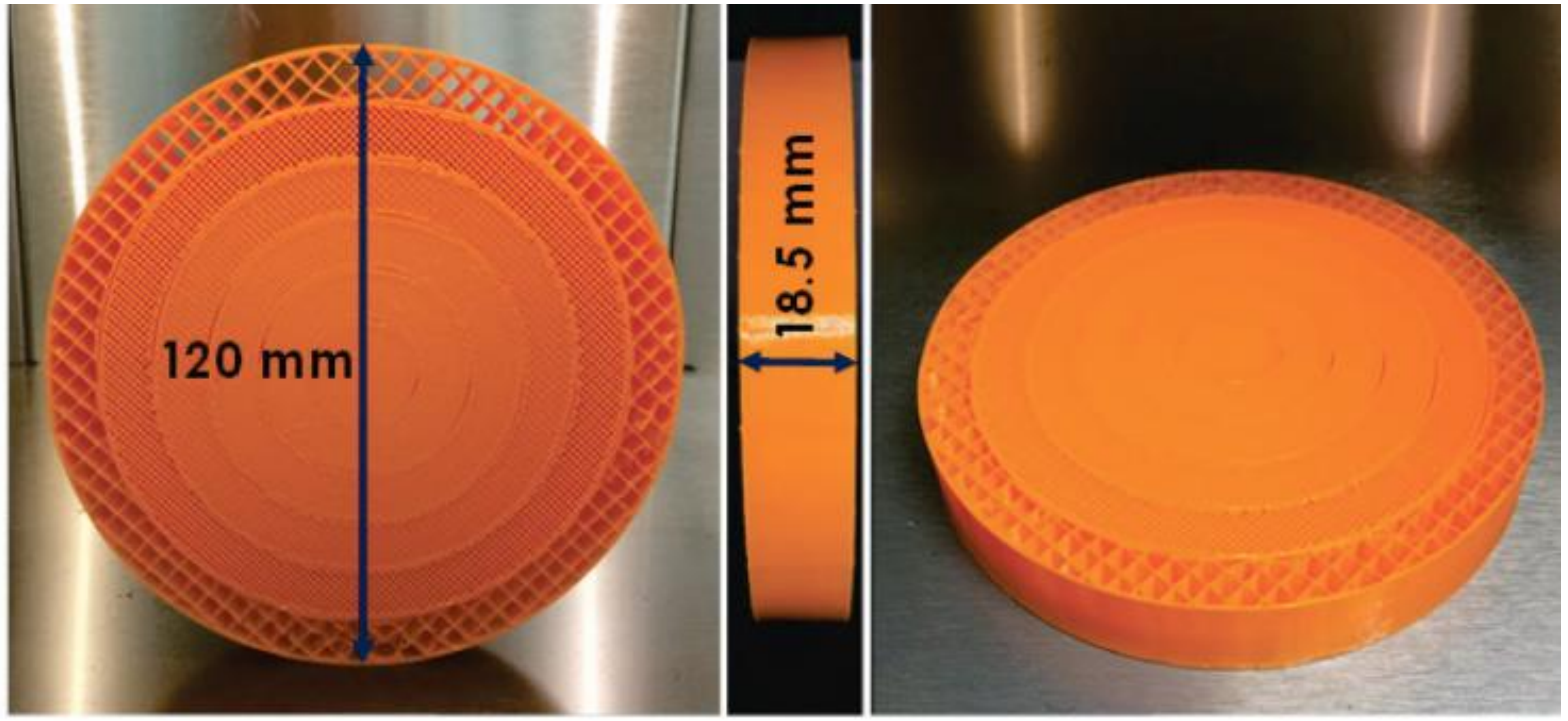


Make your own GRIN lens: 3D-printed planar lens (for 13-18 GHz) made from PLA, price: ±10 dollar



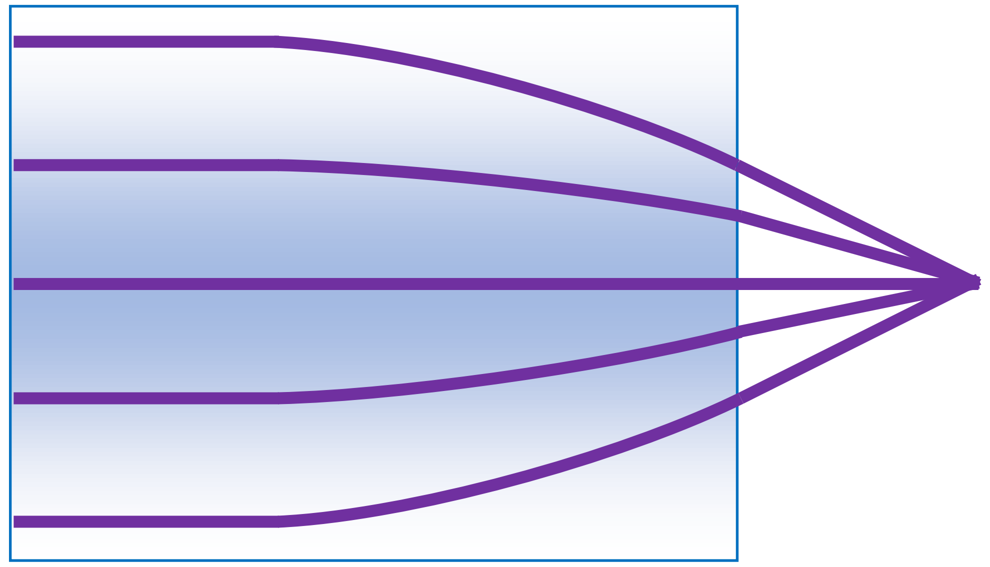
Shiyu Zhang et al. 3D-Printed Graded Index Lens for RF Applications, Proceedings of ISAP2016, Okinawa, Japan

REFRACTIVE ELEMENTS: GRIN ROD LENS

- Diffusion of positive ions (Ag^+) into cylindrical rod
- Concentration of ions in time: $C(r, t)$

$$\Delta n(r) = n_0 \left(1 - \frac{1}{2} g^2 r^2\right)$$

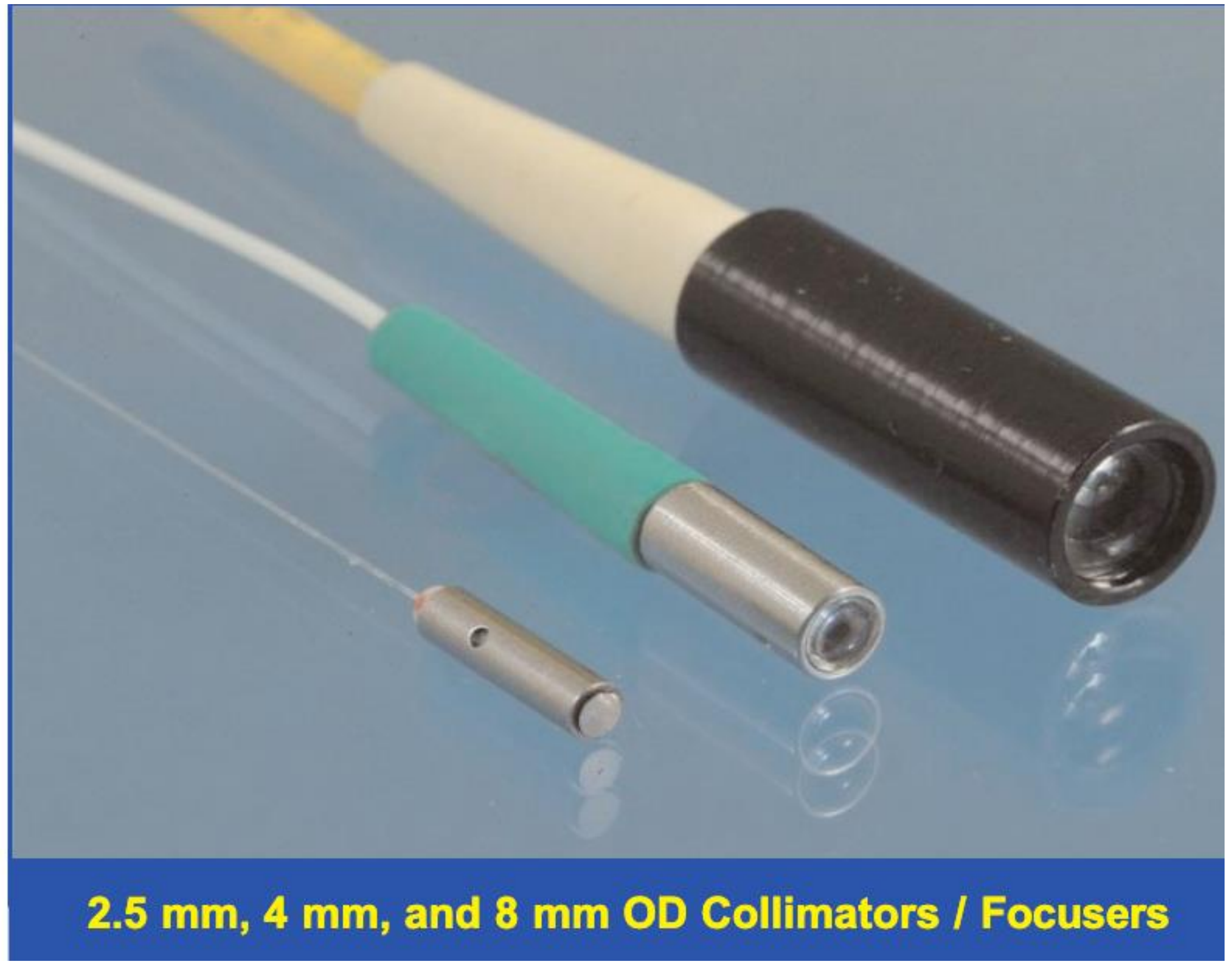
- Stronger refraction index gradient – strong magnefication



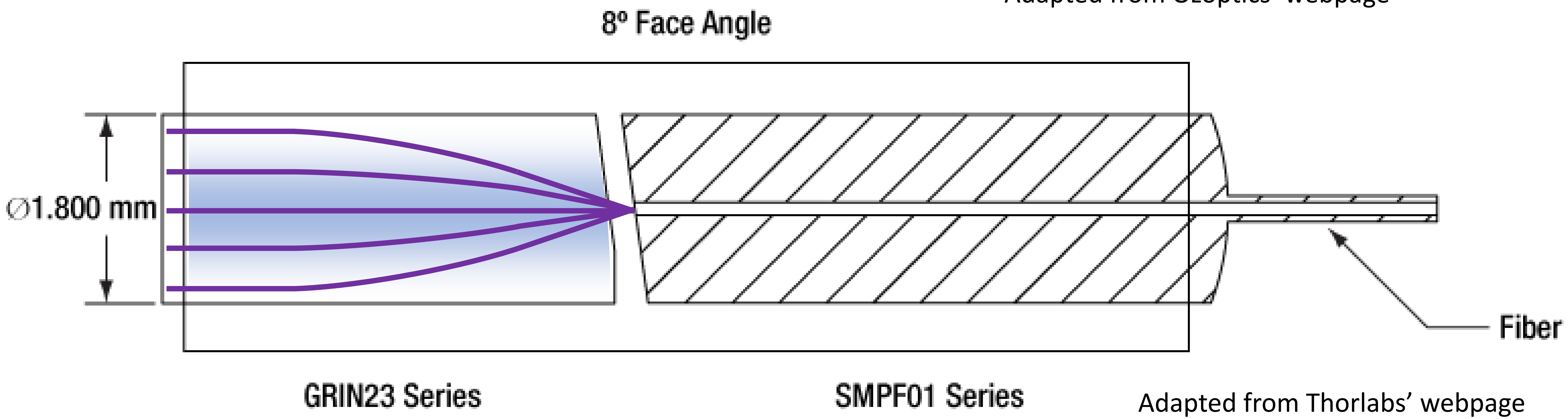
GRIN ROD LENS APPLICATIONS

- Collimation of light from a fiber
- Coupling laser source into fiber
- Coupling (focusing) into waveguide

- Non-normal angle to prevent reflection



Adapted from Ozoptics' webpage



Adapted from Thorlabs' webpage