PHOT 451: Microscale optical system design
LECTURE 08

Michaël Barbier, Fall semester (2025-2026)

OVERVIEW OF THE COURSE

week	Topic
Week 1	Introduction to micro-scale optical components
Week 2	Light propagation in free space
Week 3	Geometric optics and raycasting
Week 4	Diffraction limit & Abberations
Week 5	**Quiz** + Beam propagation
Week 6	Refractive optical elements Microlenses
Week 7	Blazed Fresnel lenses
Week 8	Digital lenses
Week 9	Diffractive optical elements
Week 10	**Quiz** + Wave guides and beam propagation
Week 11	Wave mixing
Week 12	Gratings, periodic structures
Week 13	photonic crystals
Week 14	Whole optical system optimization

Methods of Light Propagation

Some insight in different models

SUMMARY LIGHT PROPAGATION

- Full wave electromagnetic field

- Wave equation: Amplitude and Phase
  - Near field effects

- Paraxial approximation
  - Geometrical Optics: spherical (thick) lenses
  - **Paraxial Helmholtz equation & Beam solution**

- Geometrical optics
  - short wavelengths
  - Optical design dimensions larger than wavelength

Beam Propagation

Paraxial approx.: Beam solutions are good approximations for lasers

SOLUTIONS TO THE WAVE EQUATION

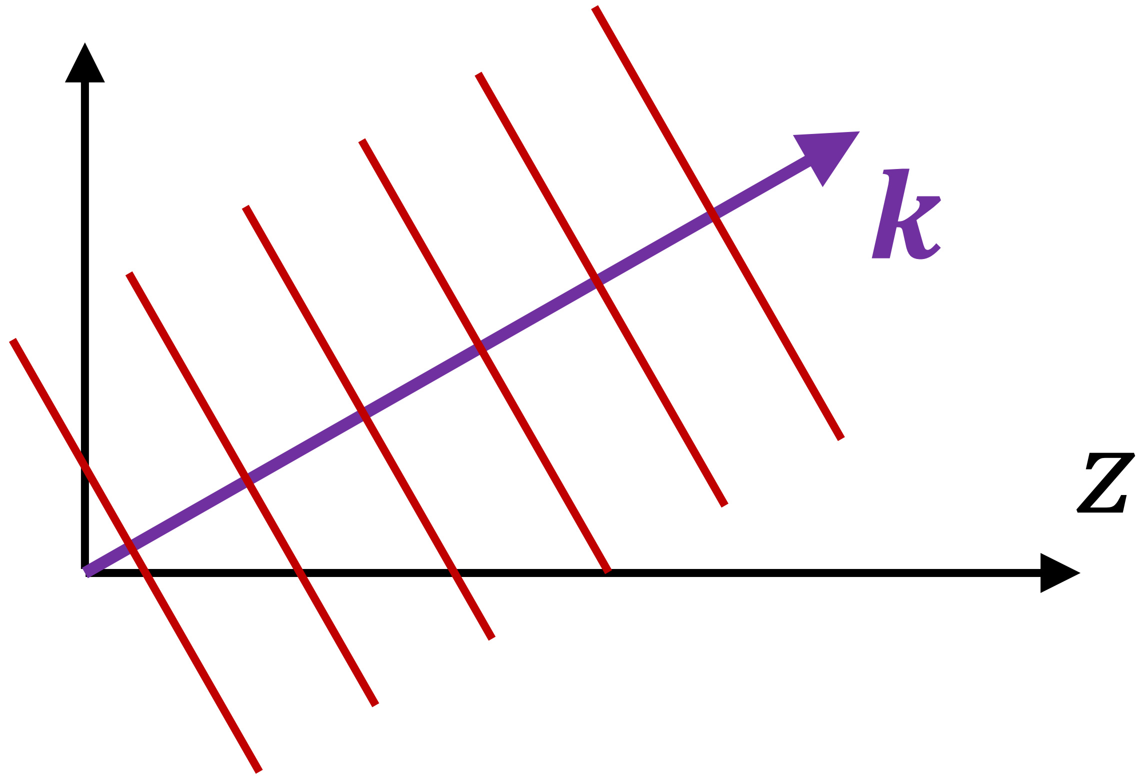
$$\nabla^2 U(\boldsymbol{r},t) - \frac{1}{c^2}\frac{\partial^2 U(\boldsymbol{r},t)}{\partial t^2}$$

Plane waves

$$U(\boldsymbol{r},t) = U_0 e^{ik(r \pm ct)}$$

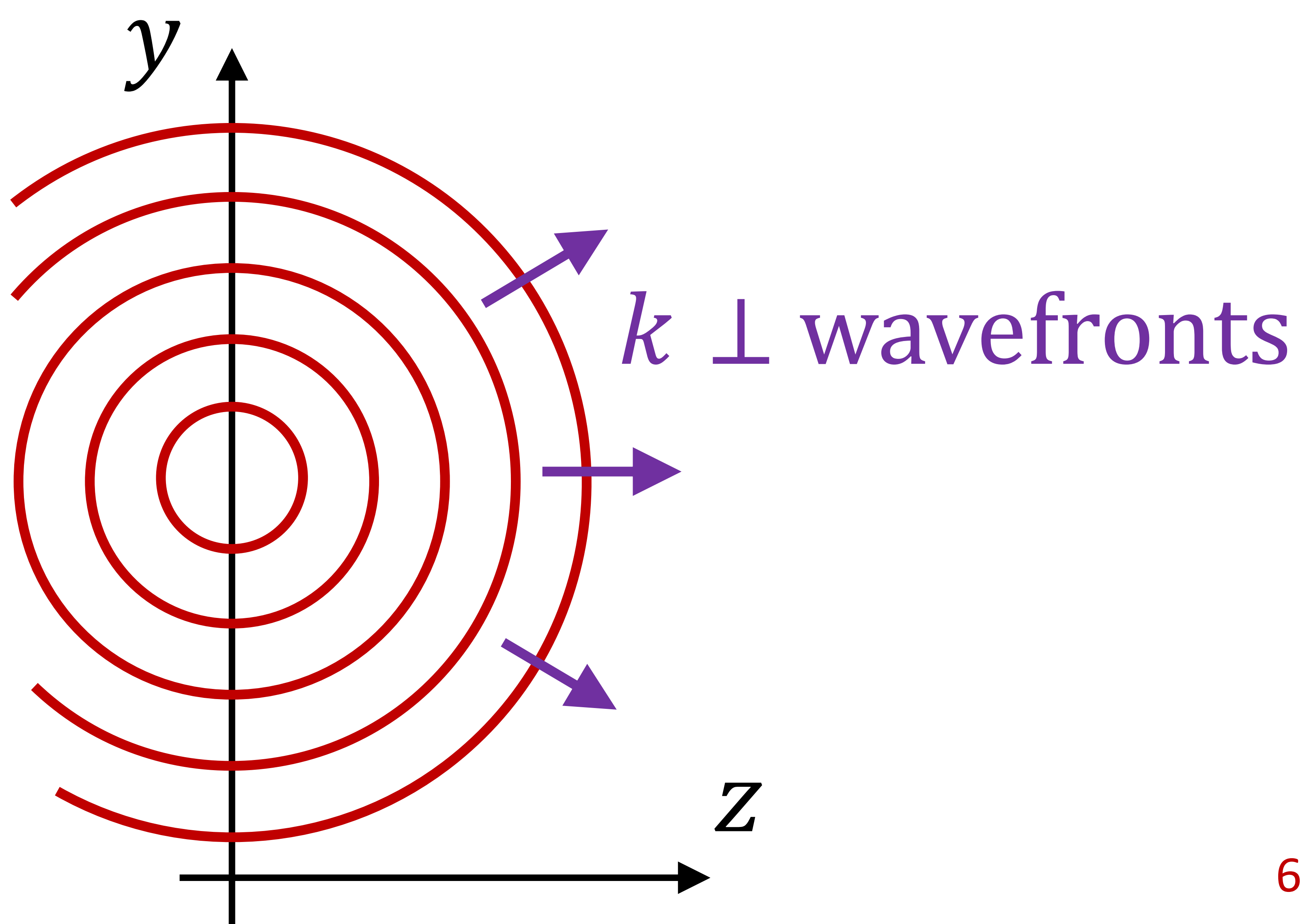
Planes:

$\boldsymbol{k} \cdot \boldsymbol{r} = \text{constant}$



Spherical waves

$$U(\boldsymbol{r},t) = U_0 \frac{e^{ik(r \pm ct)}}{r}$$



PARAXIAL HELMHOLTZ EQUATION

Wave equation:

$$\nabla^2 U(\boldsymbol{r},t) - \frac{1}{c^2}\frac{\partial^2 U(\boldsymbol{r},t)}{\partial t^2} = 0$$

$$U(\boldsymbol{r},t) = u(\boldsymbol{r})\, e^{-ikct}$$

Helmholtz equation:

$$\nabla^2 u(\boldsymbol{r}) + k^2\, u(\boldsymbol{r}) = 0$$

OR

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] u(\boldsymbol{r}) + k^2\, u(\boldsymbol{r}) = 0$$

PARAXIAL HELMHOLTZ EQUATION

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]u(\boldsymbol{r})+k^2\,u(\boldsymbol{r})=0$$

Paraxial: assume propagation in z

$$u(\boldsymbol{r})=E(\boldsymbol{r})\,e^{-ikz}$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\left(E(\boldsymbol{r})\,e^{ikz}\right)+\ k^2E(\boldsymbol{r})e^{ikz}=0$$

PARAXIAL HELMHOLTZ EQUATION

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\left(E(\boldsymbol{r})\,e^{ikz}\right)+\ k^2E(\boldsymbol{r})e^{ikz}=0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\left(E(\boldsymbol{r})\,e^{ikz}\right)+\frac{\partial^2E(\boldsymbol{r})\,e^{ikz}}{\partial z^2}+\ k^2E(\boldsymbol{r})e^{ikz}=0$$

$$\nabla_\perp^2\,E(\boldsymbol{r})e^{ikz}+\frac{\partial^2E(\boldsymbol{r})}{\partial z^2}-k^2E(\boldsymbol{r})e^{ikz}+i2k\frac{\partial E(\boldsymbol{r})}{\partial z}+k^2E(\boldsymbol{r})e^{ikz}=0$$

PARAXIAL HELMHOLTZ EQUATION

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\left(E(\boldsymbol{r})\,e^{ikz}\right)+\;k^2E(\boldsymbol{r})e^{ikz}=0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\left(E(\boldsymbol{r})\,e^{ikz}\right)+\frac{\partial^2 E(\boldsymbol{r})\,e^{ikz}}{\partial z^2}+\;k^2E(\boldsymbol{r})e^{ikz}=0$$

$$\nabla_\perp^2\,E(\boldsymbol{r})e^{ikz}+\frac{\partial^2E(\boldsymbol{r})}{\partial z^2}-\cancel{k^2E(\boldsymbol{r})e^{ikz}}+i2k\,\frac{\partial E(\boldsymbol{r})}{\partial z}+\cancel{k^2E(\boldsymbol{r})e^{ikz}}=0$$

0

PARAXIAL HELMHOLTZ EQUATION

Paraxial Helmholtz equation:

$$\nabla_\perp^2 E(\boldsymbol{r}) + i2k\frac{\partial E(\boldsymbol{r})}{\partial z} = 0$$

Assumption:

$$\frac{\partial^2 E(\boldsymbol{r})}{\partial z^2} \ll \frac{\partial^2 E(\boldsymbol{r})}{\partial x^2}, \frac{\partial^2 E(\boldsymbol{r})}{\partial y^2}$$

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Beam solution

$$E(\boldsymbol{r}) = A\, e^{-\frac{ik\left(x^2+y_2\right)}{2\,q(z)}}\, e^{-i\,p(z)}$$

With :

$$\begin{cases} q(z) = z + iz_0 \\ p(z) = -i\ln\left(1 - \frac{iz}{z_0}\right) \end{cases}$$

BEAM SOLUTION: STANDARD FORM

$$E(\boldsymbol{r}) = A\frac{w_0}{w(z)} e^{-\frac{(x^2+y^2)}{w(z)^2}} e^{-ikz} e^{-i\frac{k(x^2+y^2)}{2R(z)}} e^{i\phi(z)}$$

With :

Amplitude **Phase**

$$R(z) = z\left(1+\left(\frac{z_0}{z}\right)^2\right),$$

Curvature radius

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}, \qquad w_0 = \sqrt{\frac{z_0\lambda}{\pi}},$$

Beam width

$$\phi(z) = \text{atan}\left(\frac{z}{z_0}\right),$$

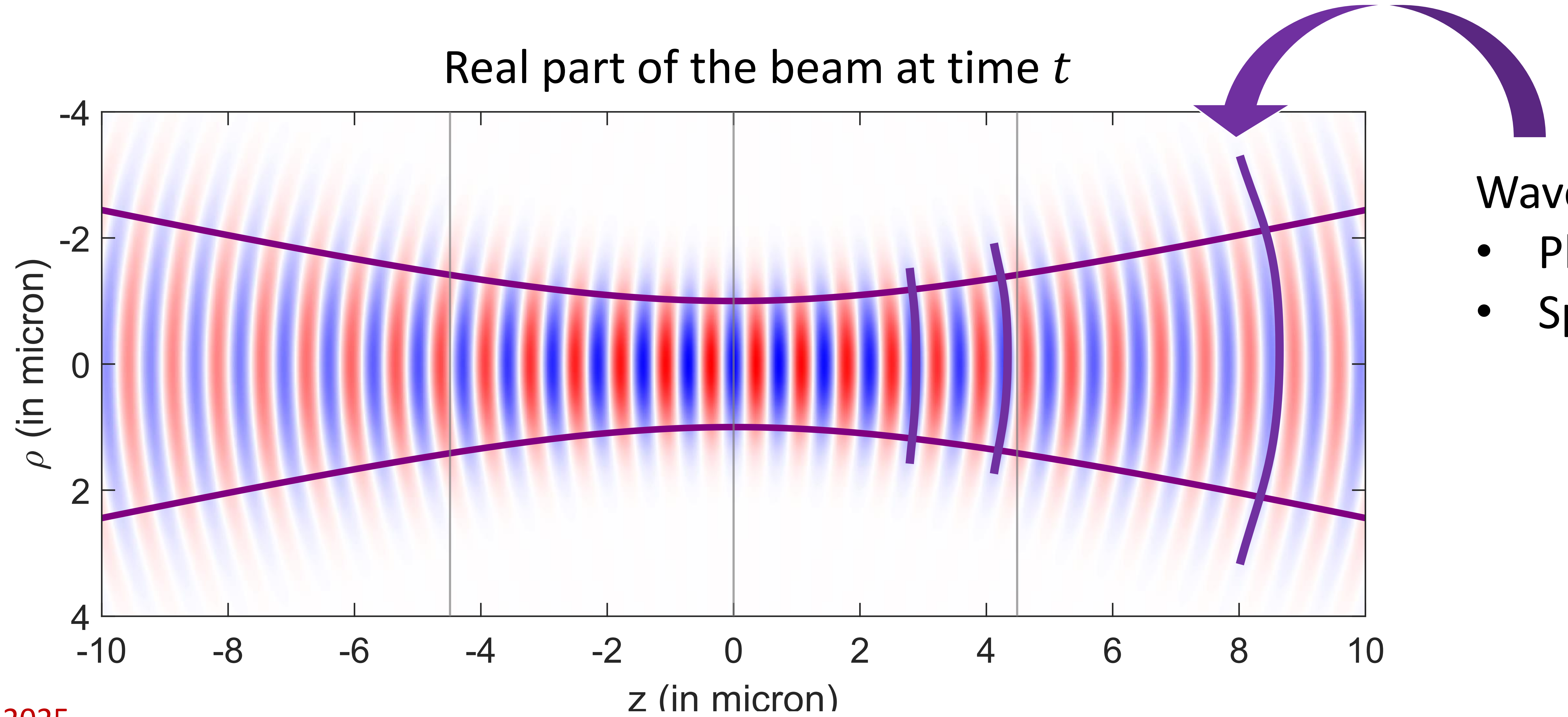
Original phase of U(r)

BEAM SOLUTION: STANDARD FORM

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{A}\frac{w_0}{w(z)} e^{-\frac{(x^2+y^2)}{w(z)^2}} e^{-ikz} e^{-i\frac{k(x^2+y^2)}{2R(z)}} e^{i\phi(z)}$$

Amplitude **Phase**

Real part of the beam at time t



Wavefronts:

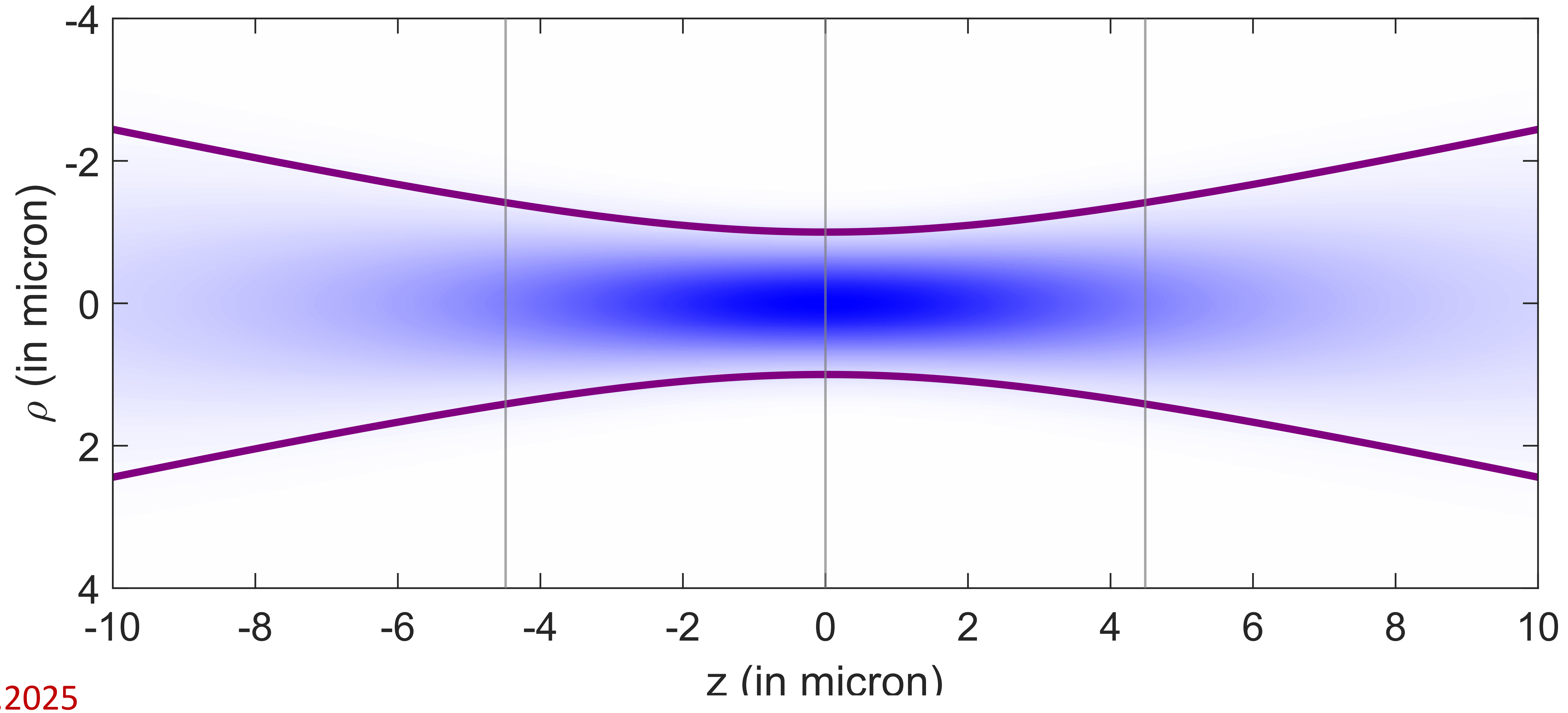
- Planar close to beam waist
- Spherical at far-field

BEAM SOLUTION: STANDARD FORM

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{A}\frac{w_0}{w(z)} e^{-\frac{(x^2+y^2)}{w(z)^2}} e^{-ikz} e^{-i\frac{k(x^2+y^2)}{2R(z)}} e^{i\phi(z)}$$

Amplitude **Phase**

Intensity of the beam



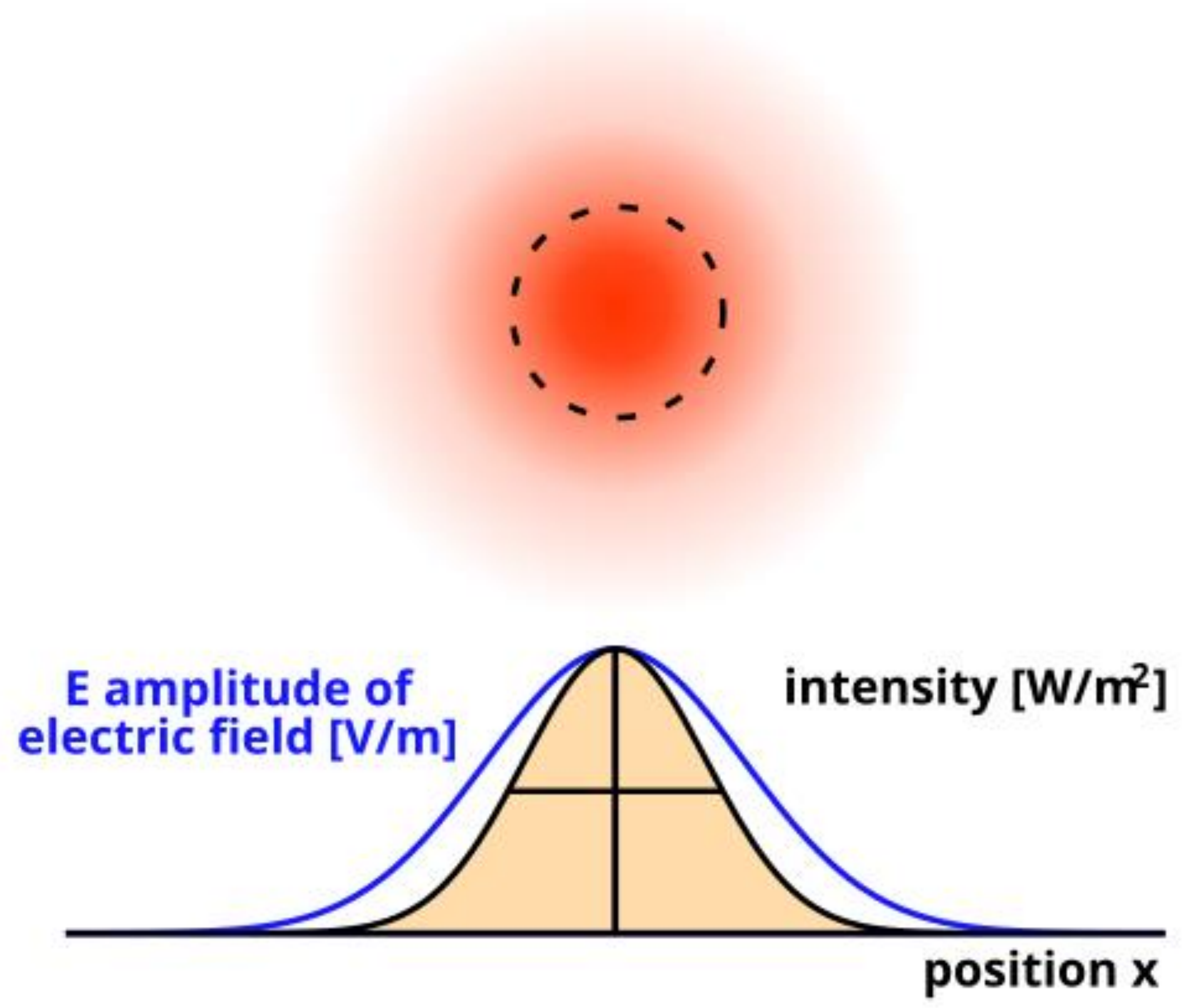
Intensity of the beam:

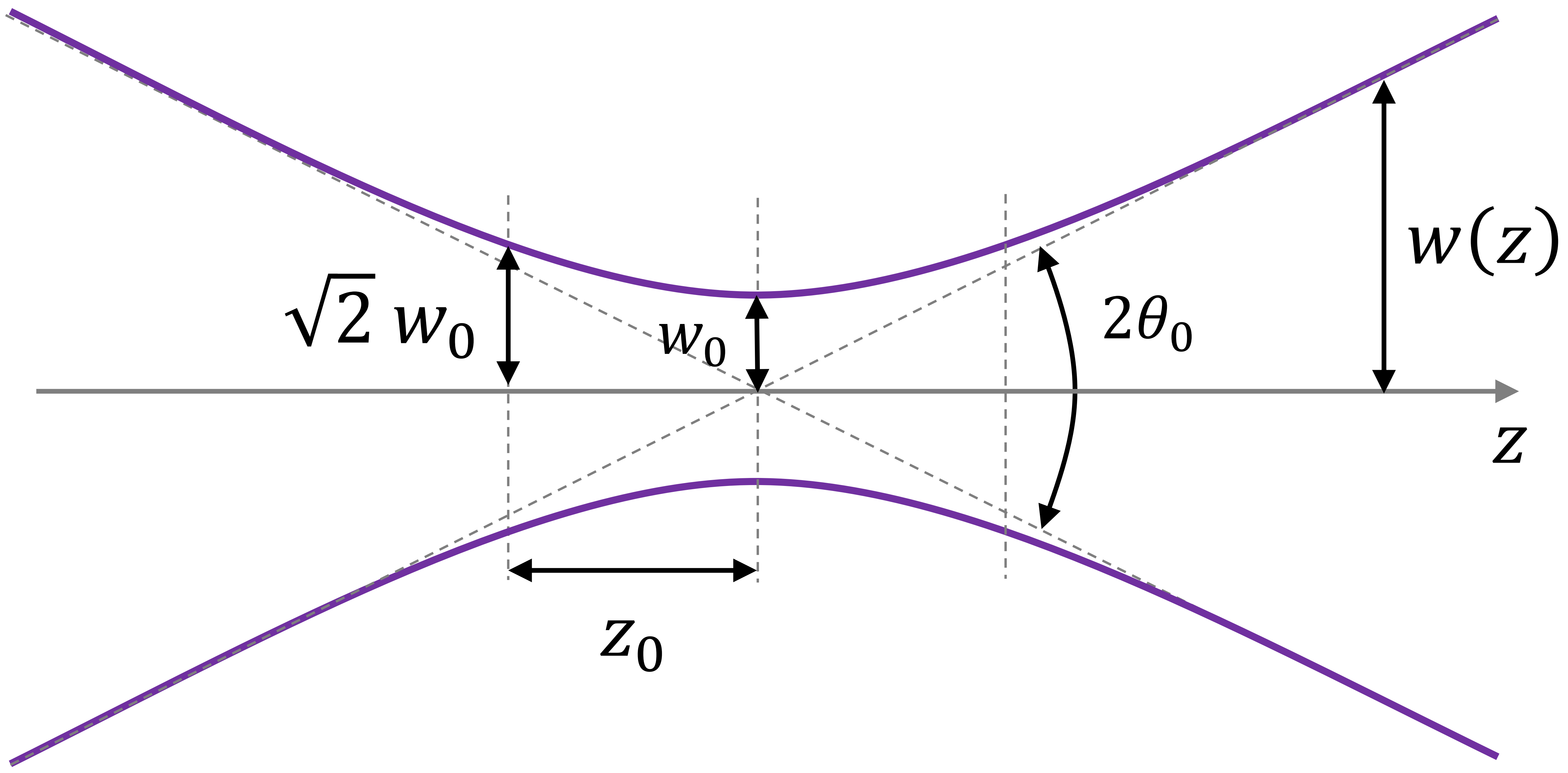
$$I(\boldsymbol{r}) = |\boldsymbol{E}(\boldsymbol{r})|\,|\boldsymbol{E}^*(\boldsymbol{r})|$$

$$= |\boldsymbol{A}|^2 \left(\frac{w_0}{w(z)}\right)^2 e^{-\frac{2(x^2+y^2)}{w(z)^2}}$$

BEAM PARAMETERS

$$I(\boldsymbol{r}) = |\boldsymbol{A}|^2 \left(\frac{w_0}{w(z)}\right)^2 e^{-\frac{2(x^2+y^2)}{w(z)^2}}$$





BEAM RADIUS & POWER

Power of the beam

$$P = \int_0^{+\infty} I(\rho, z)\, 2\pi\rho\, d\rho = \frac{1}{2} I_0\, \pi w_0^2$$

Rewrite the intensity:

$$I(\rho, z) = |\boldsymbol{A}|^2 \left(\frac{w_0}{w(z)}\right)^2 e^{-\frac{2\rho^2}{w^2(x)}} \quad \Rightarrow \quad I(\rho, z) = \frac{2P^2}{\pi w^2(z)} e^{-\frac{2\rho^2}{w^2(z)}}$$

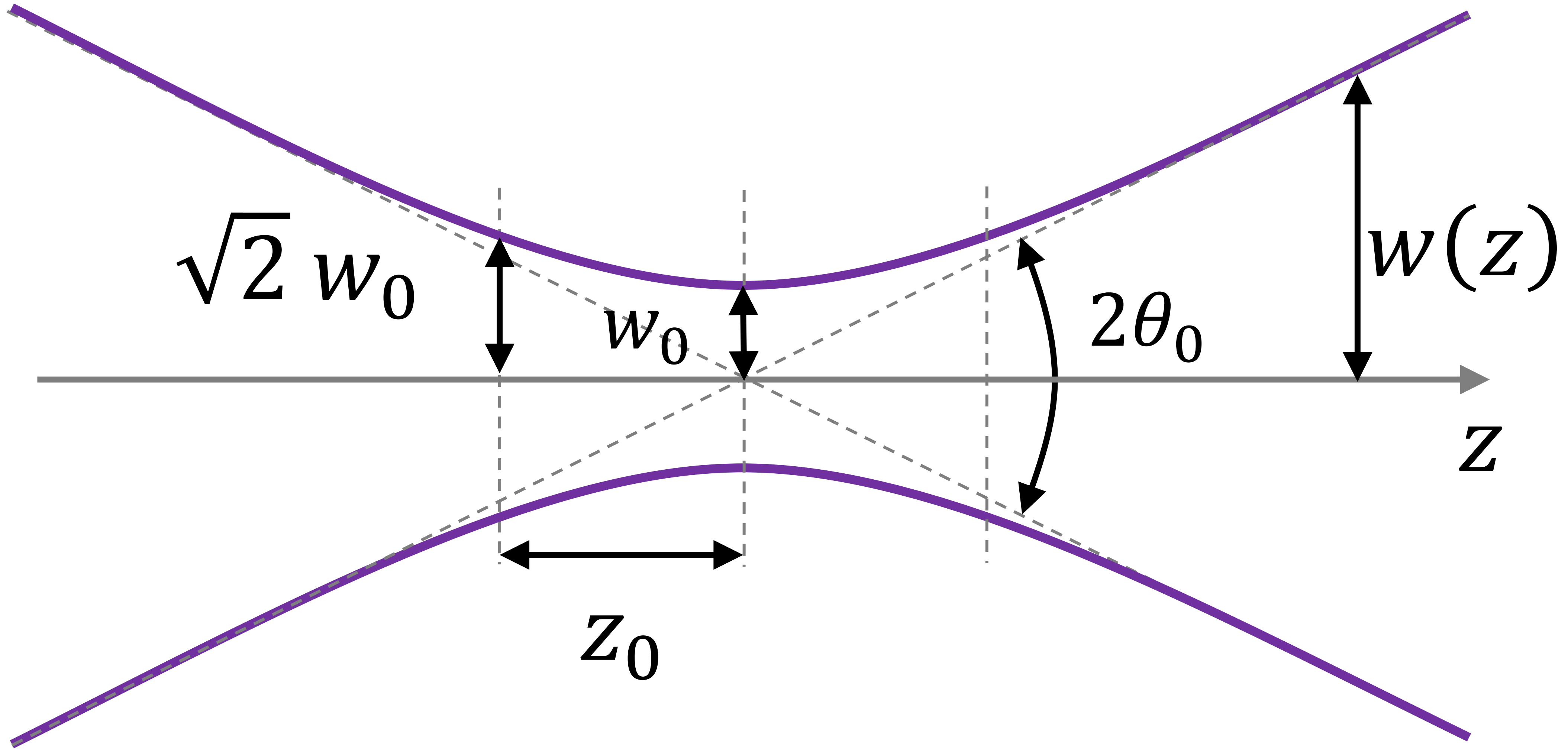
Power within radius ρ_0 :

$$\frac{1}{P}(\rho, z) = \int_0^{\rho_0} I(\rho, z)\, 2\pi\rho\, d\rho = 1 - e^{-\frac{2\rho_0^2}{w^2(z)}}$$

BEAM DEPTH OF FOCUS AND DIVERGENCE

Limited divergence within Rayleigh distance $-z_0 \rightarrow z_0$

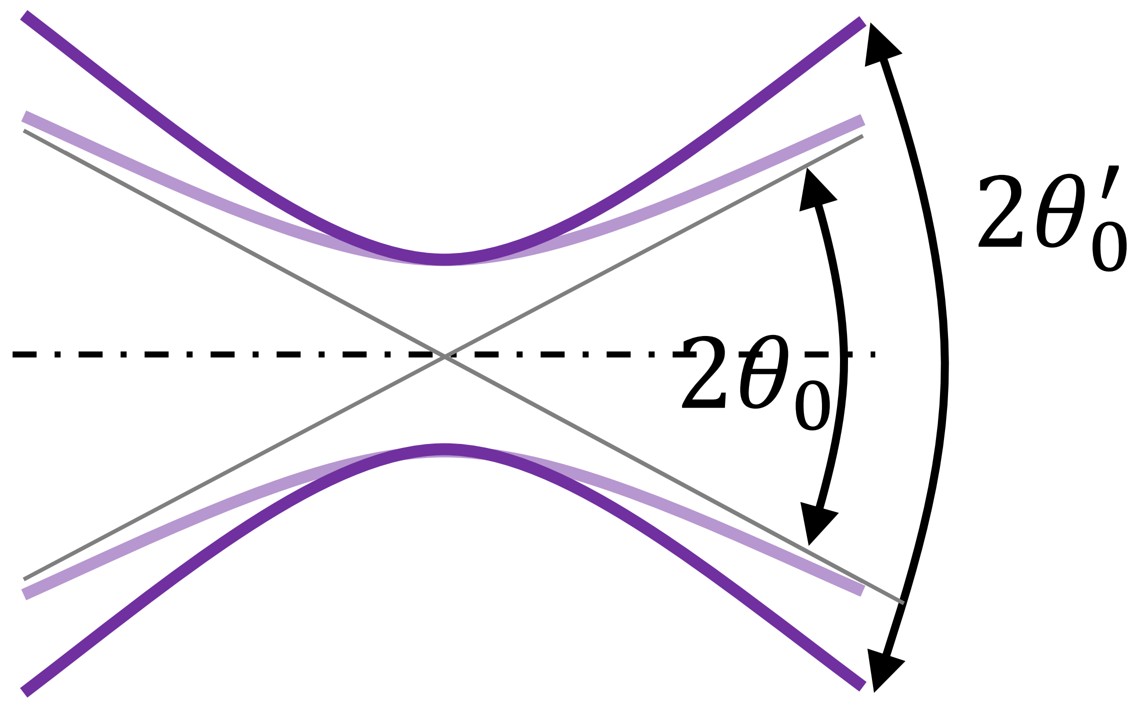
- Depth of focus = $2z_0 = \frac{2\pi w_0^2}{\lambda}$
- Divergence $\theta_0 = \frac{w_0}{z_0} = \frac{2\lambda}{(2w_0)\pi}$



BEAM QUALITY

Beam quality M^2 parameter :

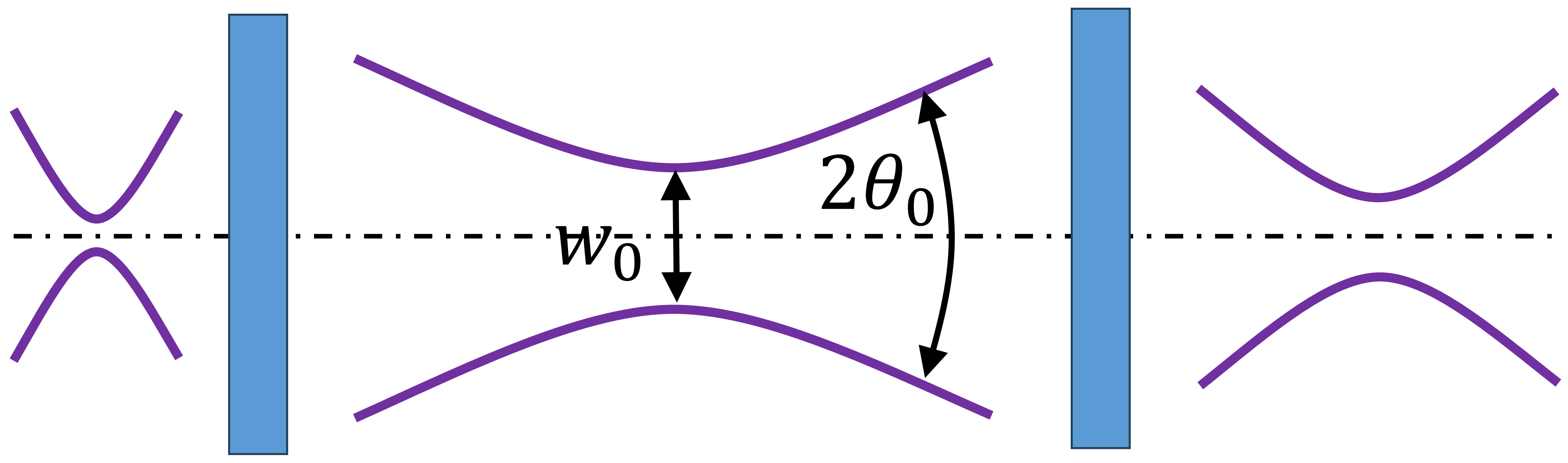
$$M^2 = \frac{\theta_0'}{\theta_0} = \frac{\theta_0' \pi w_0}{\lambda} = \frac{\text{real divergence}}{\text{ideal Gaussian divergence}}$$



Beam Parameter Product $\mathrm{BPP} = \theta_0 w_0$ is a constant $\propto \frac{\lambda}{\pi}$

Decreasing beam waist

Increasing divergence angle



ABCD-Matrices for Gaussian Beams

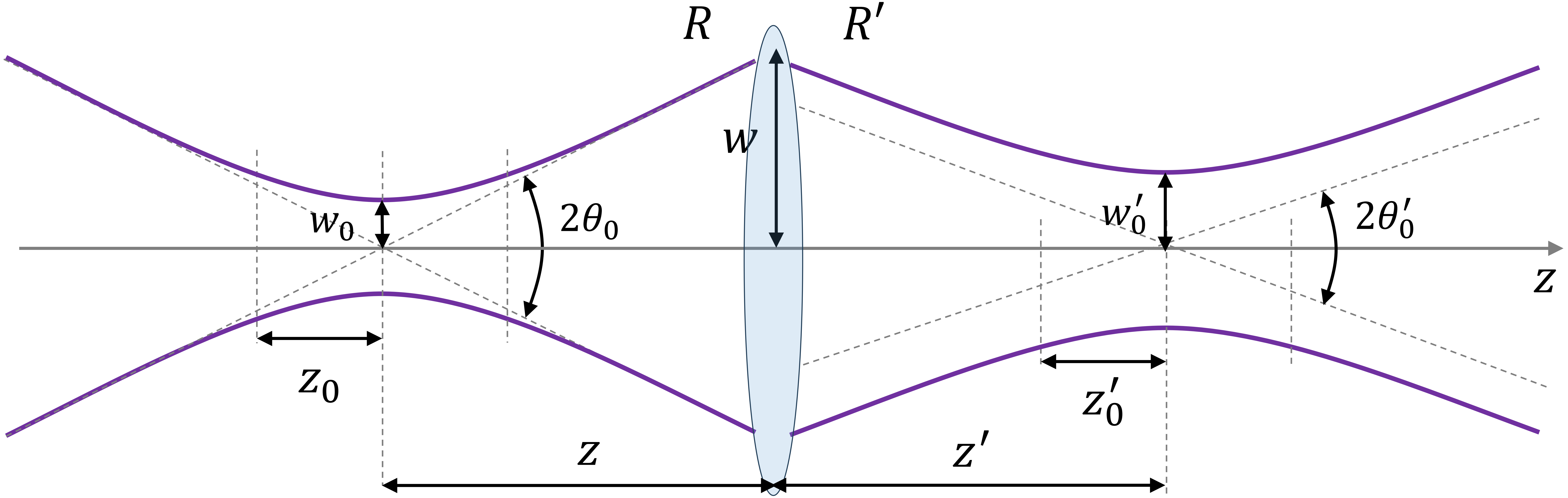
ABCD MATRICES FOR GAUSSIAN BEAMS

ABCD-matrix: $\begin{bmatrix} q' \\ 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} q \\ 1 \end{bmatrix},$

$$\frac{1}{q(z)} = \frac{1}{\mathrm{R(z)}} - \frac{\mathrm{i}\lambda}{\pi w^2(z)}$$

$$q(z) = z + iz_0$$

$q(z)$ defines beam: $E(\boldsymbol{r}) = A\, e^{-\frac{ik(x^2+y_2)}{2\,q(z)}}\, e^{-i\,p(z)}$



ABCD MATRIX: FREE SPACE PROPAGATION

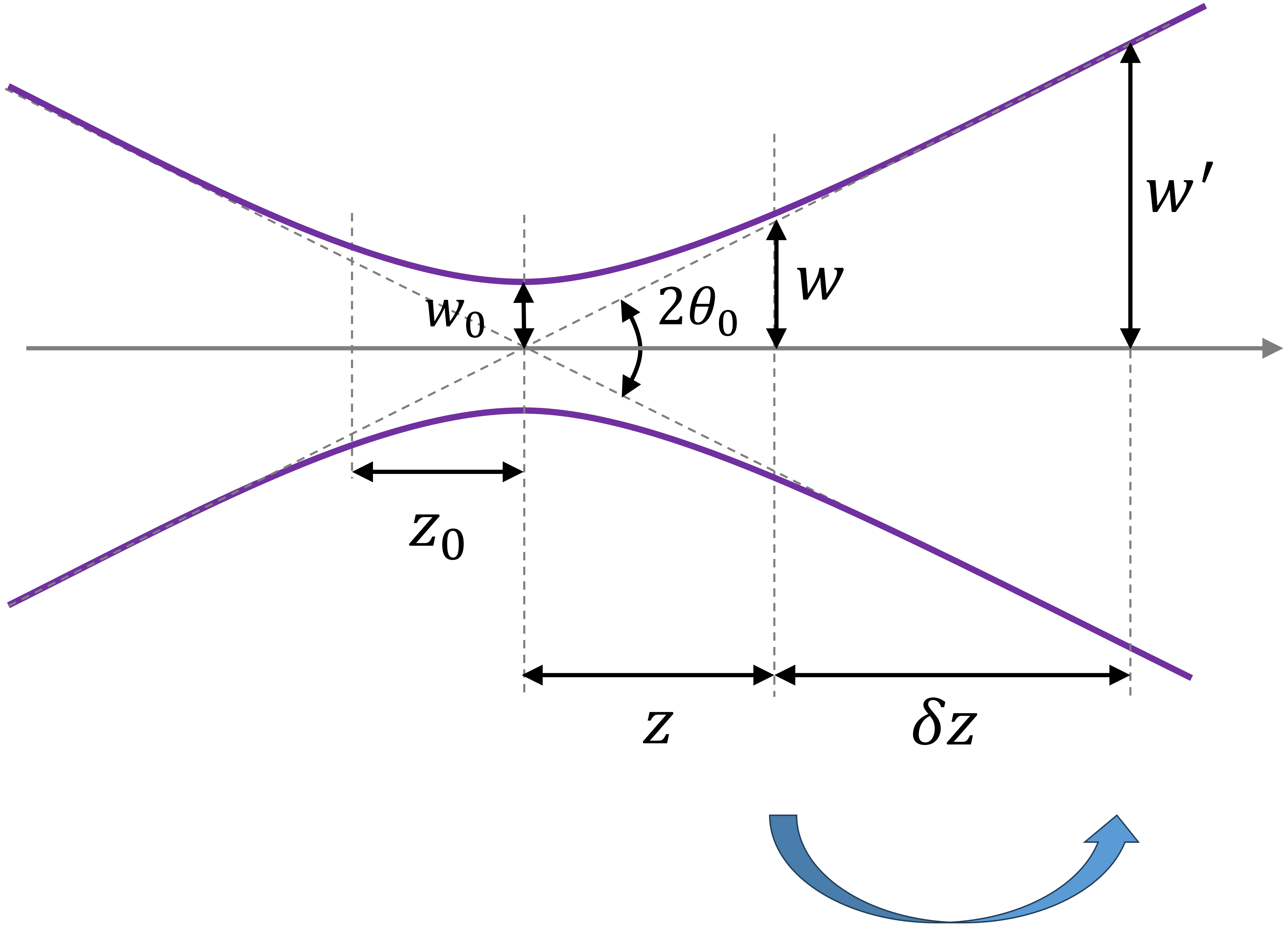
ABCD-matrix:

$$\begin{bmatrix} q' \\ 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} q \\ 1 \end{bmatrix}$$

Free space propagation

$$q' = z + \delta z + i z_0$$

$$\begin{bmatrix} q' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{\delta z}{n} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q \\ 1 \end{bmatrix}$$

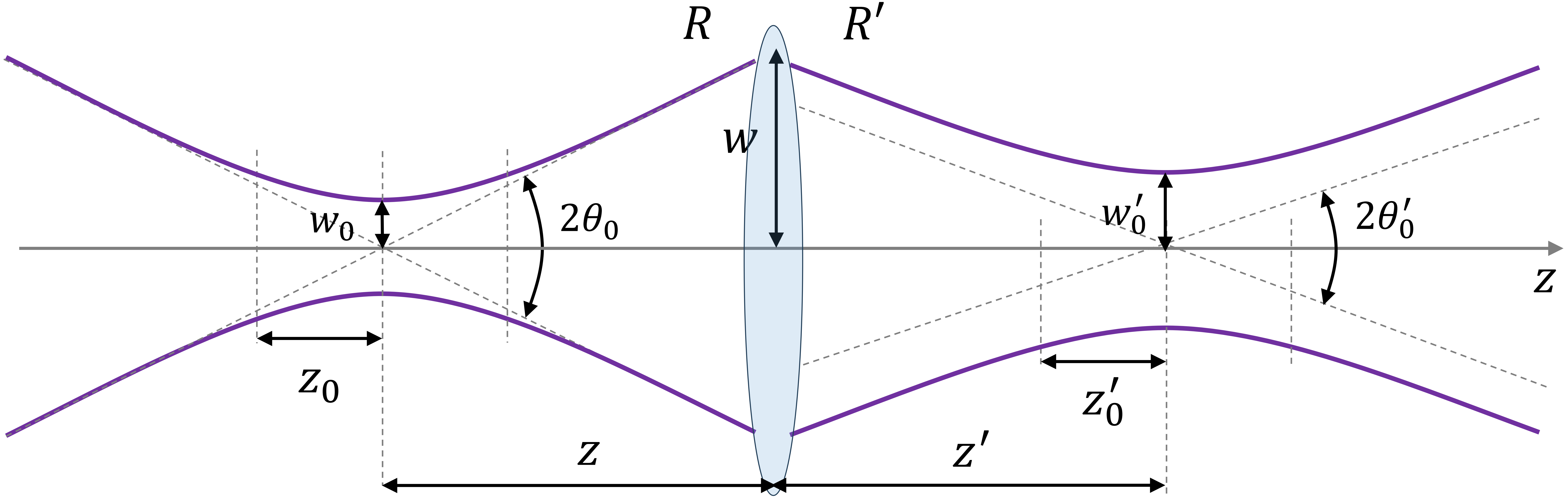


ABCD MATRIX: SINGLE LENS

ABCD-matrix for a thin lens:

$$\begin{bmatrix} q' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} q \\ 1 \end{bmatrix},$$

$$\begin{cases} \dfrac{1}{q(z)} = \dfrac{1}{\mathrm{R(z)}} - \dfrac{\mathrm{i}\lambda}{\pi w^2(z)} \\ q(z) = z + iz_0 \end{cases}$$

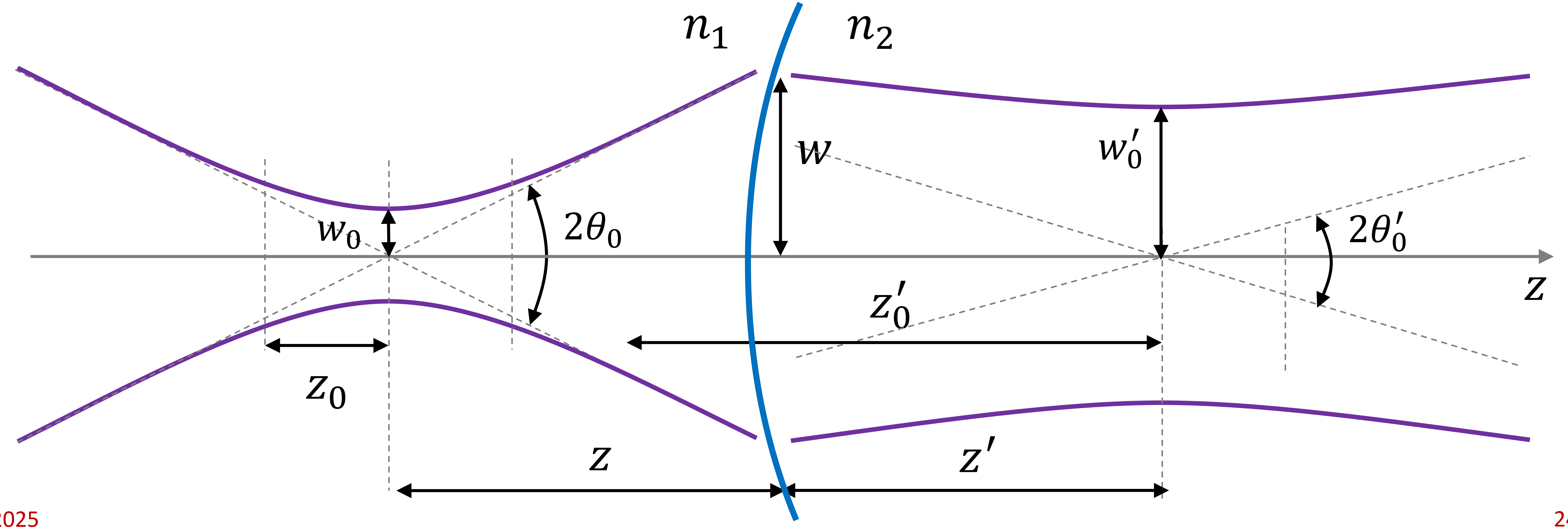


ABCD MATRIX: SINGLE SPHERICAL SURFACE

ABCD-matrix for a single surface:

$$\begin{bmatrix} q' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{n_1 - n_2}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} q \\ 1 \end{bmatrix},$$

$$\begin{cases} \dfrac{1}{q(z)} = \dfrac{1}{\mathrm{R(z)}} - \dfrac{\mathrm{i}\lambda}{\pi w^2(z)} \\ q(z) = z + i z_0 \end{cases}$$

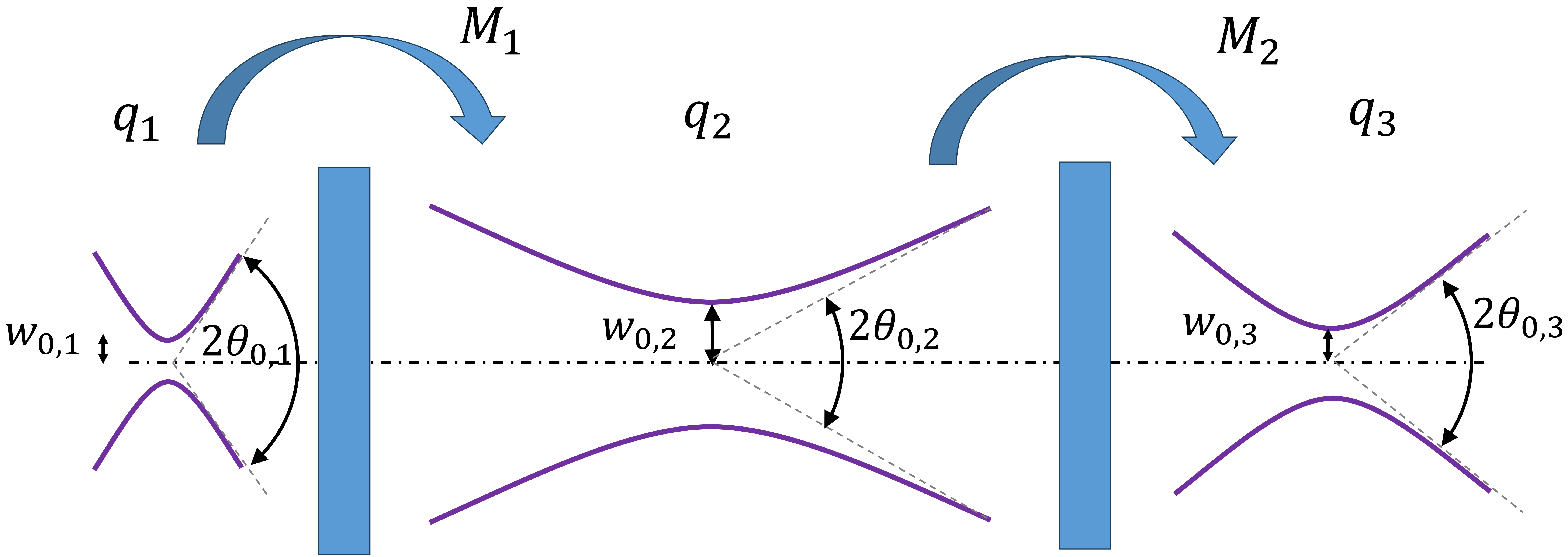


ABCD MATRICES FOR GAUSSIAN BEAMS

ABCD-matrix for multiple surfaces:

$$\begin{bmatrix} q_{N+1} \\ 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} q_1 \\ 1 \end{bmatrix} = M_N \dots M_2 M_1 \begin{bmatrix} q_1 \\ 1 \end{bmatrix}$$

$$\begin{cases} \dfrac{1}{q(z)} = \dfrac{1}{\mathrm{R(z)}} - \dfrac{\mathrm{i}\lambda}{\pi w^2(z)} \\ q(z) = z + iz_0 \end{cases}$$



ABCD MATRICES FOR GAUSSIAN BEAMS

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$$\begin{cases} \dfrac{1}{q(z)} = \dfrac{1}{\mathrm{R(z)}} - \dfrac{\mathrm{i}\lambda}{\pi w^2(z)} \\ q(z) = z + i z_0 \end{cases}$$

- ABCD matrices are same as for geometric rays
- BUT … different interpretation parameter $\begin{bmatrix} q_n \\ 1 \end{bmatrix}$

ZEMAX Practical session