PHOT 451: Microscale optical system design
LECTURE 07

Michaël Barbier, Fall semester (2025-2026)

OVERVIEW OF THE COURSE

week	Topic
Week 1	Introduction to micro-scale optical components
Week 2	Light propagation in free space
Week 3	Geometric optics and raycasting
Week 4	Diffraction limit & Abberations
Week 5	**Quiz** + Beam propagation
Week 6	Refractive optical elements Microlenses
Week 7	Blazed Fresnel lenses
Week 8	Digital lenses
Week 9	Diffractive optical elements
Week 10	**Quiz** + Wave guides and beam propagation
Week 11	Wave mixing
Week 12	Gratings, periodic structures
Week 13	photonic crystals
Week 14	Whole optical system optimization

Methods of Light Propagation

Some insight in different models

SUMMARY LIGHT PROPAGATION

- Full wave electromagnetic field

- Wave equation: Amplitude and Phase
  - Near field effects

- Paraxial approximation
  - Geometrical Optics: spherical lenses
  - Paraxial Helmholtz equation & Beam solution

- Geometrical optics
  - short wavelengths
  - Optical design dimensions larger than wavelength

SUMMARY LIGHT PROPAGATION

- Full wave electromagnetic field

- Wave equation: Amplitude and Phase
  - Near field effects

- Paraxial approximation
  - **Geometrical Optics**: spherical lenses
  - **Paraxial Helmholtz equation** & **Beam solution**

- Geometrical optics
  - short wavelengths
  - Optical design dimensions larger than wavelength

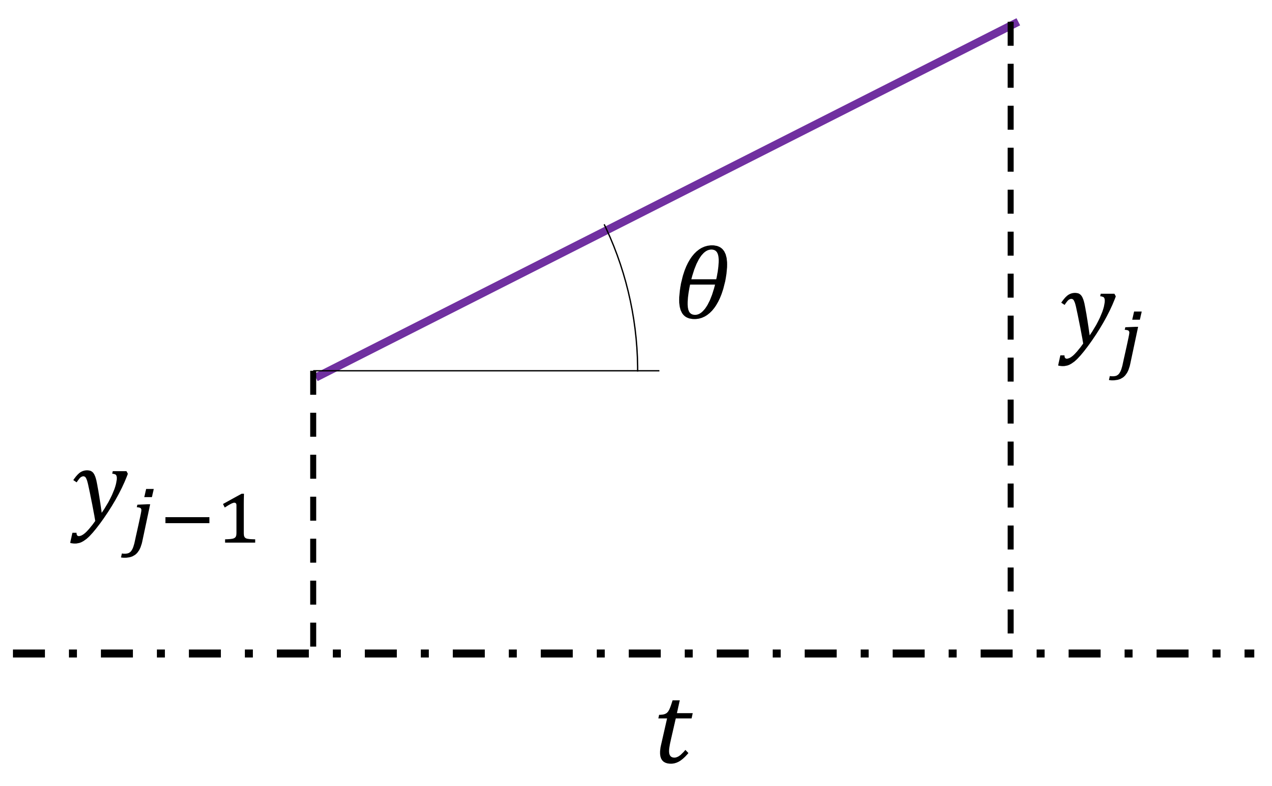
ABCD Matrices

Propagation of light through “thick” lenses BUT paraxial approx.

RAY TRACING

Conditions

- Circularly symmetric around optical axis.
- Ray are **Paraxial** & **Meridional**



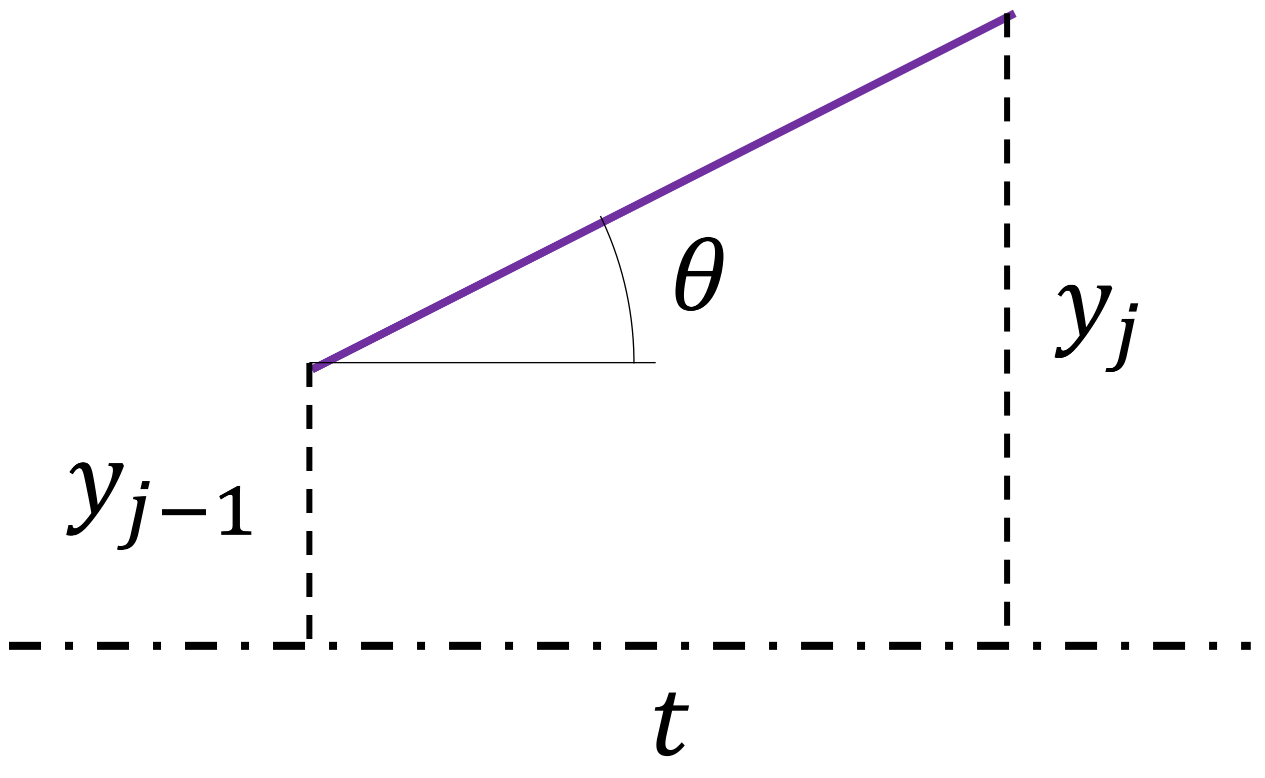
Rays are defined by:

- Angle θ
- Height from optical axis y

$$y_j = y_{j-1} + \theta\ t$$

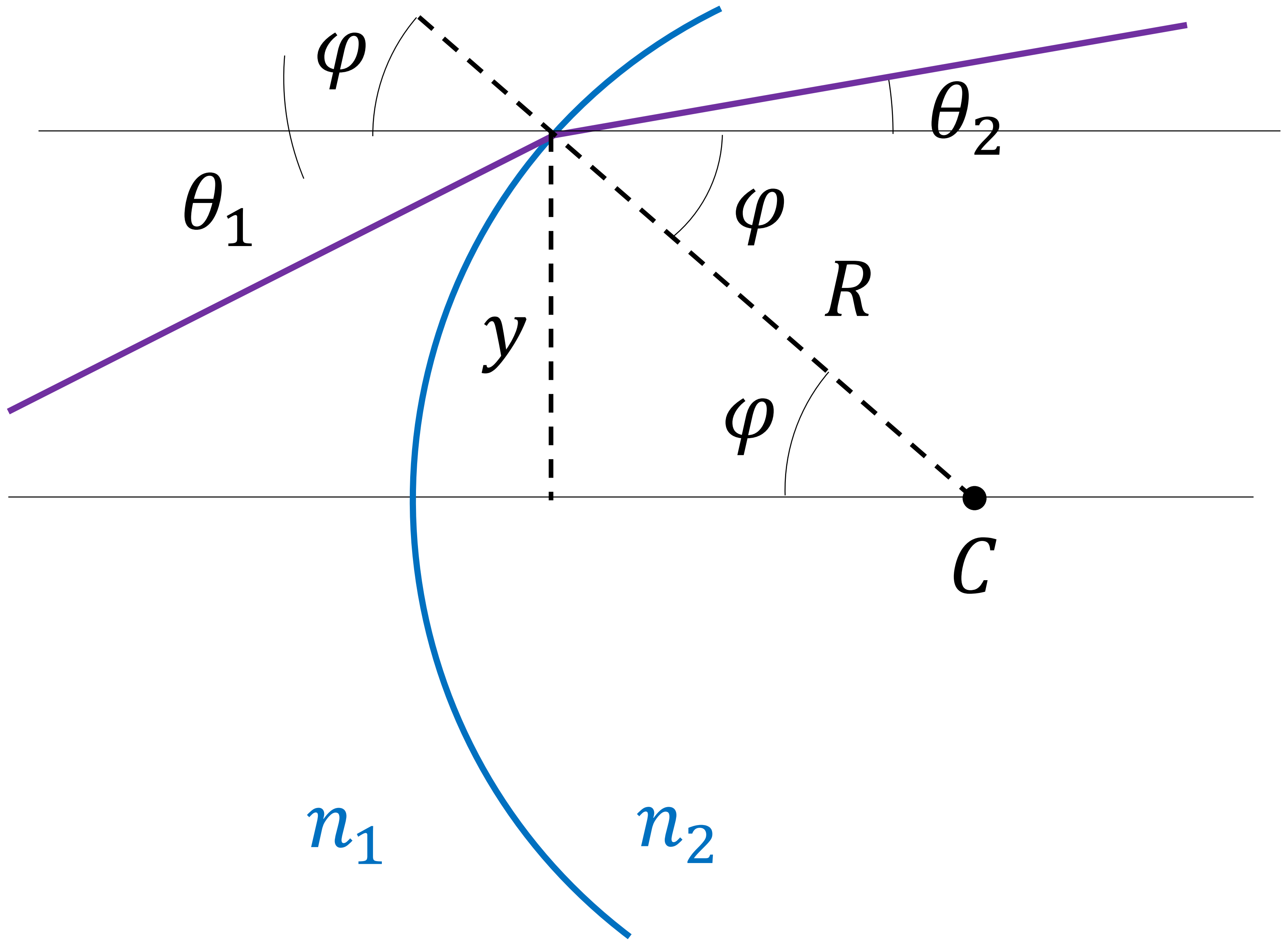
RAY TRACING FOR SINGLE SURFACE

- Free space: straight propagation
- At surfaces: change direction



Snell's law:

$$n_1 \sin(\varphi + \theta_1) = n_2 \sin(\varphi + \theta_2)$$

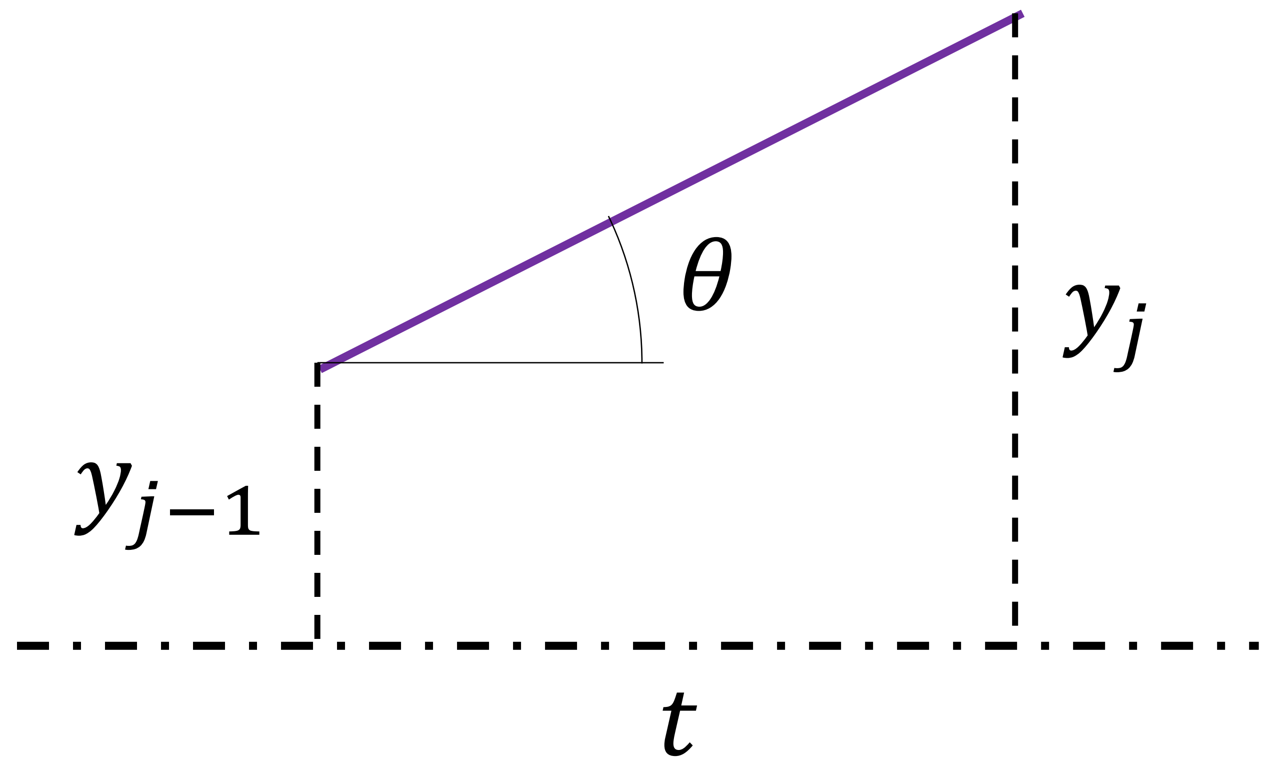


Paraxial approx: $\sin(\alpha) \approx \alpha$

$$n_1(\varphi + \theta_1) = n_2(\varphi + \theta_2)$$

RAY TRACING: THE ABCD-MATRIX

- Free space: straight propagation
- At surfaces: change direction

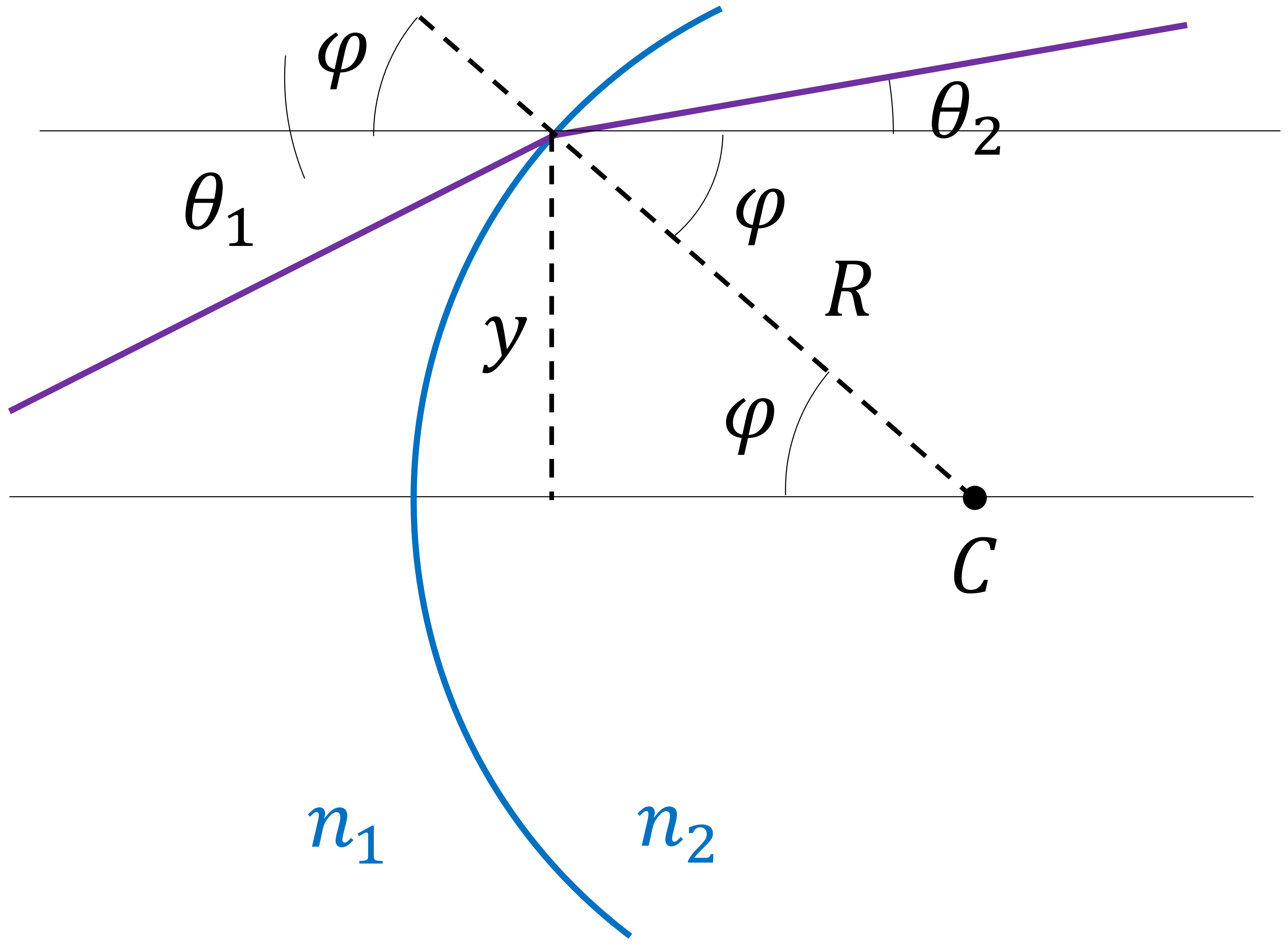


- **Relation ray heights and angles**

$$\begin{cases} y_2 = A\,y_1 + B\,\theta_1 \\ \theta_2 = C\,y_1 + D\,\theta_1 \end{cases}$$

OR

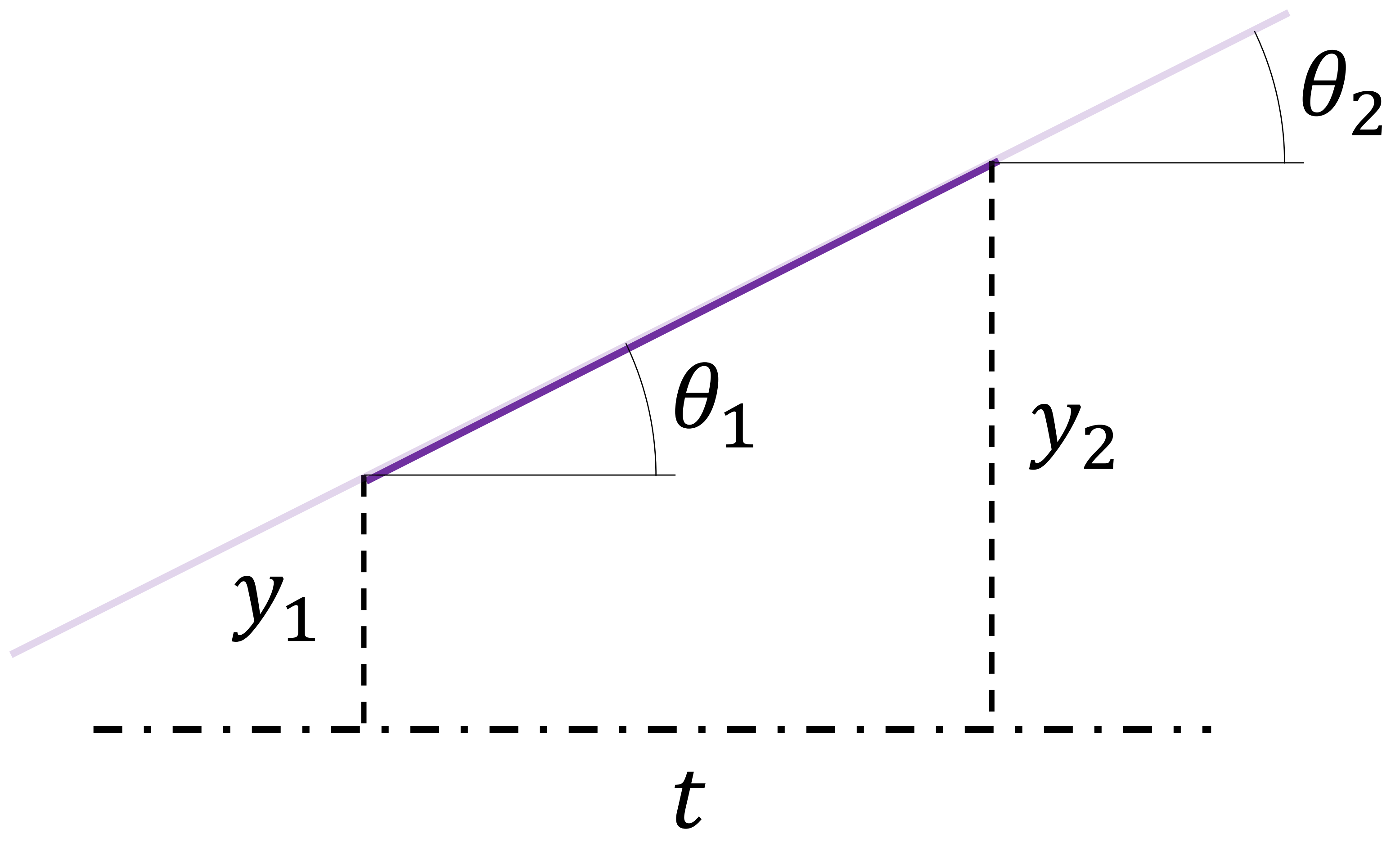
$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



ABCD-MATRIX: FREE SPACE

What are A, B, C, D?

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



In free space:

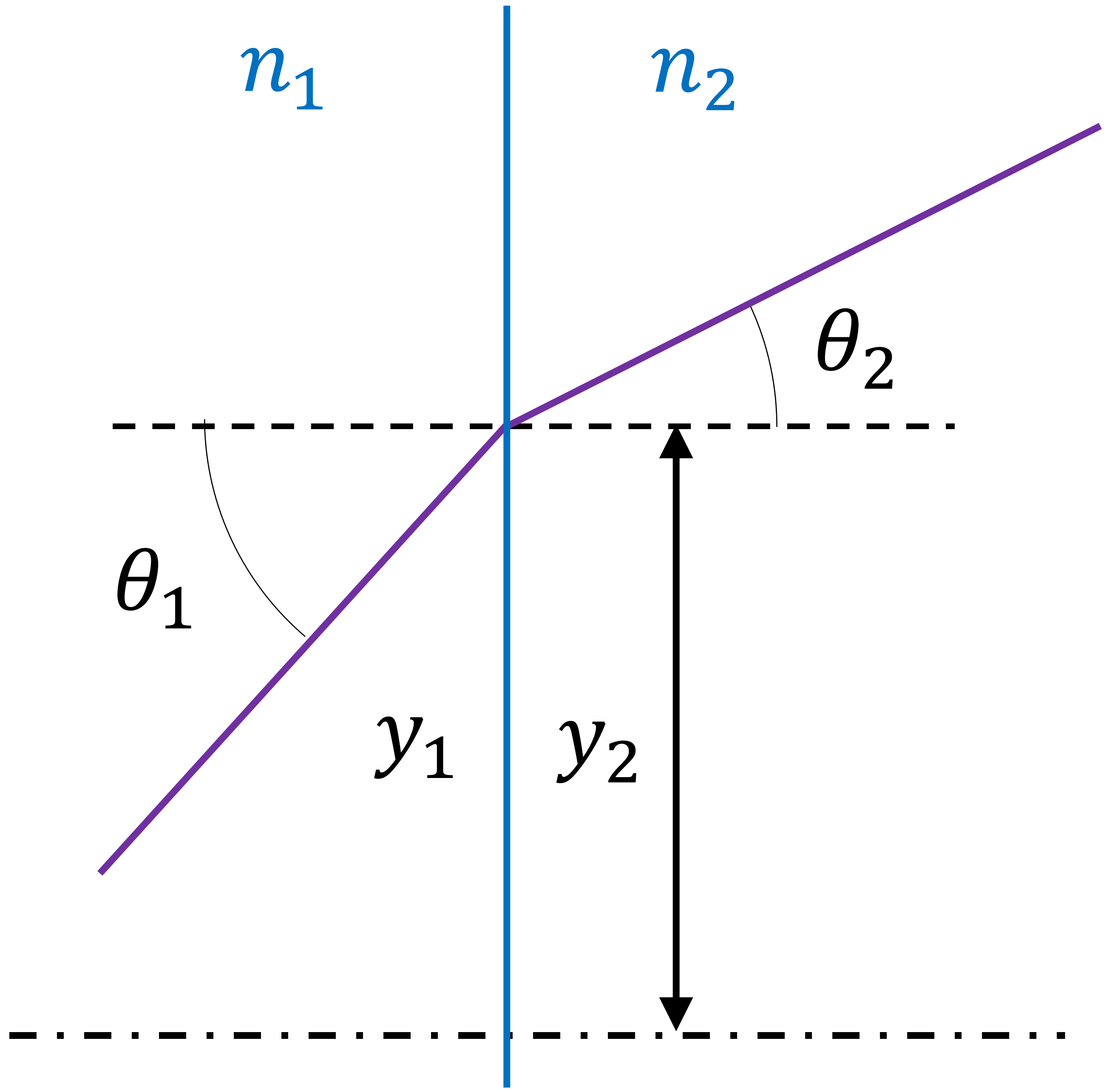
$$\begin{cases} y_2 = y_1 + t\theta_1 \\ \theta_2 = \theta_1 \end{cases}$$

$$\Rightarrow \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

ABCD-MATRIX: PLANAR SURFACE

What are A, B, C, D?

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



Snell's law (**paraxial** approx.)

$$n_2\theta_2 = n_1\theta_1$$

$$\begin{cases} y_2 = y_1 \\ \theta_2 = \dfrac{n_1}{n_2}\theta_1 \end{cases}$$

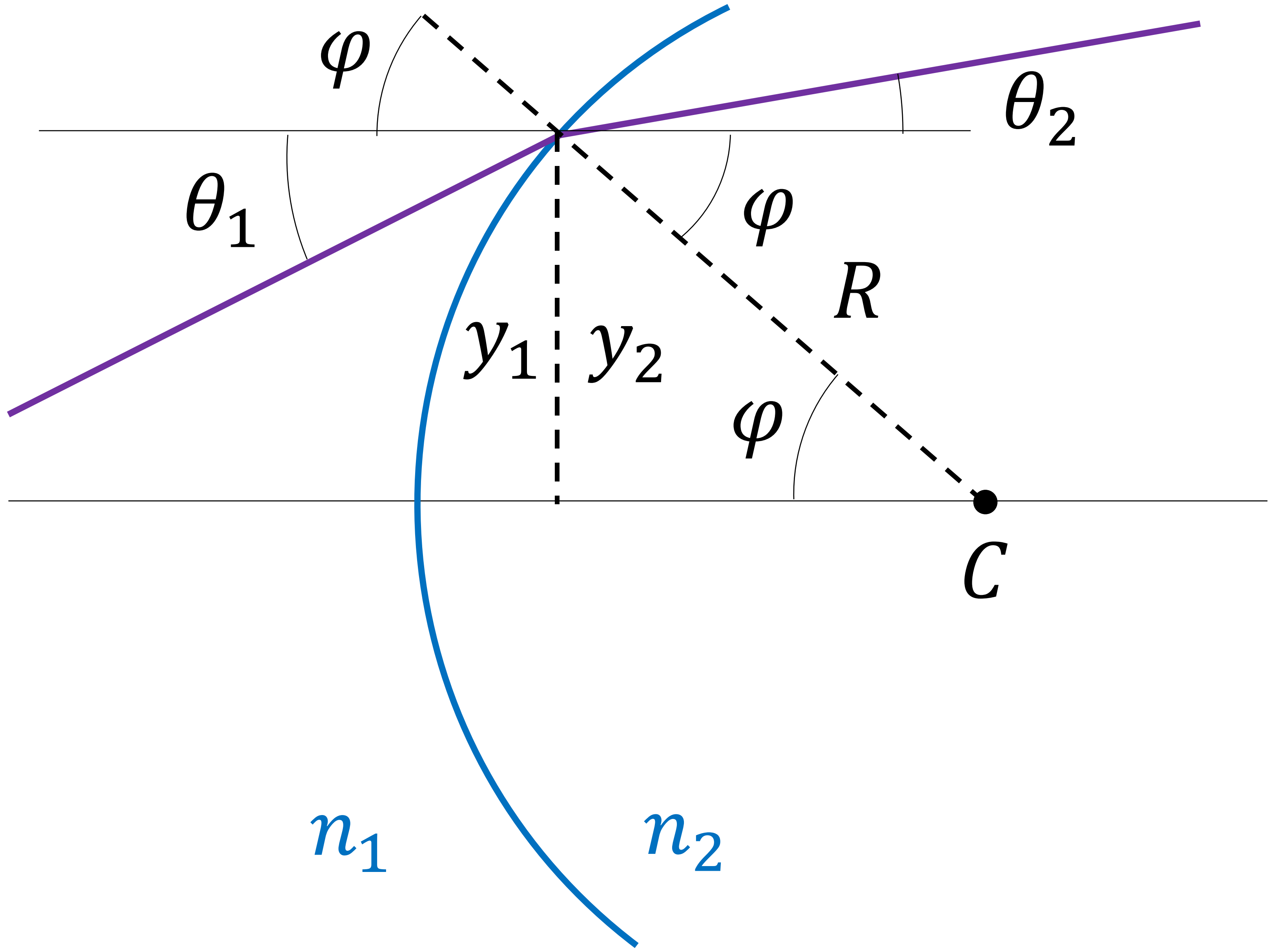
$\Rightarrow$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

ABCD-MATRIX: SPHERICAL SURFACE

What are A, B, C, D?

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



Snell's law & **paraxial** approx.

$$n_1(\varphi + \theta_1) = n_2(\varphi + \theta_2)$$

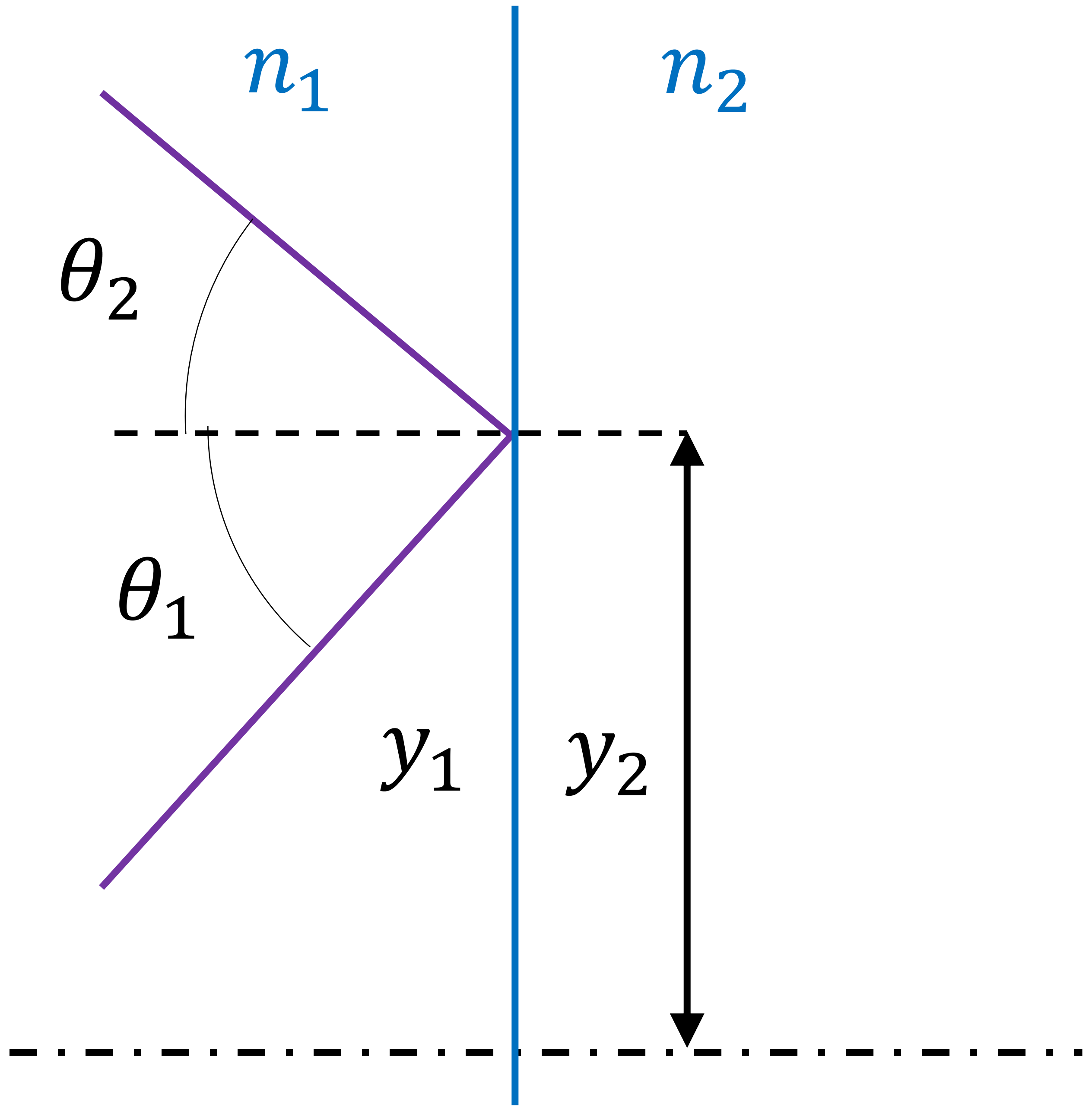
$$\varphi \approx \frac{y_1}{R}$$

$$\Rightarrow \quad \theta_2 = \frac{-(n_2 - n_1)}{n_2}\frac{y_1}{R} + \frac{n_1}{n_2}\theta_1 \quad \Rightarrow \quad \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

ABCD-MATRIX: REFLECTION ON A PLANAR SURFACE

What are A, B, C, D?

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



Angle changes sign $\theta_2 = -\theta_1$?

but **z-axis also flips**

$$\begin{cases} y_2 = y_1 \\ \theta_2 = \theta_1 \end{cases} \quad \Rightarrow \quad \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

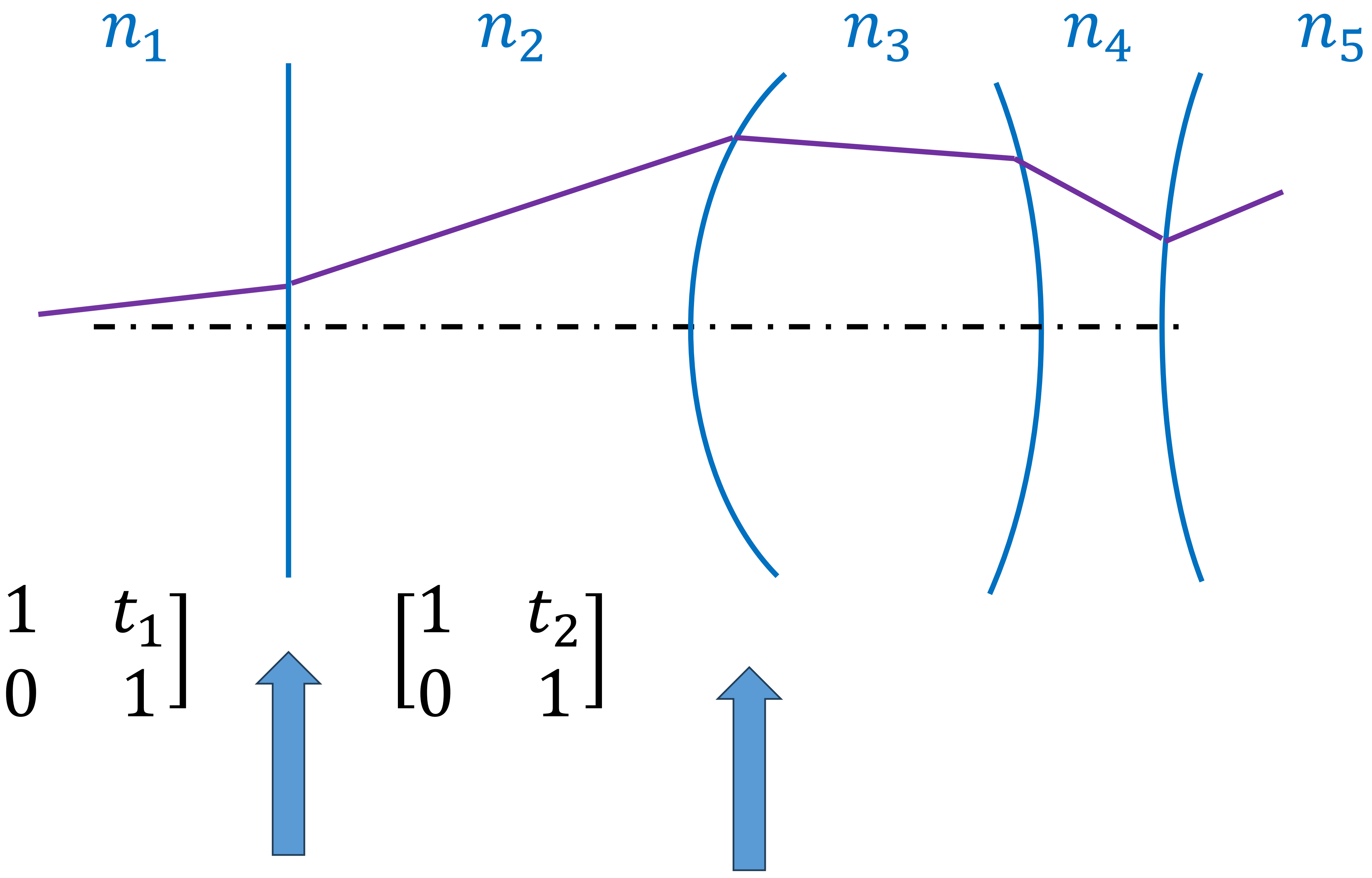
ABCD-MATRICES: PROPAGATION

ABCD-matrix for single surface:

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = M_1 \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

System with multiple surfaces:

$$\begin{bmatrix} y_{N+1} \\ \theta_{N+1} \end{bmatrix} = M_N \ldots M_2 M_1 \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & t_1 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & t_2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ \frac{-(n_3 - n_2)}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix}$$

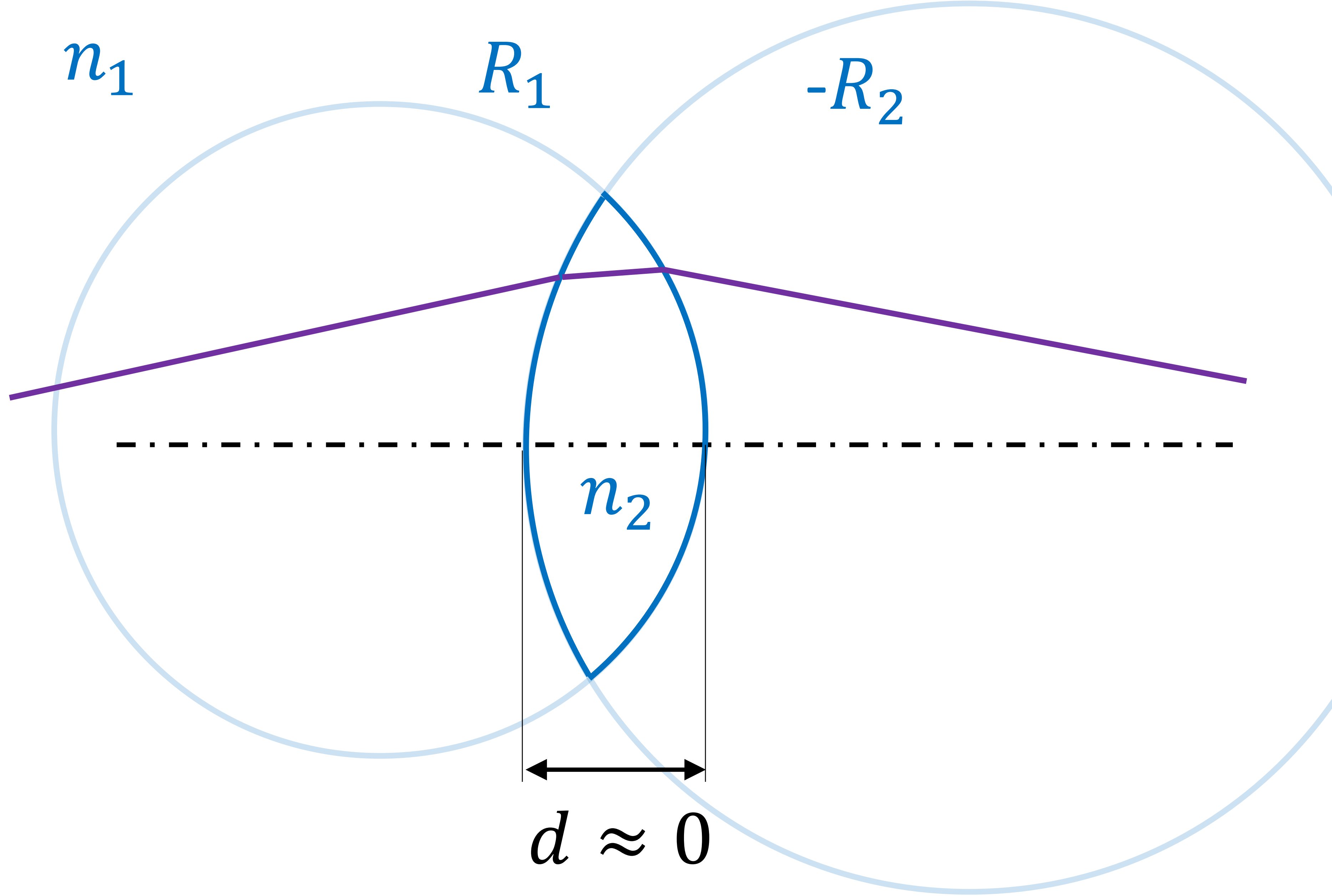
ABCD-MATRICES: THIN LENS

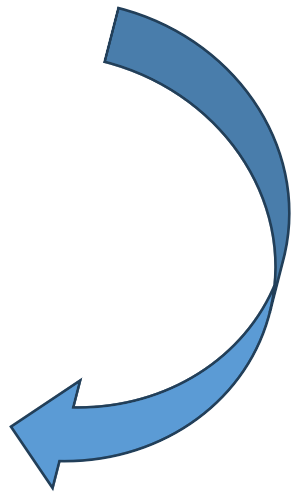
Two curved surfaces:

$$M = \begin{bmatrix} 1 & 0 \\ \frac{-(n_1 - n_2)}{n_1 R_2} & \frac{n_2}{n_1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{-(n_2 - n_1)}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{-(n_2-n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$





Lensmaker's formula

$$\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

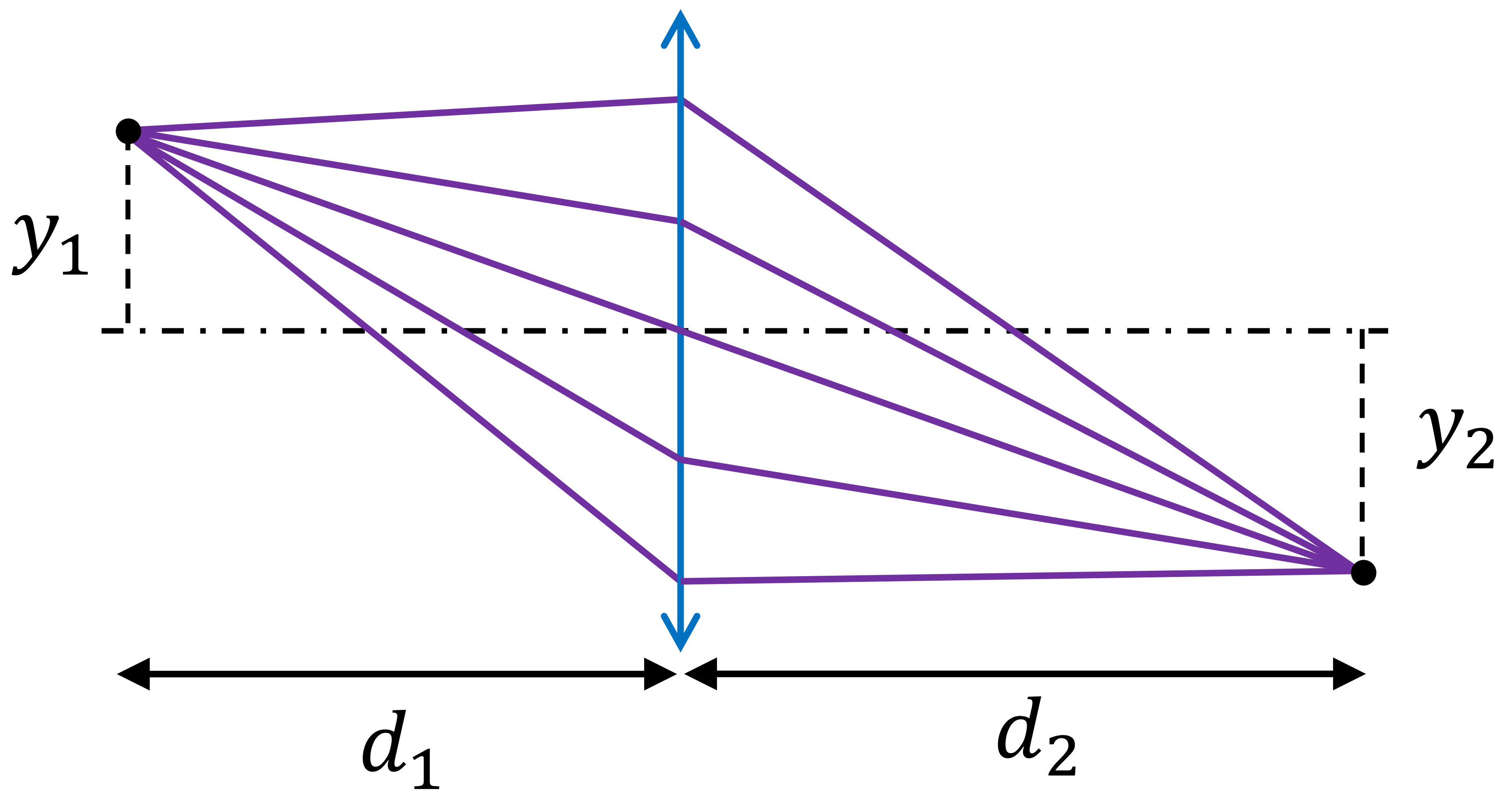
ABCD-MATRICES: THIN LENS IMAGING

Free space 2 — Thin lens — Free space 1

$$M = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{d_2}{f} & d_1 + d_2 - \frac{d_1 d_2}{f} \\ -\frac{1}{f} & 1 - \frac{d_1}{f} \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \frac{d_2}{f} & 0 \\ -\frac{1}{f} & 1 - \frac{d_1}{f} \end{bmatrix}$$



Thin lens condition

$$\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2}$$

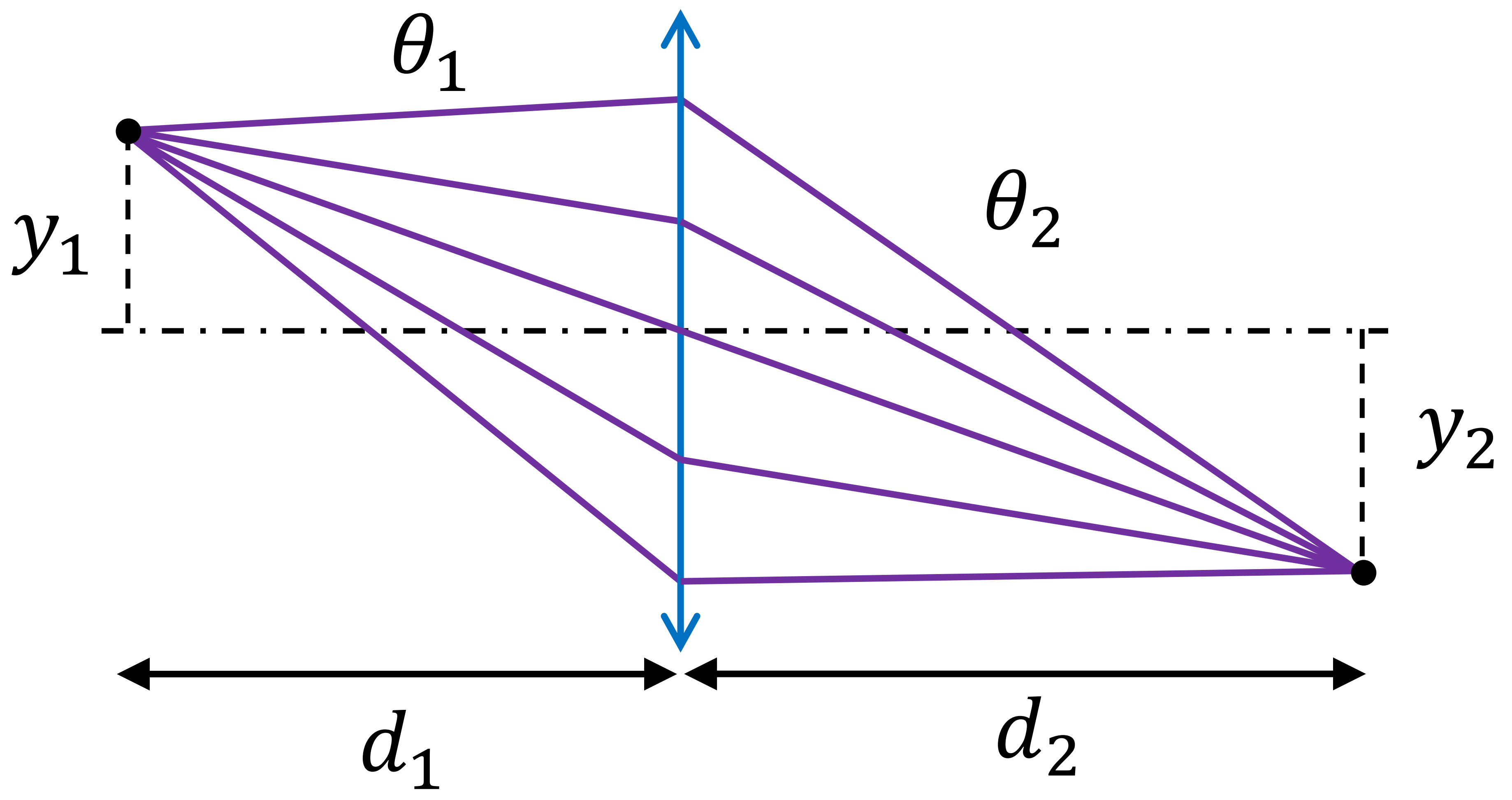
ABCD-MATRICES: THIN LENS IMAGING

Relation ray angles & heigths:

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 - \frac{d_2}{f} & 0 \\ -\frac{1}{f} & 1 - \frac{d_1}{f} \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

Ray height independent of angle

$$y_2 = y_1 \left(1 - \frac{d_2}{f} \right)$$



ABCD-MATRICES: THIN LENS IMAGING

Relation ray angles & heigths:

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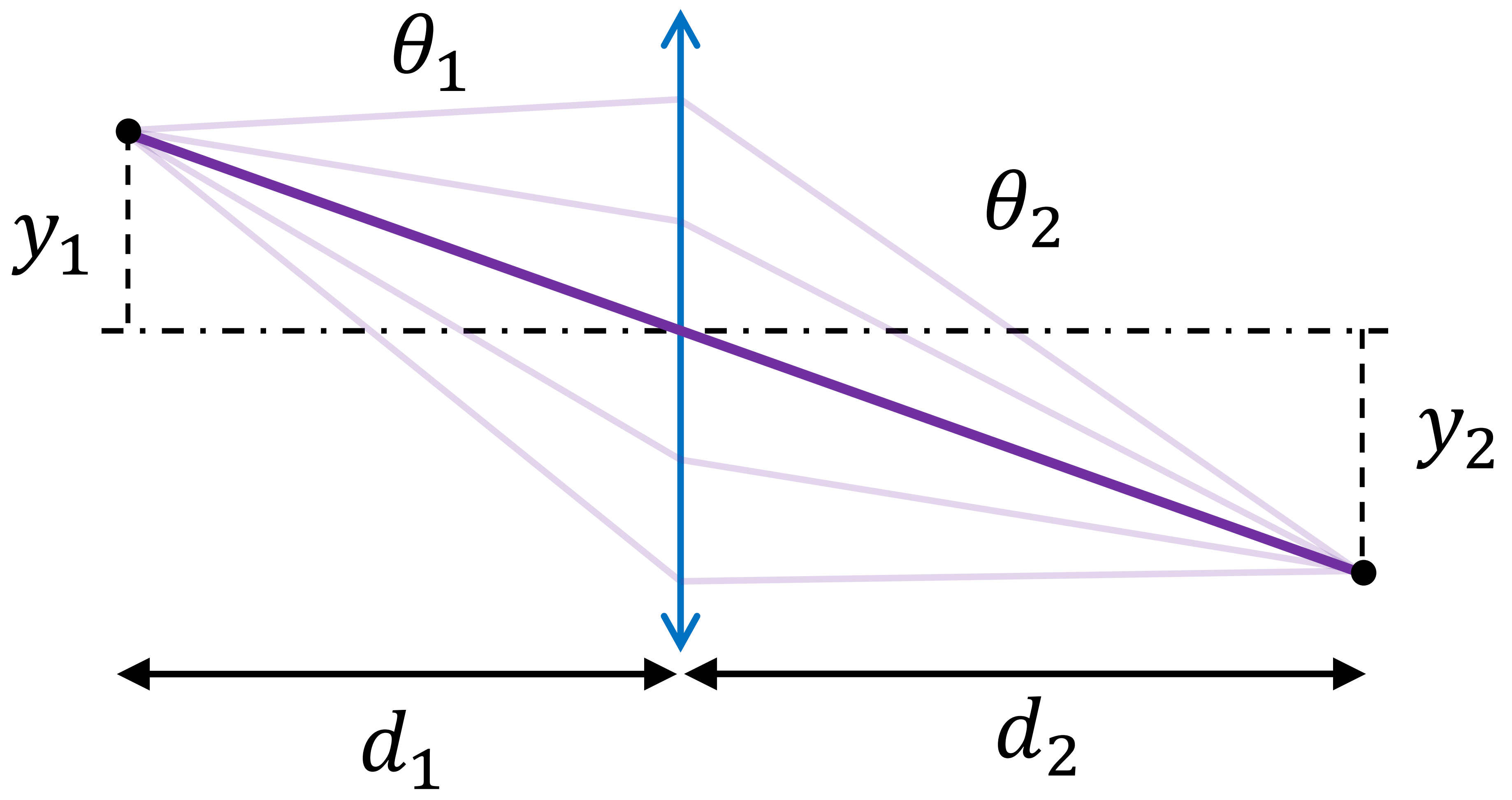
Ray height independent of angle

$$y_2 = y_1 \left(1 - \frac{d_2}{f}\right)$$



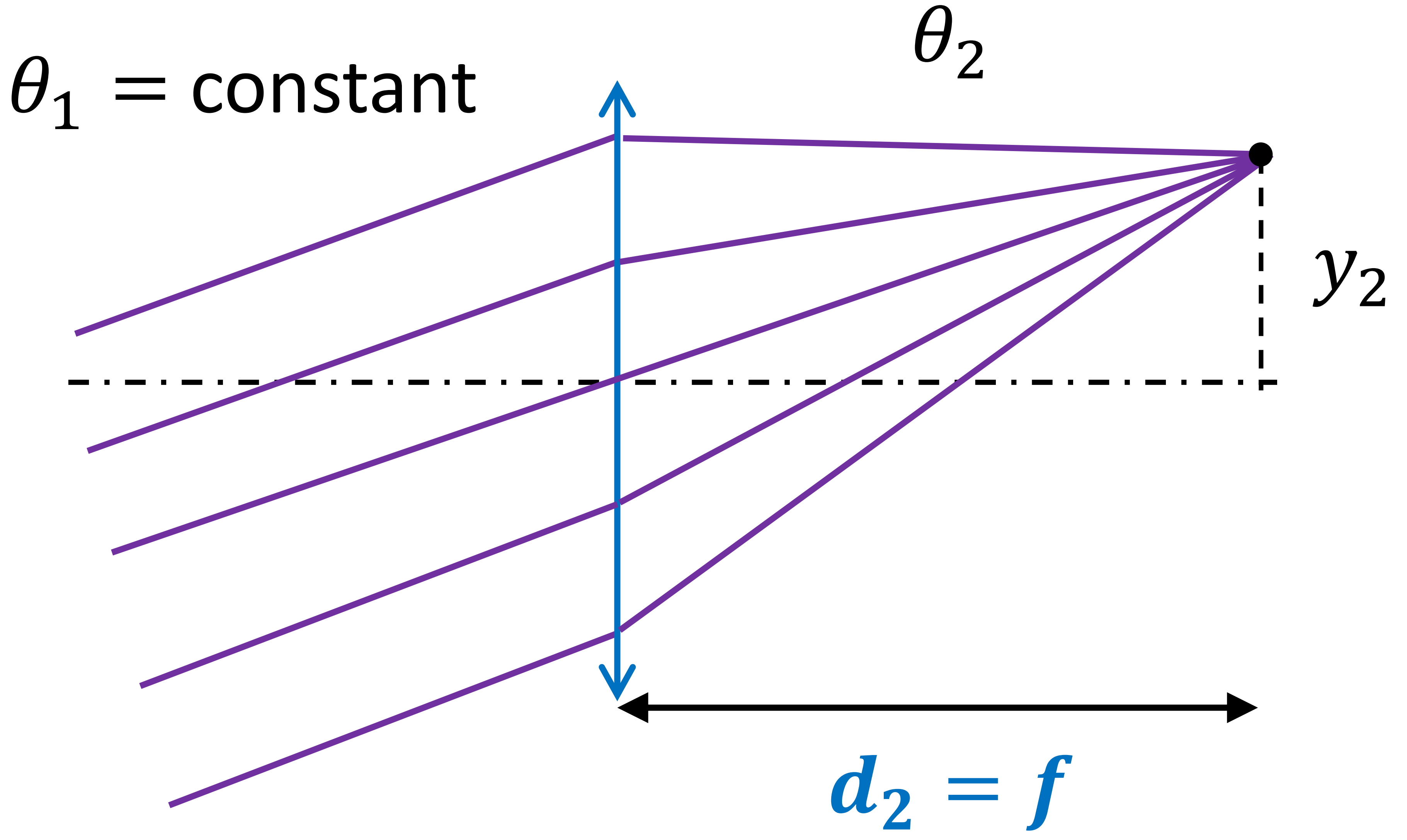
Ray through center: relation between heights and distances

$$\frac{y_2}{d_2} = \frac{y_1}{d_1}$$

ABCD-MATRICES: THIN LENS IMAGING

Relation ray angles & heights:

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \overset{0}{1-\frac{d_2}{f}} & \overset{d_2}{d_1+d_2-\frac{d_1 d_2}{f}} \\ -\frac{1}{f} & 1-\frac{d_1}{f} \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



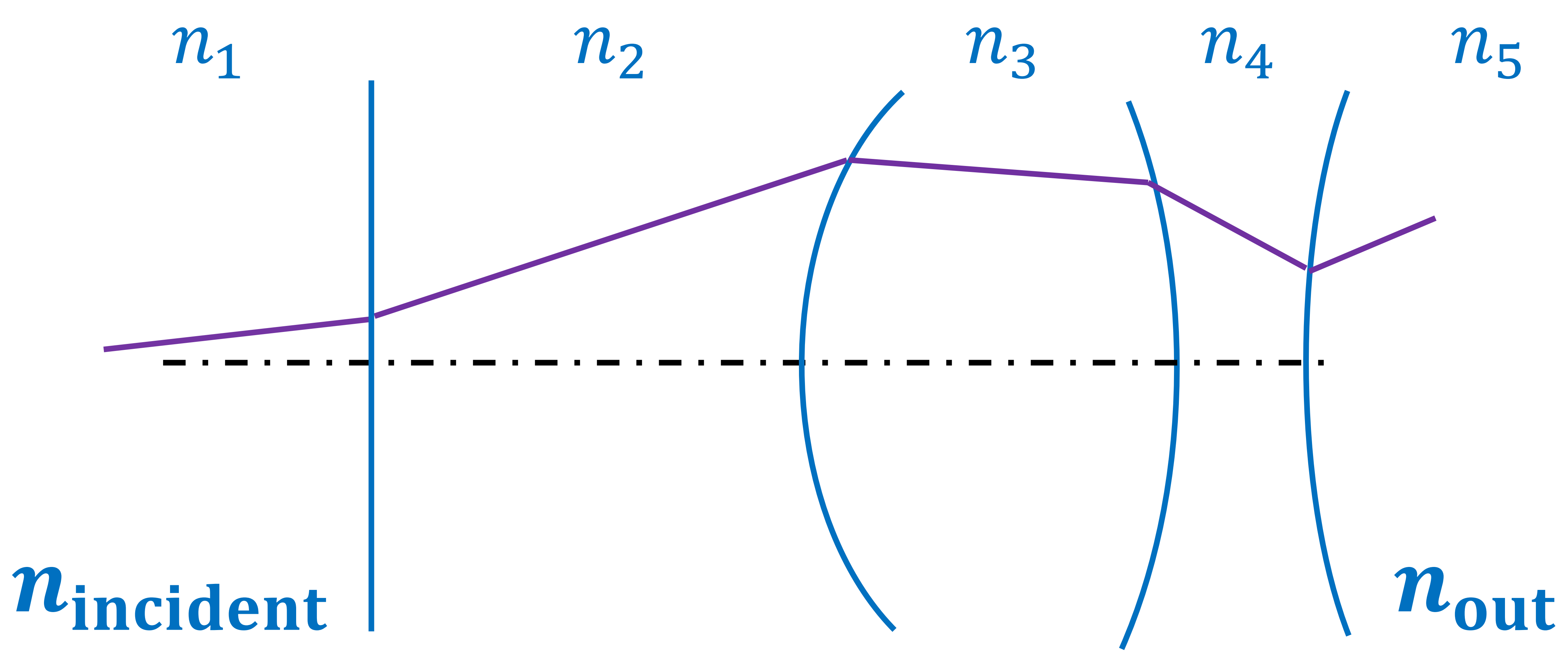
Ray height y_2 independent of height y_1

$$y_2 = B\,\theta_1 = d_2\theta_1$$

THE LAGRANGE INVARIANT

System multiple surfaces

$$\begin{bmatrix} y_{N+1} \\ \theta_{N+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



ABCD-matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M = M_N \dots M_2 M_1$$

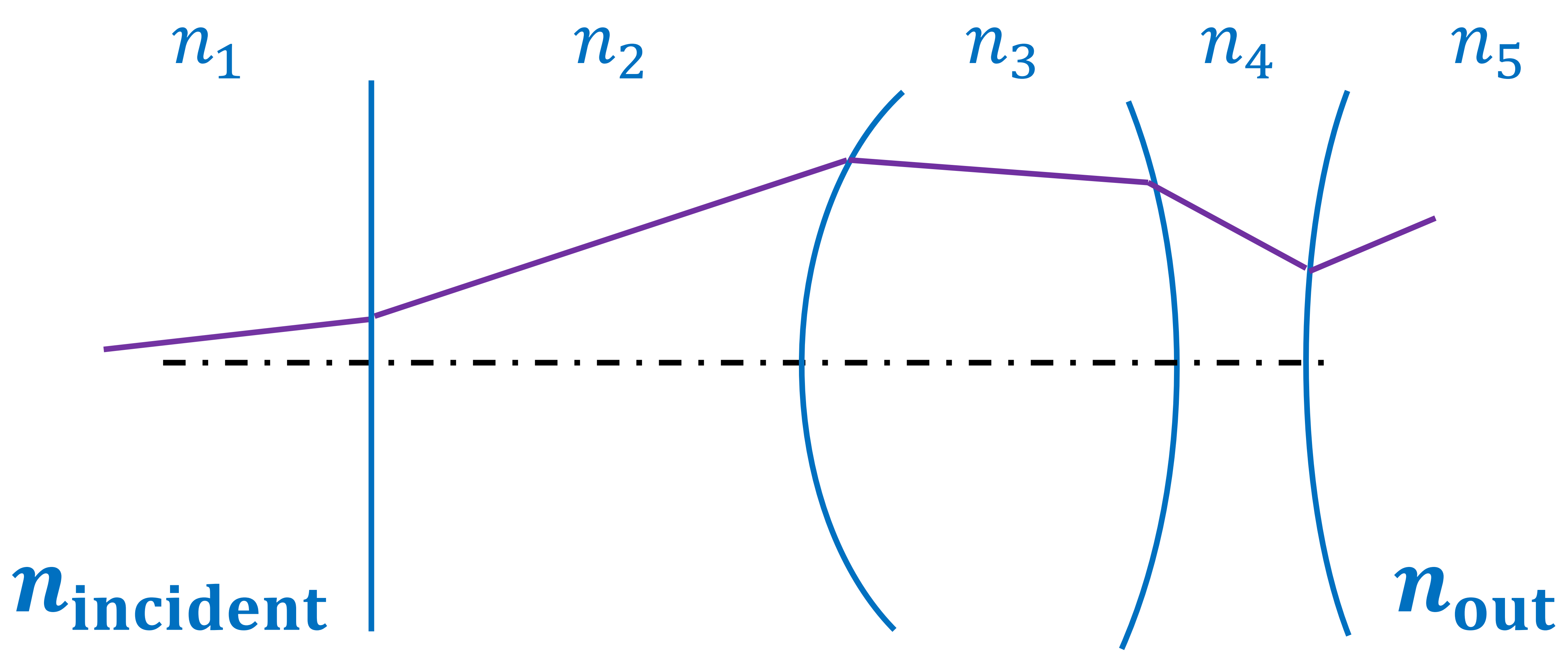
Constant:

$$\det(M) = \frac{n_{\text{incident}}}{n_{\text{out}}}$$

THE LAGRANGE INVARIANT

System multiple surfaces

$$\begin{bmatrix} y_{N+1} \\ \theta_{N+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$



ABCD-matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M = M_N \dots M_2 M_1$$

$$\det(M) = \det(M_N) \cdot \dots \cdot \det(M_2) \cdot \det(M_1)$$

Constant:

$$\det(M) = \frac{n_{\text{incident}}}{n_{\text{out}}}$$

THE LAGRANGE INVARIANT

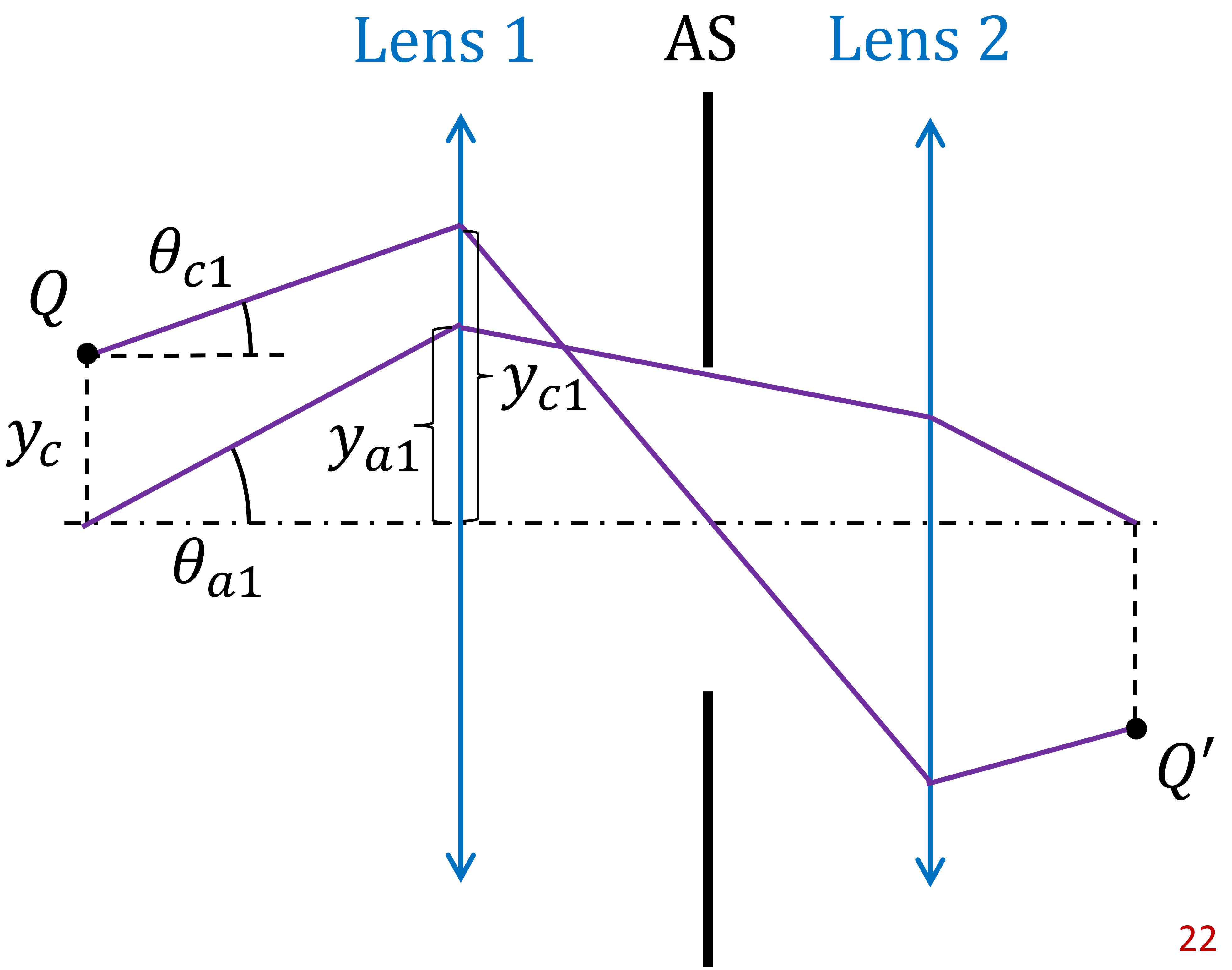
Langrange invariant: $\mathrm{LI} = y_{c1}\theta_{a1} - y_{a1}\theta_{c1}$ is a constant

Two independent rays:

- Chief ray (y_{c1}, θ_{c1})
- Axial ray (y_{a1}, θ_{a1})

Both rays obey the ABCD-matrix

$$\begin{bmatrix} y_{N+1} \\ \theta_{N+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

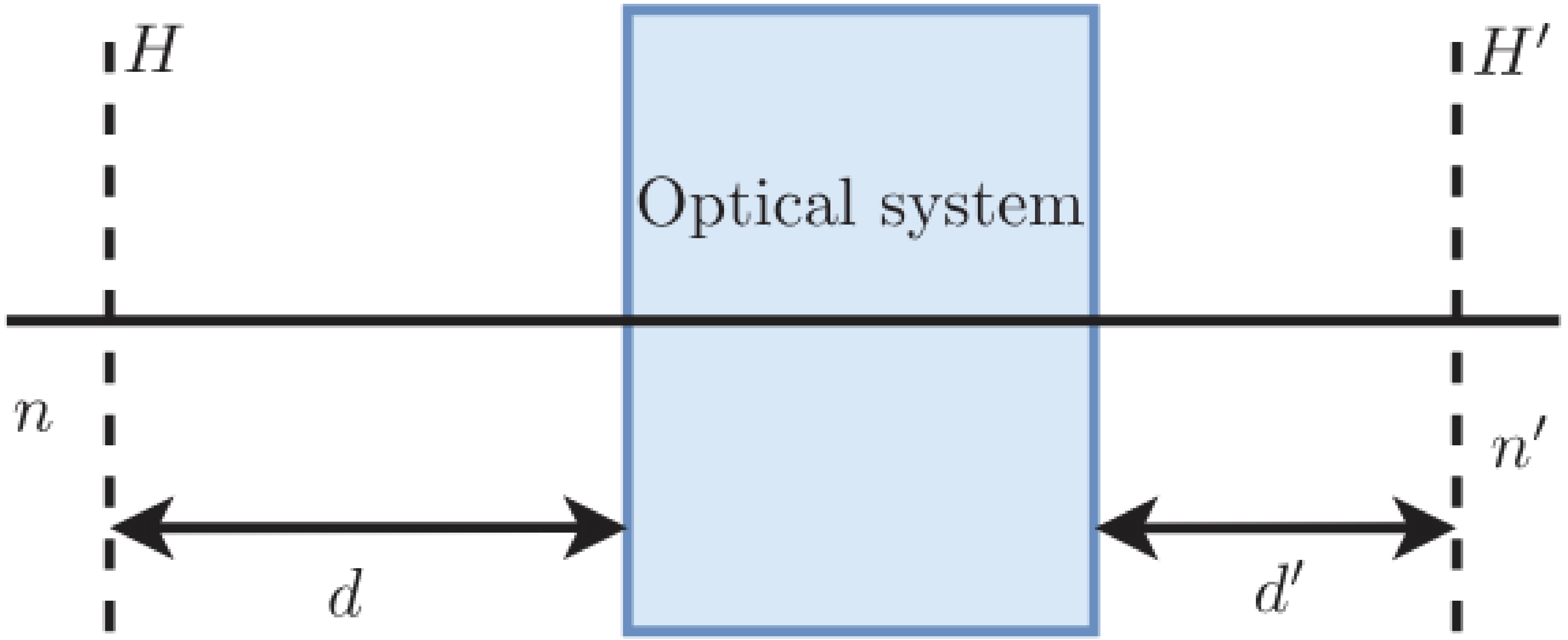


OPTICAL SYSTEM AND PRINCIPAL PLANES

- Matrix calculations less insight

- Simplifying ABCD-matrix

$$\begin{bmatrix} y' \\ \theta' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix}$$



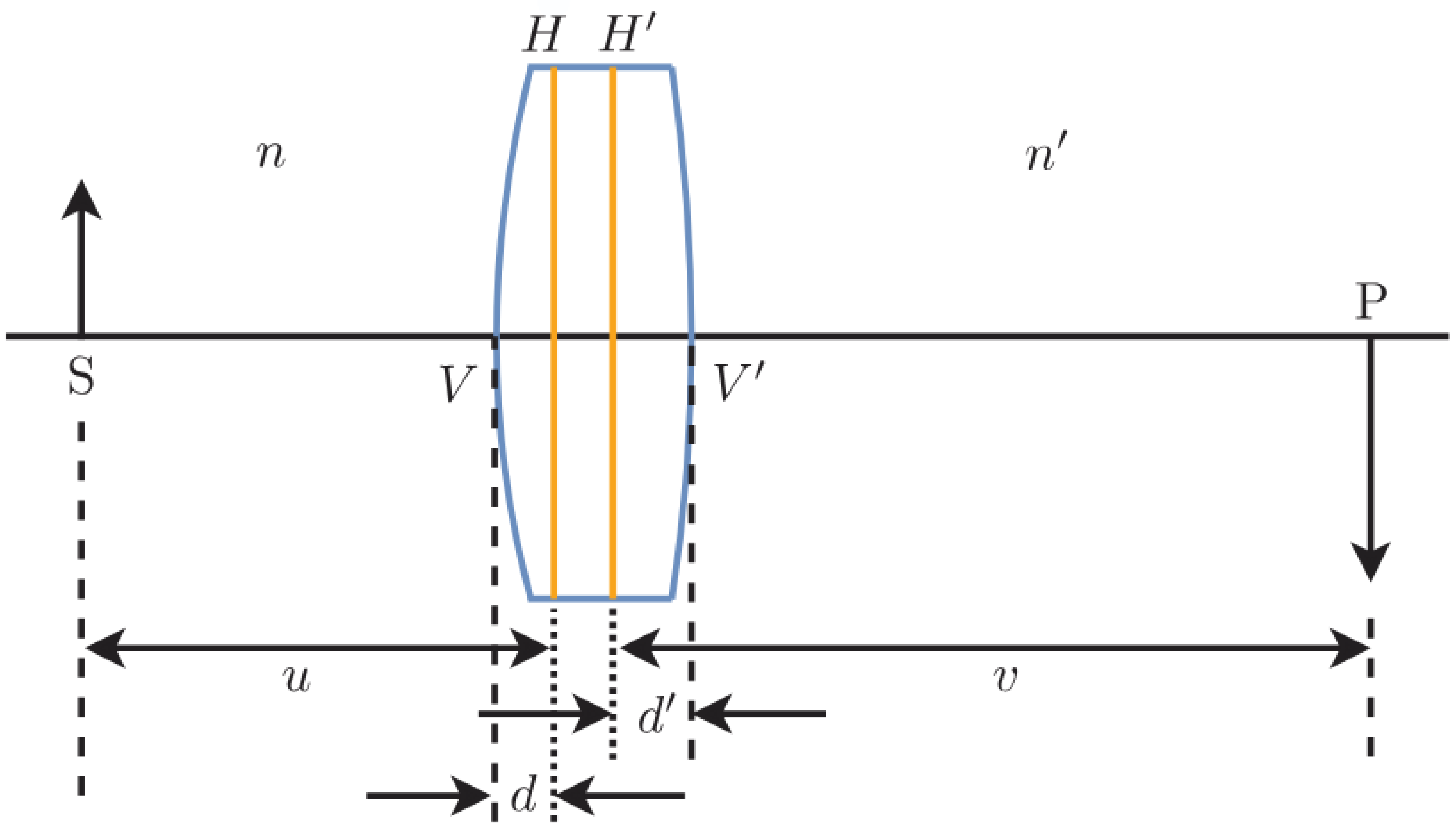
$$M_{HH'} = \begin{bmatrix} 1 & d' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

OPTICAL SYSTEM AND PRINCIPAL PLANES

- Matrix calculations less insight

- Simplifying ABCD-matrix

$$\begin{bmatrix} y' \\ \theta' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix}$$

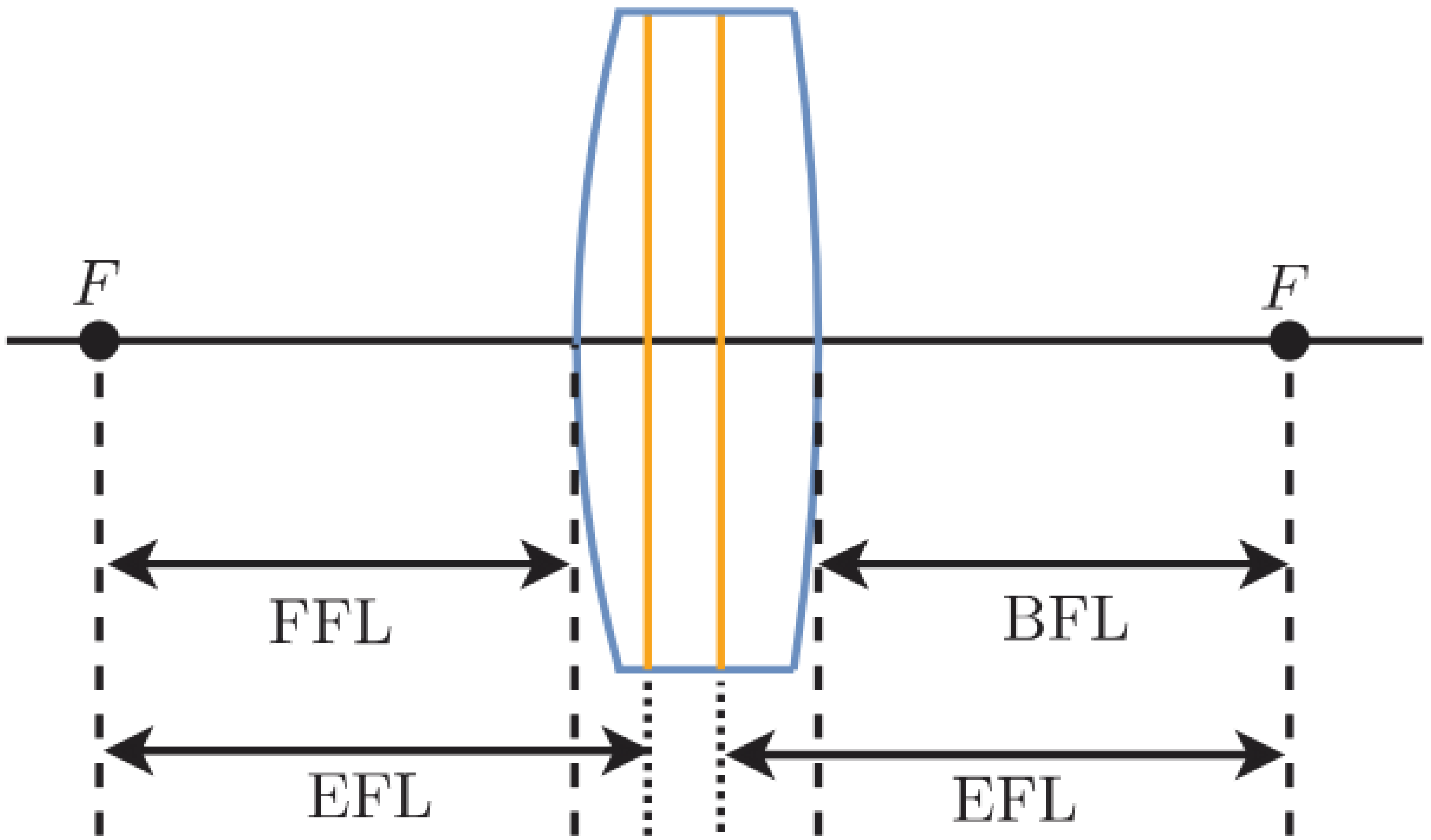


$$\begin{bmatrix} A & 0 \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & v \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{P}{n'} & \frac{n}{n'} \end{bmatrix} \begin{bmatrix} 1 & u \\ 0 & 1 \end{bmatrix}$$

PRINCIPAL PLANES

ABCD-matrix

$$\begin{bmatrix} y' \\ \theta' \end{bmatrix} = \begin{bmatrix} 1 - \frac{vP}{n'} & u - \frac{uvP}{n'} + \frac{vn}{n'} \\ -\frac{P}{n'} & -\frac{uP}{n'} + \frac{n}{n'} \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix}$$



- Imaging system:

$$B = u - \frac{uvP}{n'} + \frac{vn}{n'} = 0 \Rightarrow \boldsymbol{\frac{n}{u} + \frac{n'}{v} = P}$$

- BFL: Back Focal Length
- FFL: Front Focal Length
- EFL: Effective Focal Length

Practical Calculations

Matrix multiplications, determinants, ABCD matrices, Principal Planes

ZEMAX Practical session