PHOT 451: Microscale optical system design
LECTURE 02

Michaël Barbier, Fall semester (2025-2026)

OVERVIEW OF THE COURSE

week	Topic
Week 1	Introduction to micro-scale optical components
Week 2	Light propagation in free space
Week 3	Geometric optics and raycasting
Week 4	Diffraction limit & Abberations
Week 5	**Quiz** + Beam propagation
Week 6	Refractive optical elements Microlenses
Week 7	Blazed Fresnel lenses
Week 8	Digital lenses
Week 9	Diffractive optical elements
Week 10	**Quiz** + Wave guides and beam propagation
Week 11	Wave mixing
Week 12	Gratings, periodic structures
Week 13	photonic crystals
Week 14	Whole optical system optimization

Light as electromagnetic waves

WAVES & MAXWELL'S EQUATIONS

- Maxwell's equations

  for light in vacuum

- Vector fields $\boldsymbol{E}(\boldsymbol{r}, t)$ and $\boldsymbol{B}(\boldsymbol{r}, t)$

- Charge $\rho(\boldsymbol{r}, t)$ and current $\boldsymbol{j}(\boldsymbol{r}, t)$ density

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \boldsymbol{B} = \varepsilon_0 \mu_0 \frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{j}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

WAVES & MAXWELL'S EQUATIONS

- Maxwell's equations

  for light in vacuum

- Vector fields $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$

- Charge $\rho(\boldsymbol{r},t)$ and current $\boldsymbol{j}(\boldsymbol{r},t)$ density

Electric field $\boldsymbol{E}$

Magnetic induction $\boldsymbol{B}$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

charge ρ (density)

$$\nabla \times \boldsymbol{B} = \varepsilon_0 \mu_0 \frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{j}$$

current $\boldsymbol{j}$ (density)

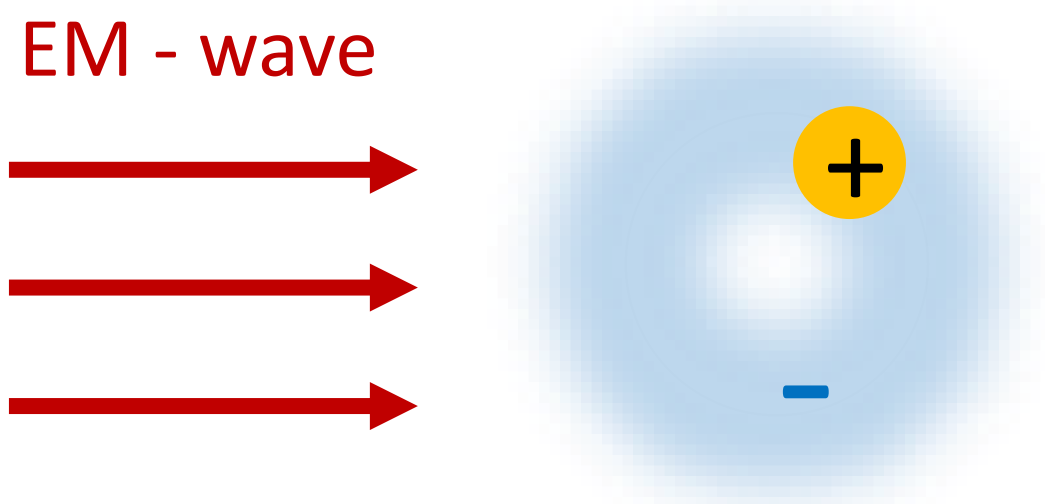
$$\nabla \cdot \boldsymbol{B} = 0$$

$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{\mathrm{A\,s}}{\mathrm{V\,m}}$ permittivity in vacuum

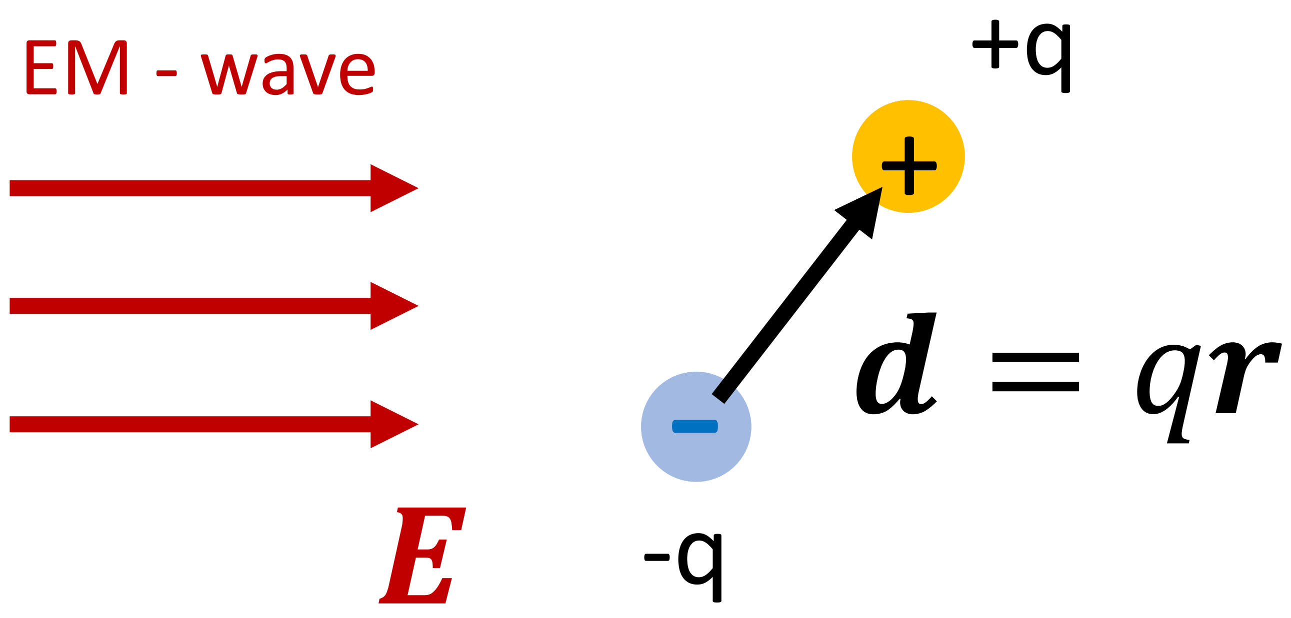
$\mu_0 = 12.6 \times 10^{-7}\ \frac{\mathrm{A\,s}}{\mathrm{V\,m}}$ permeability in vacuum

MAXWELL'S EQUATIONS IN A MEDIUM

- Electromagnetic wave interact with electron clouds



- Electric force stronger/faster than magnetic force
- Electric field oscillates:
  - Electrons react faster than nucleus
  - Dipole creation & polarization



Polarization $\boldsymbol{P}$

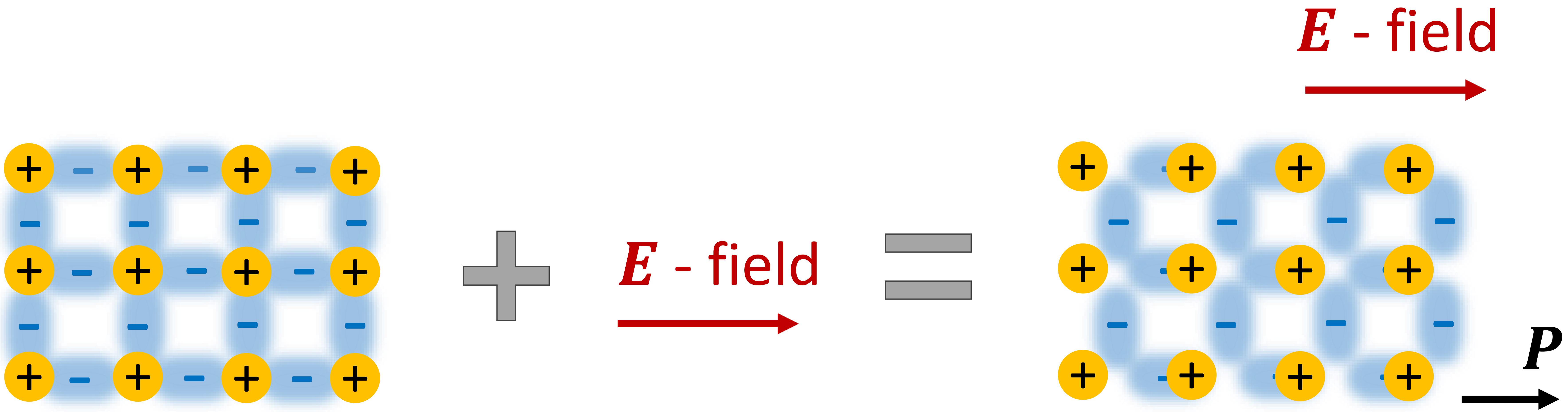
MAXWELL'S EQUATIONS IN A MEDIUM

- Polarization $\boldsymbol{P}$: Dipole moment per volume
- **Linear/isotropic** medium:

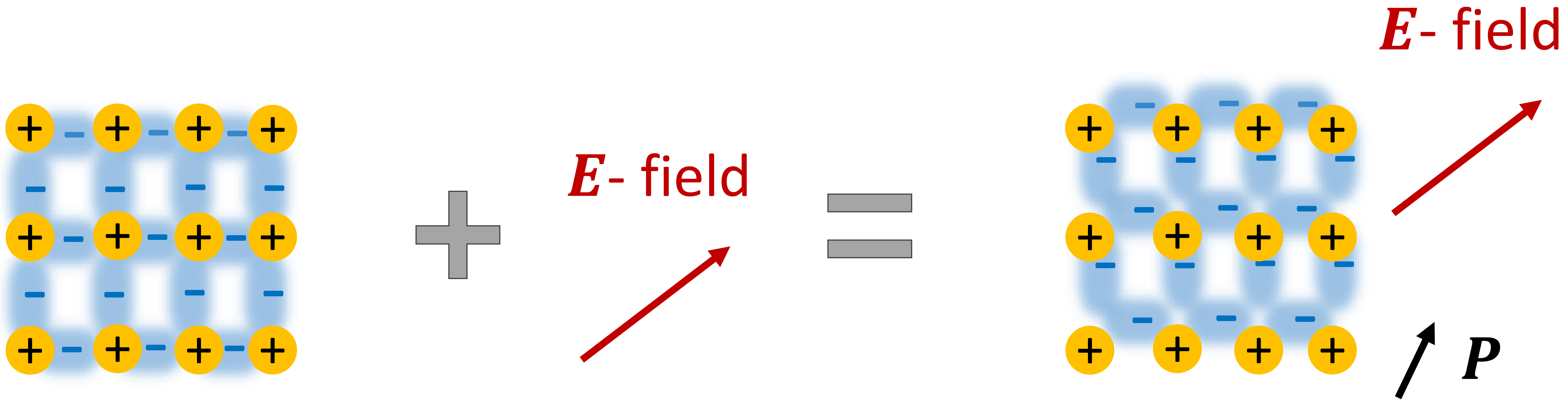
$$\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E} = \varepsilon_0 (\varepsilon_r - 1) \boldsymbol{E}$$

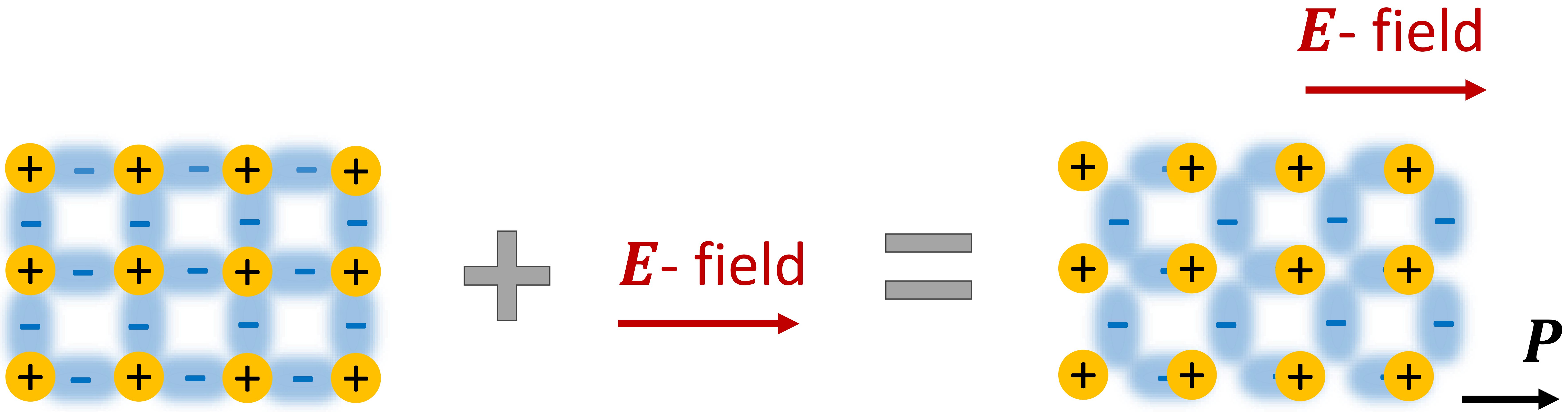
- Displacement $\boldsymbol{D}$:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_0 \varepsilon_r \boldsymbol{E} = \varepsilon \boldsymbol{E}$$



ANISOTROPIC MEDIA





WAVES & MAXWELL'S EQUATIONS

- Light in vacuum

- Light in a dielectric medium
  - Linear/isotropic medium
  - No Charges, no current

Electric field $\boldsymbol{E}$

Magnetic induction $\boldsymbol{B}$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon}$$

charge ρ

$$\nabla \times \boldsymbol{B} = \varepsilon\mu \frac{\partial \boldsymbol{E}}{\partial t} + \mu \boldsymbol{j}$$

current $\boldsymbol{j}$

$$\nabla \cdot \boldsymbol{B} = 0$$

Permittivity in medium $\varepsilon = \varepsilon_0 \varepsilon_r$

Permeability in medium $\mu = \mu_0 \mu_r$

WAVES & MAXWELL'S EQUATIONS

- Light in vacuum

- Light in a dielectric medium
  - Linear/isotropic medium
  - No Charges, no current

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = 0$$ ← charge $\rho = 0$

$$\nabla \times \boldsymbol{B} = \varepsilon\mu\frac{\partial \boldsymbol{E}}{\partial t}$$ ← current $\boldsymbol{j} = \boldsymbol{0}$

$$\nabla \cdot \boldsymbol{B} = 0$$

WAVES & MAXWELL'S EQUATIONS

- Light in vacuum

- Light in a dielectric medium
  - Linear/isotropic medium
  - No Charges, no current

Simplified notation

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}} \quad \longleftarrow \quad \dot{\boldsymbol{B}} \stackrel{\text{def}}{=} \frac{\partial B}{\partial t}$$

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{B} = \varepsilon\mu\dot{\boldsymbol{E}} \quad \longleftarrow \quad \dot{\boldsymbol{E}} \stackrel{\text{def}}{=} \frac{\partial E}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

THE WAVE EQUATIONS

$$\dot{\boldsymbol{B}} \stackrel{\text{def}}{=} \frac{\partial B}{\partial t}, \quad \dot{\boldsymbol{E}} \stackrel{\text{def}}{=} \frac{\partial E}{\partial t}$$

- Wave equations
- E and B “decoupled”
- Velocity depends on medium

$$\Delta \boldsymbol{E} \stackrel{\text{def}}{=} \nabla^2 \boldsymbol{E} = \frac{\partial^2 \boldsymbol{E}}{\partial x^2} + \frac{\partial^2 \boldsymbol{E}}{\partial y^2} + \frac{\partial^2 \boldsymbol{E}}{\partial z^2}$$

$$c \stackrel{\text{def}}{=} \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} = \frac{c_0}{n}$$

$$\begin{cases} \nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}} \\ \nabla \cdot \boldsymbol{E} = 0 \\ \nabla \times \boldsymbol{B} = \varepsilon\mu\dot{\boldsymbol{E}} \\ \nabla \cdot \boldsymbol{B} = 0 \end{cases} \longrightarrow \begin{cases} \Delta \boldsymbol{E} - \frac{1}{c^2}\ddot{\boldsymbol{E}} = 0 \\ \Delta \boldsymbol{B} - \frac{1}{c^2}\ddot{\boldsymbol{B}} = 0 \end{cases}$$

TOWARDS THE WAVE EQUATIONS

- Start from Maxwell's equations

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}} \quad \xrightarrow{\text{Apply } \nabla \times} \quad \nabla \times \nabla \times \boldsymbol{E} = -\nabla \times \dot{\boldsymbol{B}}$$

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{B} = \varepsilon\mu\dot{\boldsymbol{E}}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

WAVES & MAXWELL'S EQUATIONS

- Start from Maxwell’s equations

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}}$$

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{B} = \varepsilon\mu\dot{\boldsymbol{E}}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

Apply $\nabla \times$ → $\nabla \times \nabla \times \boldsymbol{E} = -\nabla \times \dot{\boldsymbol{B}}$

Apply $\frac{\partial}{\partial t}$ → $\frac{\partial \nabla \times \boldsymbol{B}}{\partial t} = \nabla \times \frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \dot{\boldsymbol{B}} = \varepsilon\mu\ddot{\boldsymbol{E}}$

WAVES & MAXWELL'S EQUATIONS

- Start from Maxwell's equations

From vector calculus:

$$\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$$

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}} \quad \xrightarrow{\text{Apply } \nabla \times} \quad \nabla \times \nabla \times \boldsymbol{E} = -\nabla \times \dot{\boldsymbol{B}}$$

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{B} = \varepsilon\mu\dot{\boldsymbol{E}} \quad \xrightarrow{\text{Apply } \frac{\partial}{\partial t}} \quad \frac{\partial \nabla \times \boldsymbol{B}}{\partial t} = \nabla \times \frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \dot{\boldsymbol{B}} = \varepsilon\mu\ddot{\boldsymbol{E}}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\Longrightarrow \nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} = -\nabla \times \dot{\boldsymbol{B}} = -\varepsilon\mu\ddot{\boldsymbol{E}} \Longrightarrow -\nabla^2 \boldsymbol{E} = -\varepsilon\mu\ddot{\boldsymbol{E}}$$

$$\Longrightarrow \nabla^2 \boldsymbol{E} - \varepsilon\mu\ddot{\boldsymbol{E}} = 0$$

EXERCISE: DERIVE WAVE EQUATION FOR B-FIELD

- Start from Maxwell's equations
- **Similar procedure for the B-field**

From vector calculus:

$$\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$$

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}}$$

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \times \boldsymbol{B} = \varepsilon\mu\dot{\boldsymbol{E}}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

Apply $\nabla \times$ → $\nabla \times \nabla \times \boldsymbol{E} = -\nabla \times \dot{\boldsymbol{B}}$

Apply $\frac{\partial}{\partial t}$ → $\frac{\partial \nabla \times \boldsymbol{B}}{\partial t} = \nabla \times \frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \dot{\boldsymbol{B}} = \varepsilon\mu\ddot{\boldsymbol{E}}$

$$\Rightarrow \nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} = -\nabla \times \dot{\boldsymbol{B}} = -\varepsilon\mu\ddot{\boldsymbol{E}} \Rightarrow -\nabla^2 \boldsymbol{E} = -\varepsilon\mu\ddot{\boldsymbol{E}}$$

$$\Rightarrow \nabla^2 \boldsymbol{E} - \varepsilon\mu\ddot{\boldsymbol{E}} = 0$$

Solutions of the wave equations

SCALAR WAVE EQUATION

- Vectorial wave equations

$$\begin{cases} \Delta \boldsymbol{E} - \frac{1}{c^2}\ddot{\boldsymbol{E}} = 0 \\ \\ \Delta \boldsymbol{B} - \frac{1}{c^2}\ddot{\boldsymbol{B}} = 0 \end{cases}$$

- First calculate E-field
- Assume E-field has constant $\boldsymbol{E}_0$

$$\boldsymbol{E} = \boldsymbol{E}_0\, u(\boldsymbol{r}, t)$$

$$\Delta U - \frac{1}{c^2}\ddot{U} = 0 \quad \Longrightarrow \quad \text{Scalar wave equation}$$

SCALAR WAVE EQUATION

$$\boldsymbol{E} = \boldsymbol{E}_0 \, U(\boldsymbol{r}, t)$$

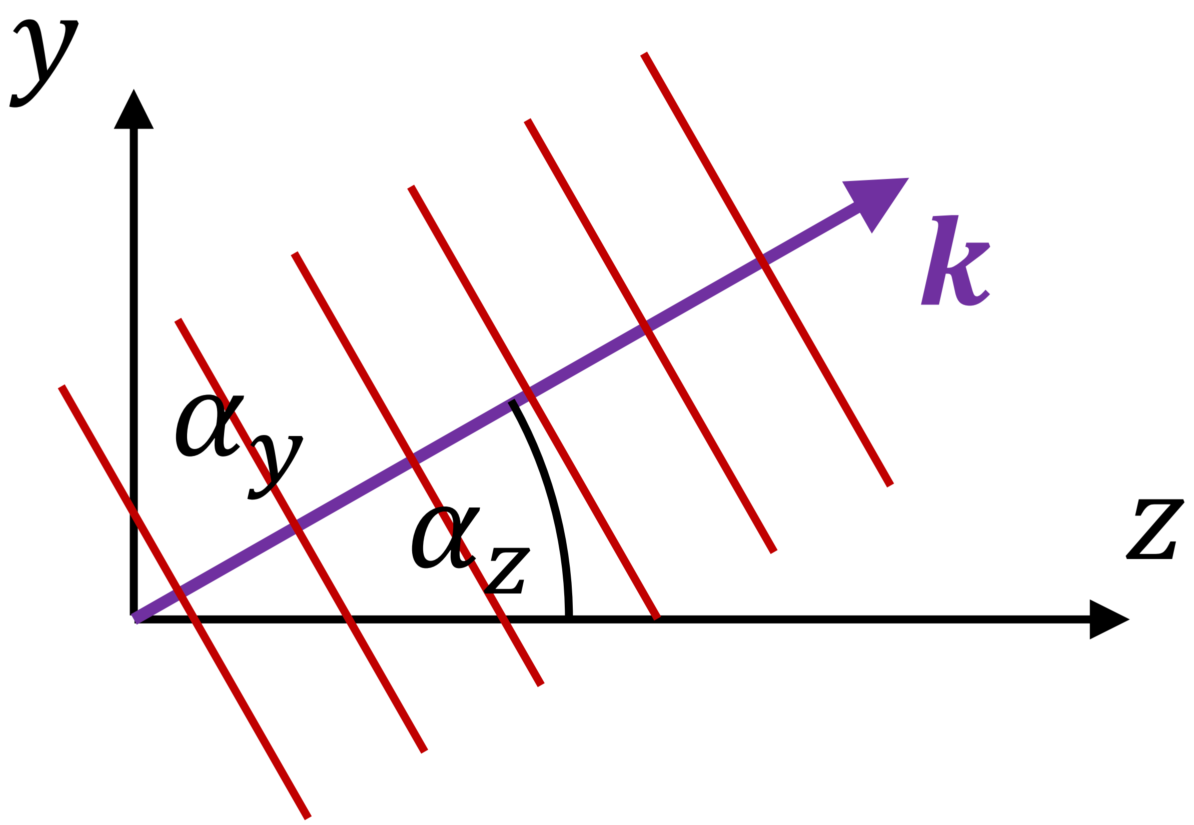
$$\Delta U - \frac{1}{c^2}\ddot{U} = 0$$

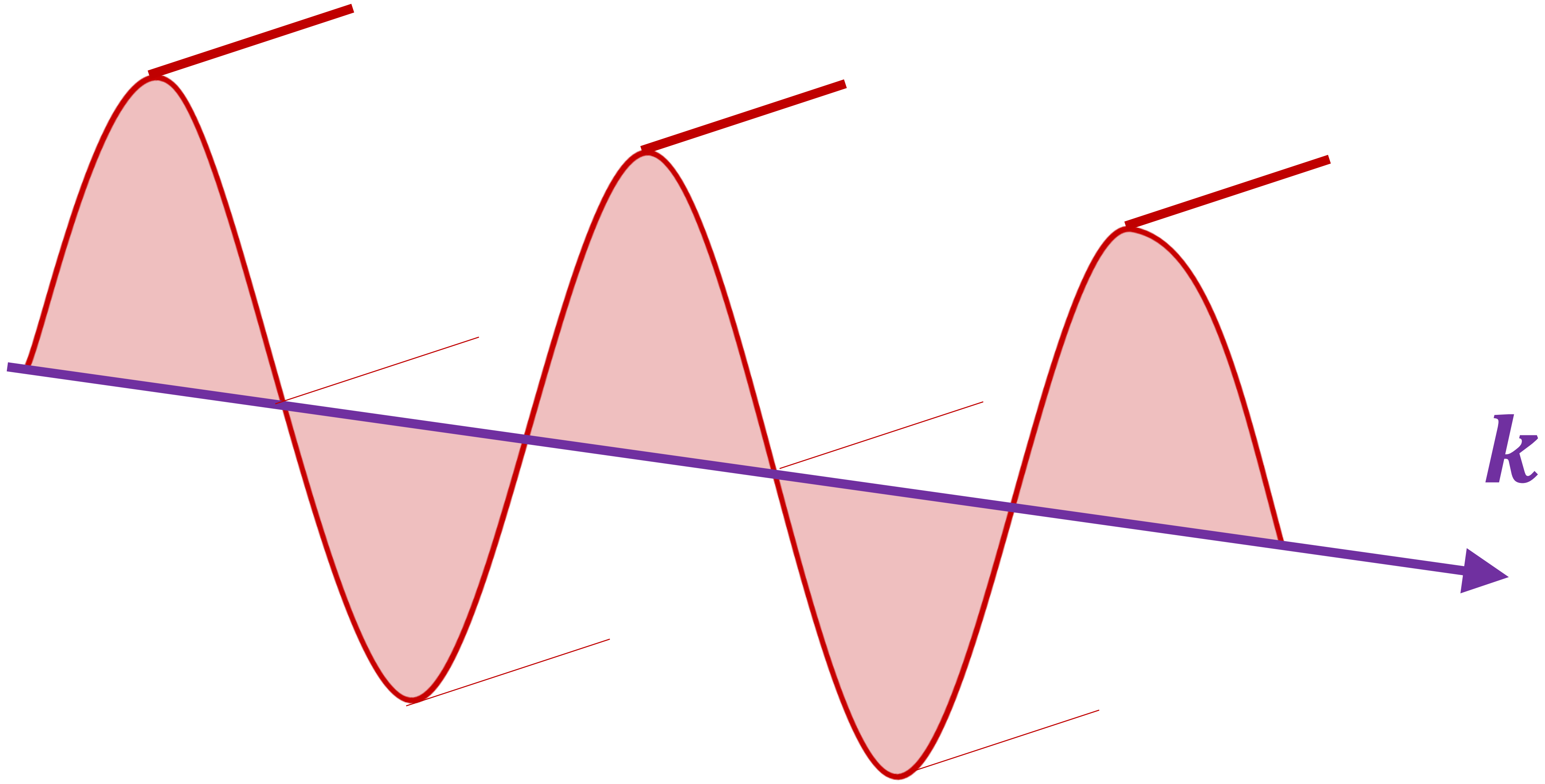
Plane waves

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$$

Planes:

$\boldsymbol{k} \cdot \boldsymbol{r} = \text{constant}$





SCALAR WAVE EQUATION: EXERCISE

$$\boldsymbol{E} = \boldsymbol{E}_0\, U(\boldsymbol{r}, t)$$

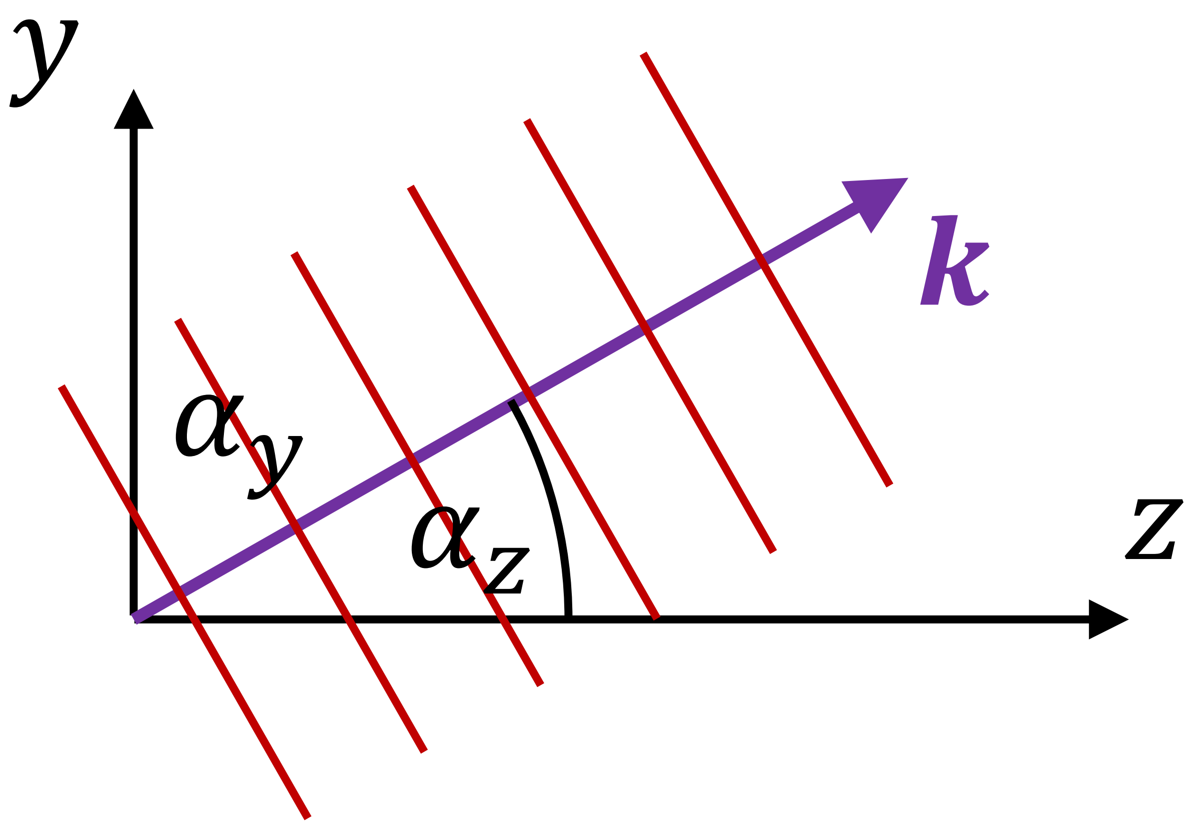
$$\Delta U - \frac{1}{c^2}\ddot{U} = 0$$

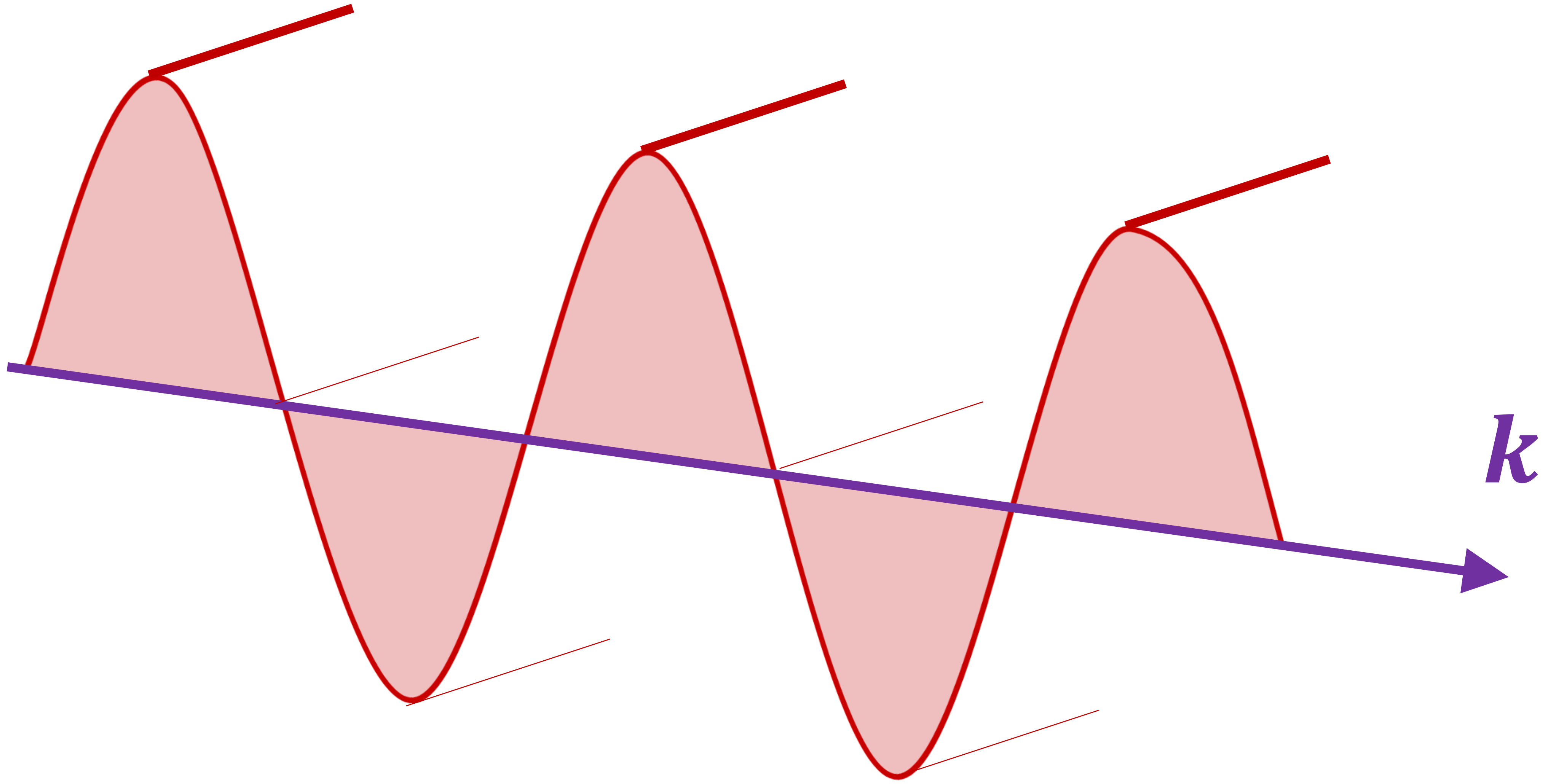
Plane waves

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)}$$

Exercise: show this is a solution of the wave equation with $\omega = ck$

Planes:
$\boldsymbol{k} \cdot \boldsymbol{r} = \text{constant}$





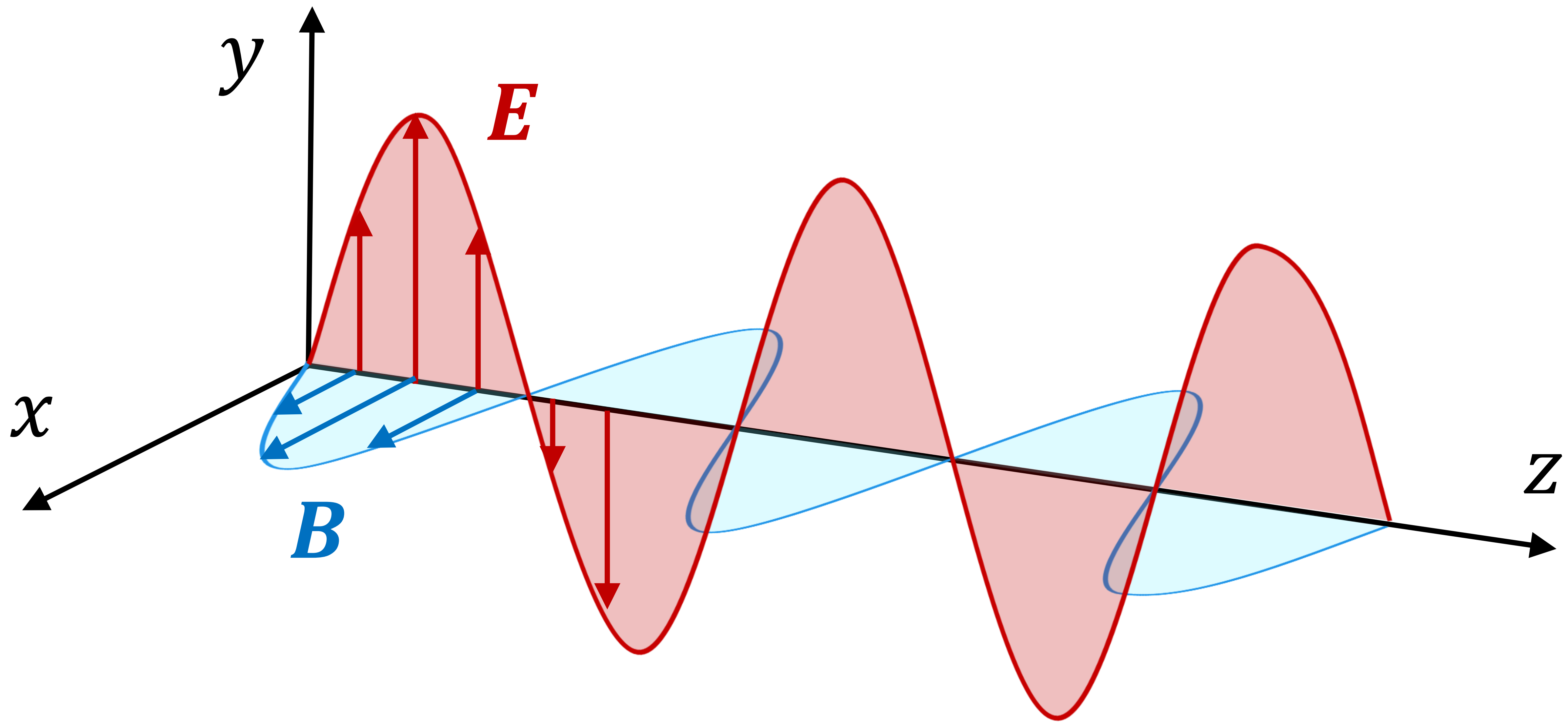
PLANE WAVES: TEM MODE

- Transverse ElectroMagnetic waves
- Linear polarization
- Propagation along z

$$\boldsymbol{E} = E_y(z,t)\boldsymbol{e_y} = E_0 \mathrm{e}^{\pm i(kz-\omega t)}\boldsymbol{e_y}$$

$$\boldsymbol{B} = B_x(z,t)\boldsymbol{e_x} = B_0 \mathrm{e}^{\pm i(kz-\omega t)}\boldsymbol{e_x}$$

$$\begin{cases} \Delta\boldsymbol{E} - \frac{1}{c^2}\ddot{\boldsymbol{E}} = 0 \\ \Delta\boldsymbol{B} - \frac{1}{c^2}\ddot{\boldsymbol{B}} = 0 \end{cases}$$



PLANE WAVES

- Start with electric field

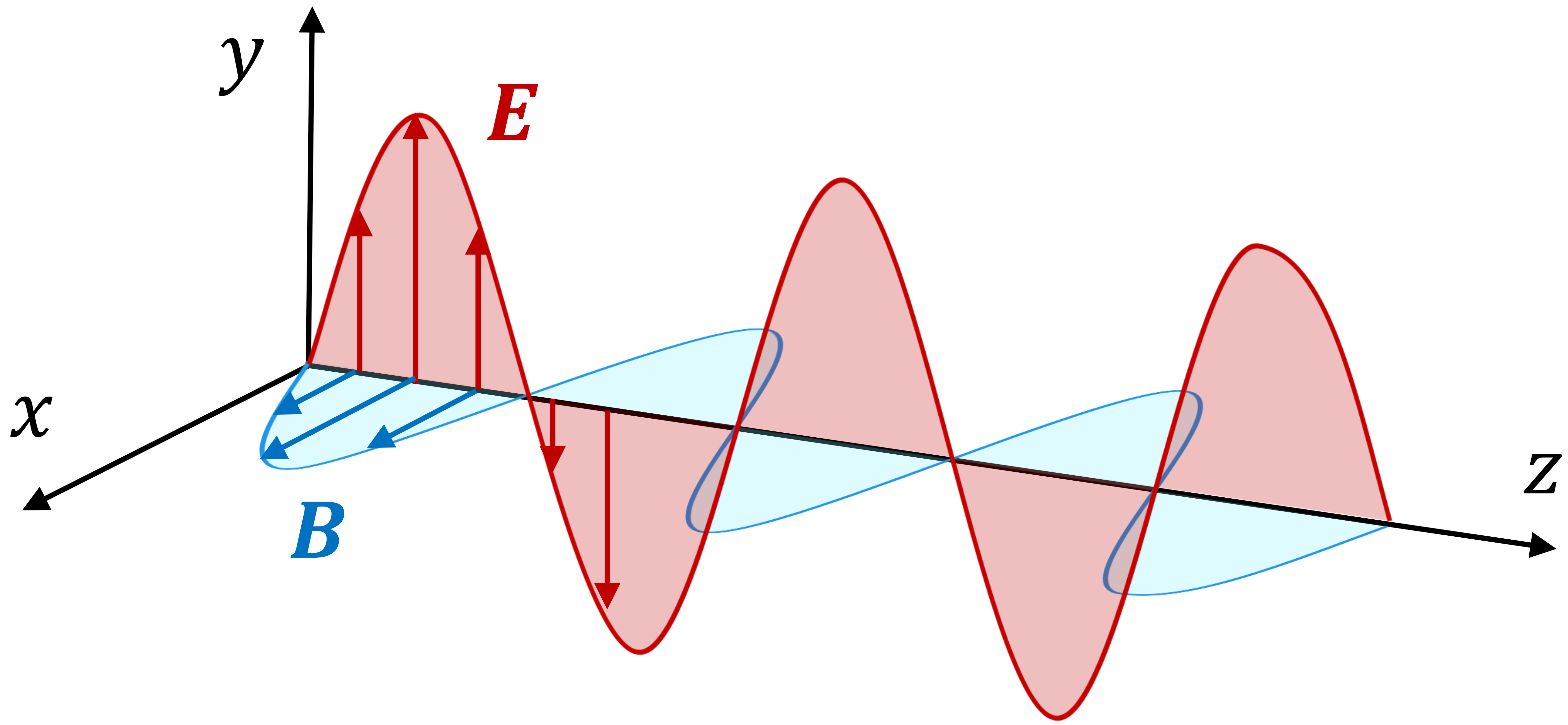
$$\Delta \boldsymbol{E} - \frac{1}{c^2}\ddot{\boldsymbol{E}} = 0$$

- First assume solution

$$\boldsymbol{E} = E_0 \mathrm{e}^{i(kz-\omega t)} \boldsymbol{e_y}$$

$$\boldsymbol{E} = E_y(z,t)\boldsymbol{e_y} = E_0 \mathrm{e}^{\pm i(kz-\omega t)} \boldsymbol{e_y}$$

$$\boldsymbol{B} = B_x(z,t)\boldsymbol{e_x} = B_0 \mathrm{e}^{\pm i(kz-\omega t)} \boldsymbol{e_x}$$



PLANE WAVES

- Start with electric field

$$\Delta \boldsymbol{E} - \frac{1}{c^2}\ddot{\boldsymbol{E}} = 0$$

- First assume solution

$$\boldsymbol{E} = E_0 \mathrm{e}^{i(kz-\omega t)}\boldsymbol{e_y}$$

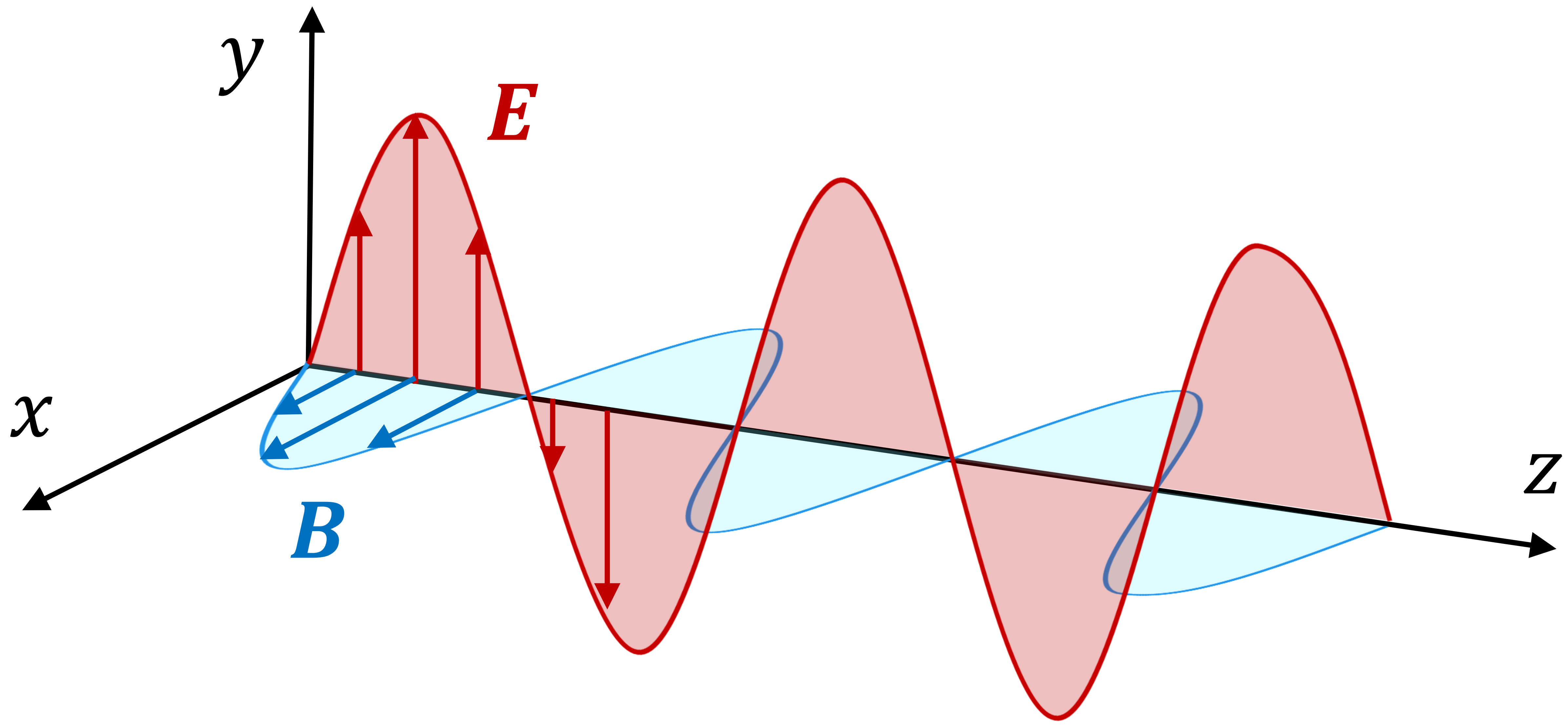
- Apply then

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}}$$

$$\Rightarrow \quad \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$$

$$\boldsymbol{E} = E_y(z,t)\boldsymbol{e_y} = E_0 \mathrm{e}^{\pm i(kz-\omega t)}\boldsymbol{e_y}$$

$$\boldsymbol{B} = B_x(z,t)\boldsymbol{e_x} = B_0 \mathrm{e}^{\pm i(kz-\omega t)}\boldsymbol{e_x}$$



PLANE WAVES: EXERCISE

- First assume solution

$$\boldsymbol{E} = E_0 \mathrm{e}^{i(kz-\omega t)} \boldsymbol{e_y}$$

- Apply then

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}}$$

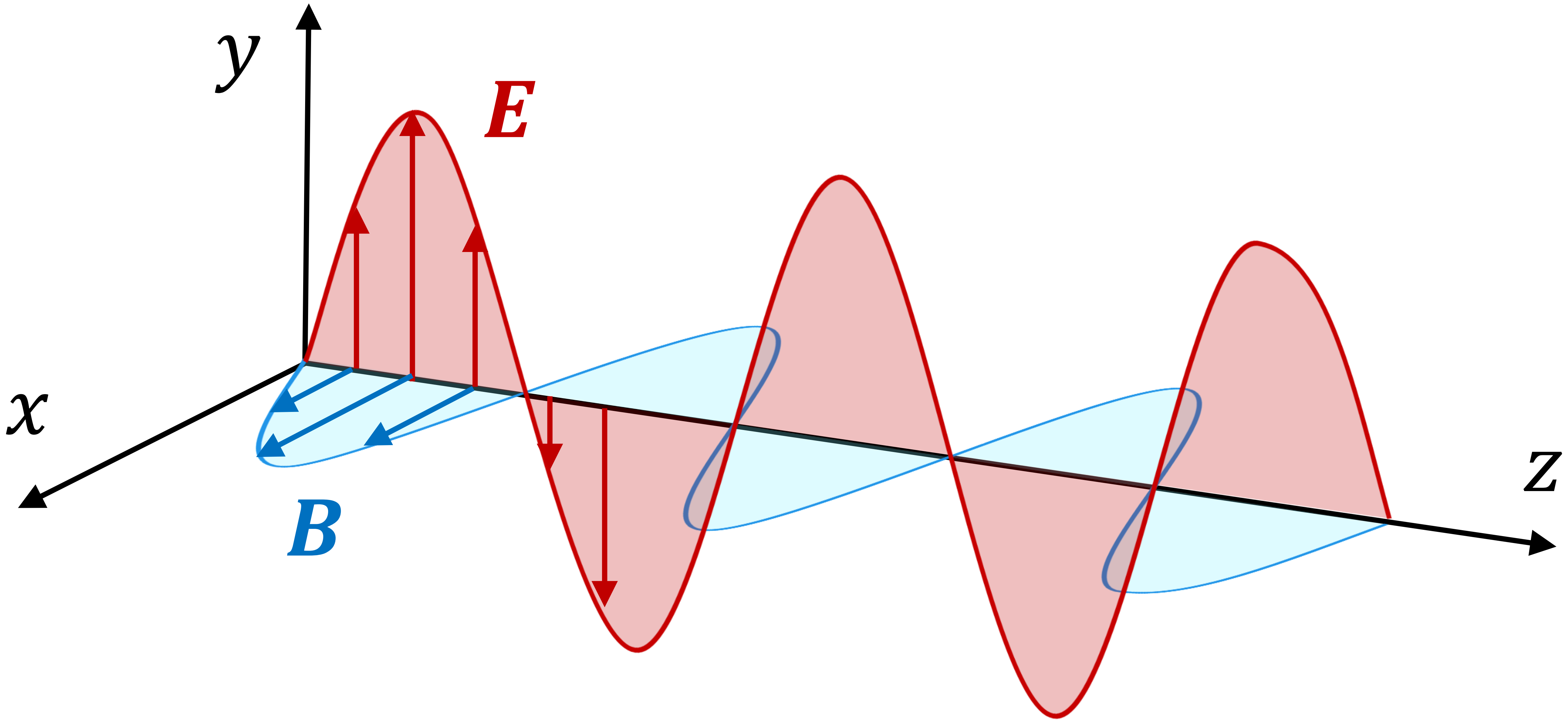
$$\Rightarrow \quad \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$$

Exercise: Show that

$$B_x = \frac{1}{c} E_y$$

$$\boldsymbol{E} = E_y(z,t)\boldsymbol{e_y} = E_0 \mathrm{e}^{\pm i(kz-\omega t)} \boldsymbol{e_y}$$

$$\boldsymbol{B} = B_x(z,t)\boldsymbol{e_x} = B_0 \mathrm{e}^{\pm i(kz-\omega t)} \boldsymbol{e_x}$$



PLANE WAVES

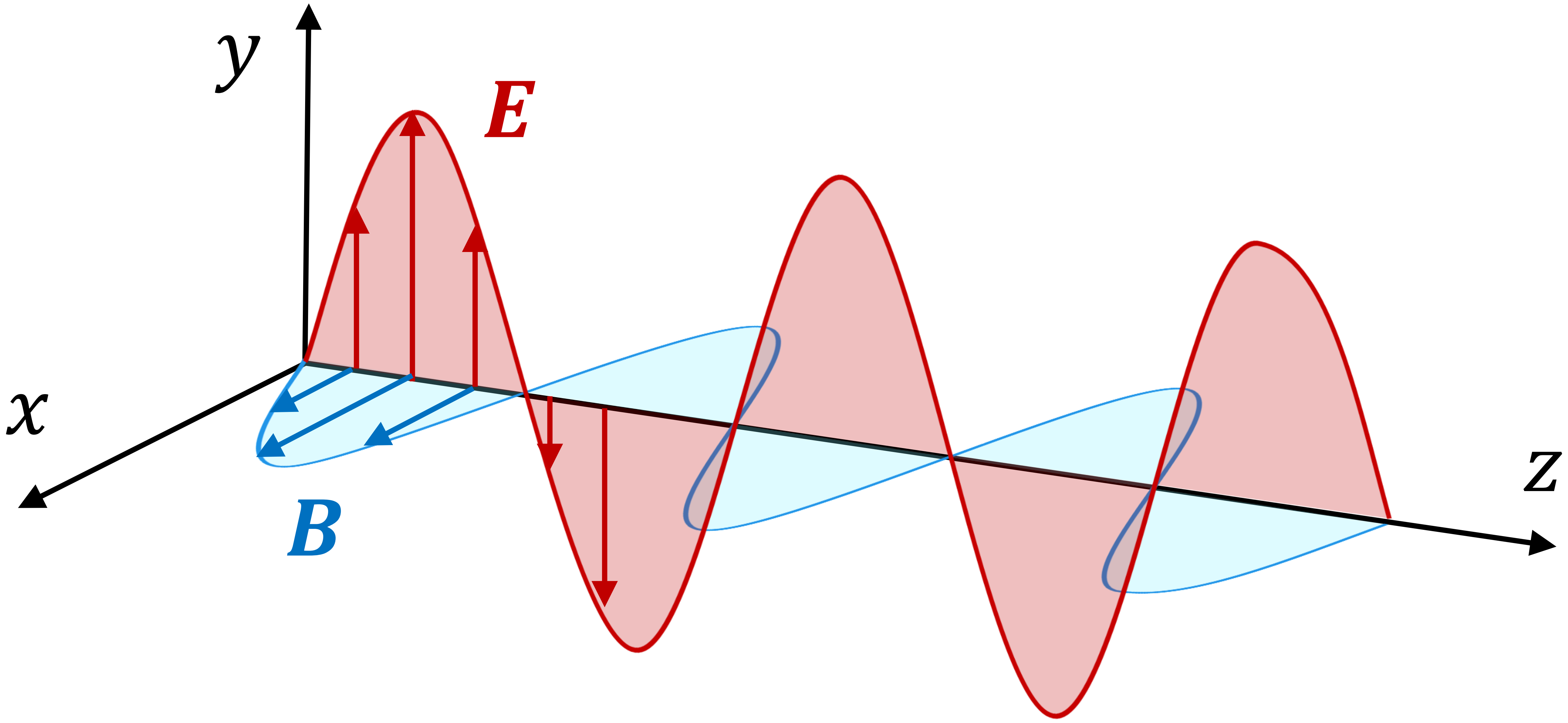
- For a general direction k

$$\boldsymbol{E} \perp \boldsymbol{k} \quad \text{and} \quad \boldsymbol{B} \perp \boldsymbol{k}$$

$$\boldsymbol{E} = E_y(z,t)\boldsymbol{e_y} = E_0 \mathrm{e}^{\pm i(kz-\omega t)}\boldsymbol{e_y}$$

$$\boldsymbol{B} = B_x(z,t)\boldsymbol{e_x} = B_0 \mathrm{e}^{\pm i(kz-\omega t)}\boldsymbol{e_x}$$

$$\begin{cases} \boldsymbol{k} \times \boldsymbol{E} = \omega \boldsymbol{B} \\ \boldsymbol{k} \times \boldsymbol{B} = \omega \varepsilon \mu \boldsymbol{B} \end{cases}$$



➡ $\boldsymbol{E} \perp \boldsymbol{B}$

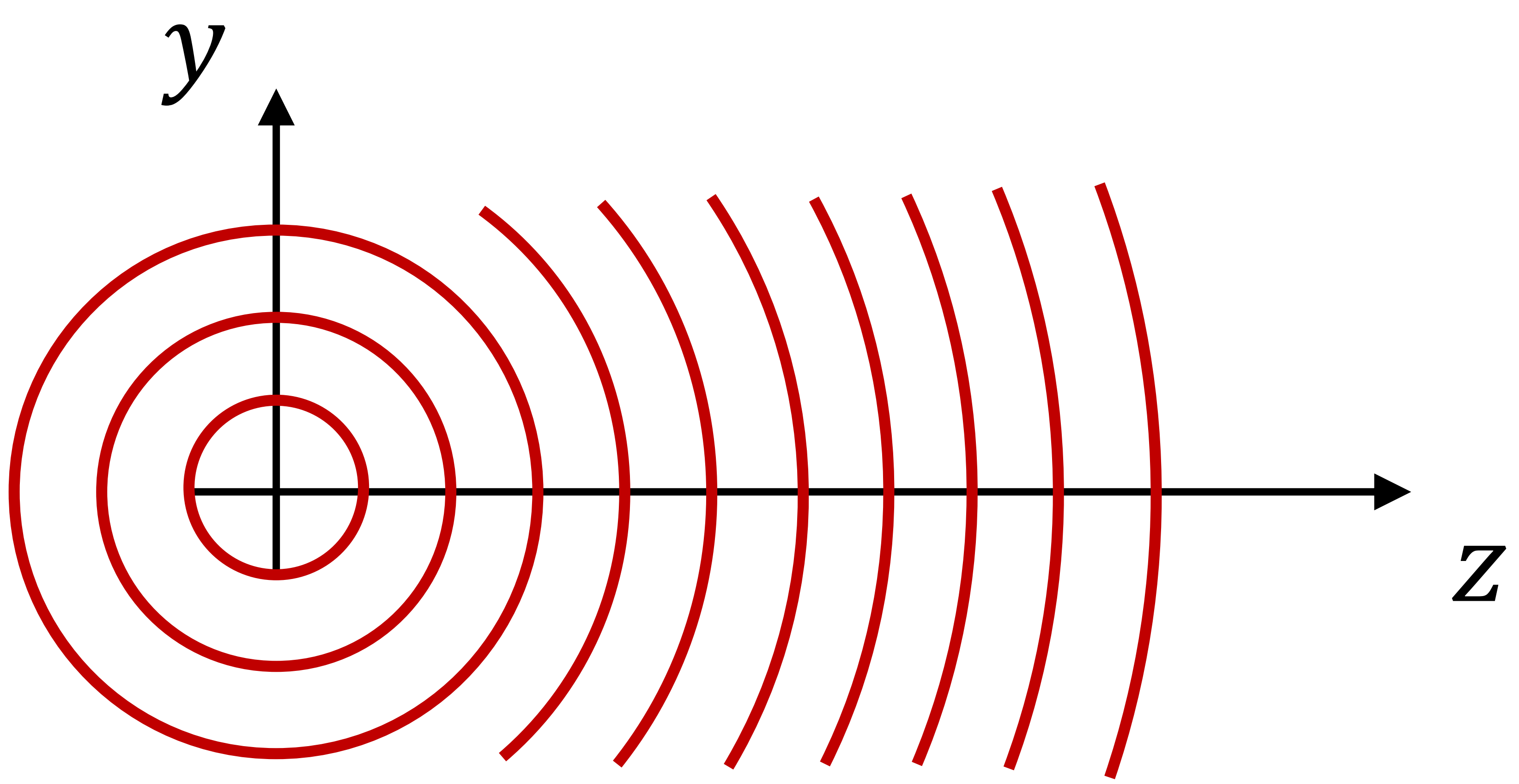
$$\boldsymbol{E} = \boldsymbol{E}_0\, U(\boldsymbol{r}, t)$$

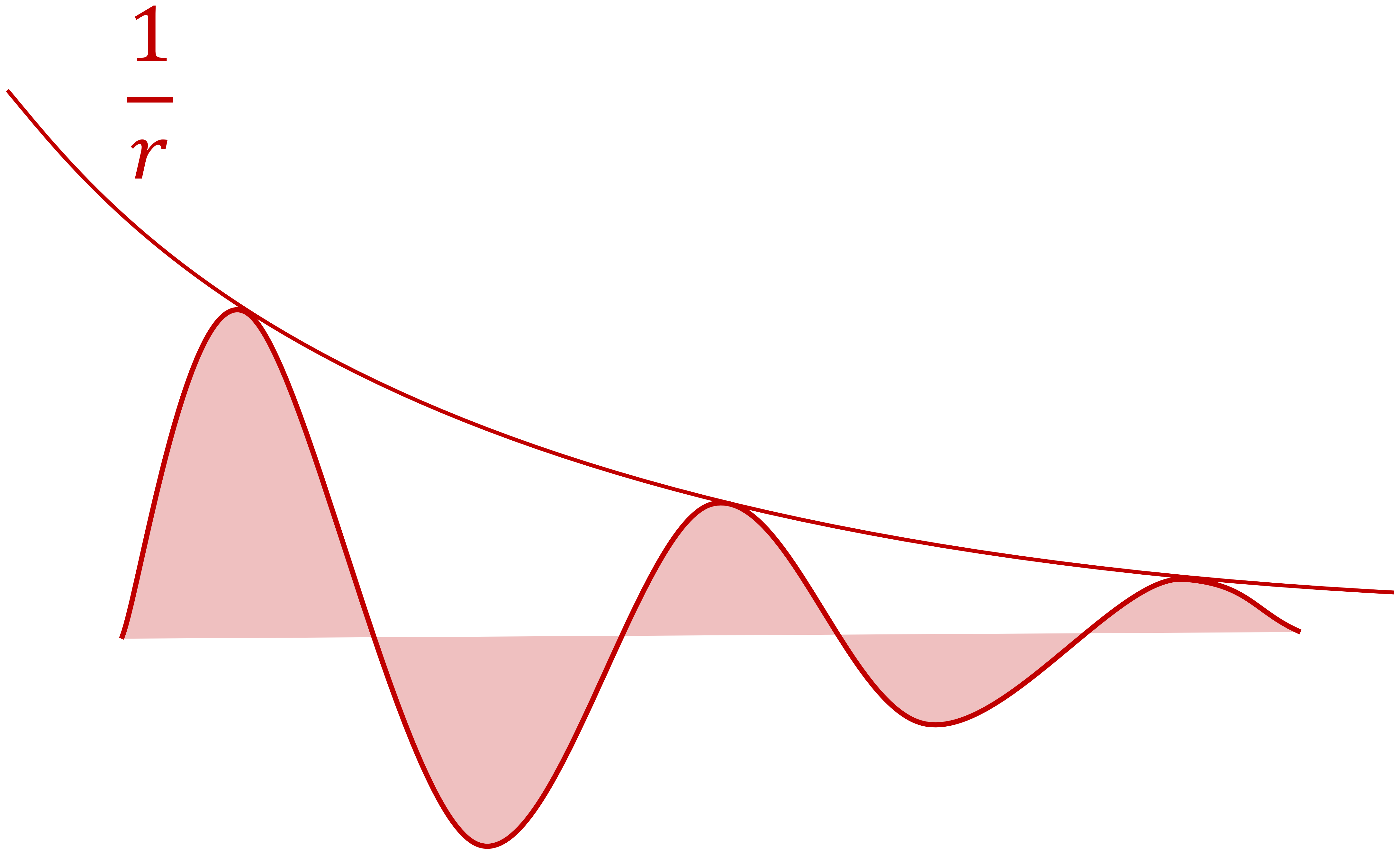
$$\Delta U - \frac{1}{c^2}\ddot{U} = 0$$

Spherical waves

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 \frac{e^{ik(r \pm ct)}}{r}$$

Spheres: r = constant





SCALAR WAVE EQUATION

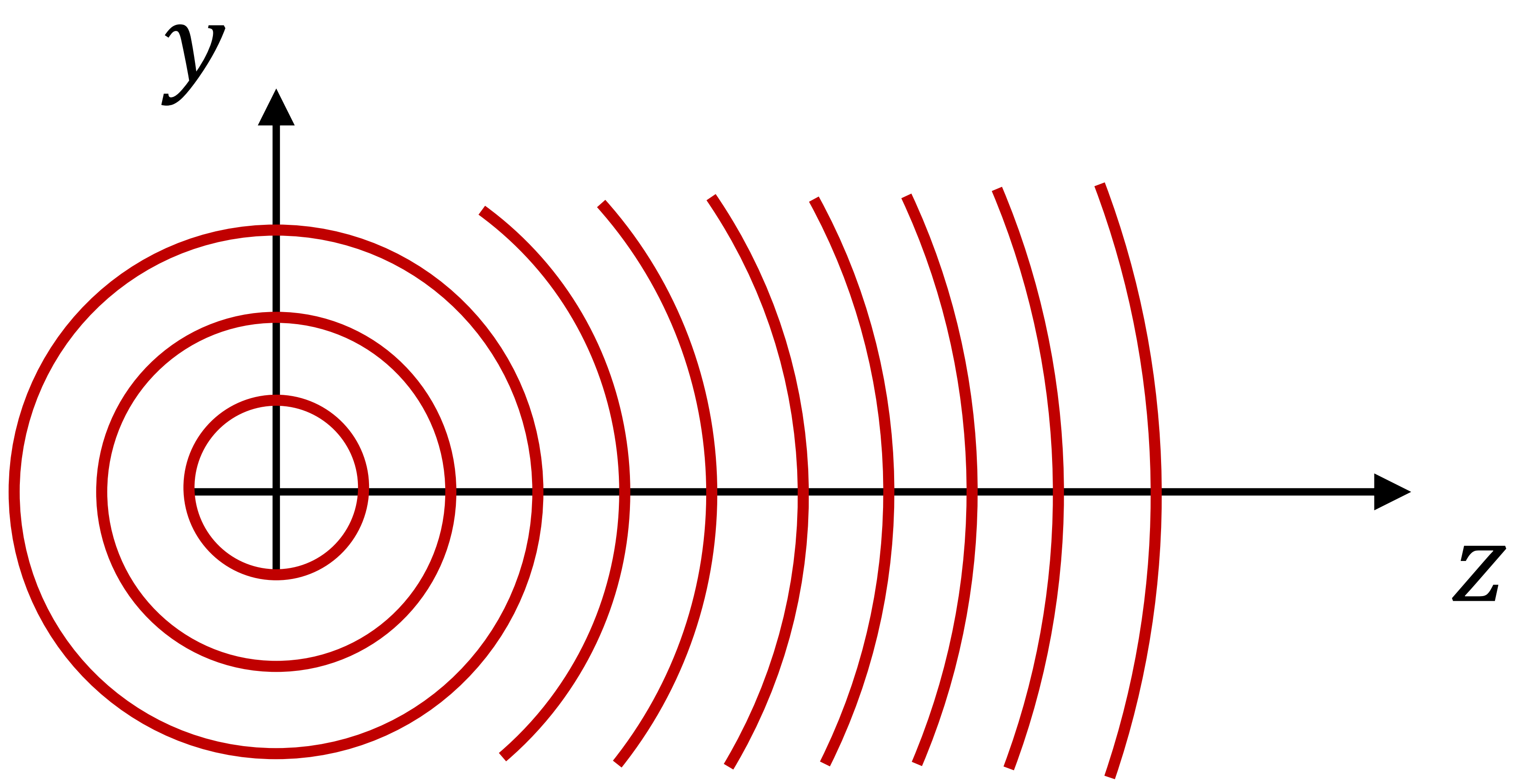
$$\boldsymbol{E} = \boldsymbol{E}_0\, U(\boldsymbol{r}, t)$$

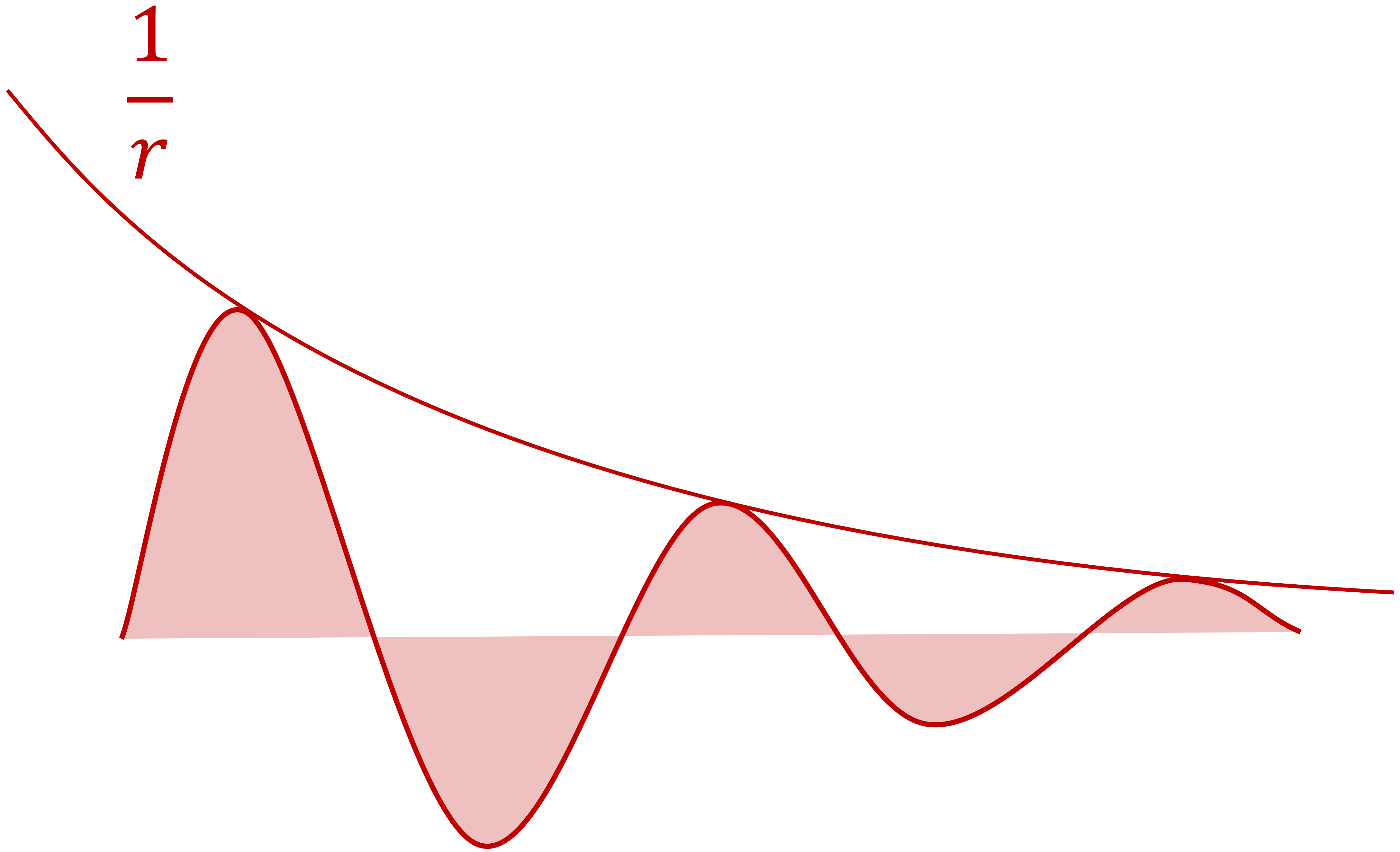
$$\Delta U - \frac{1}{c^2}\ddot{U} = 0$$

Spherical waves

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 \frac{e^{ik(r \pm ct)}}{r}$$

Spheres: $r = \text{constant}$





Geometrical Optics

GEOMETRIC OPTICS & RAY-TRACING

- Phase local wavefront (eikonal)

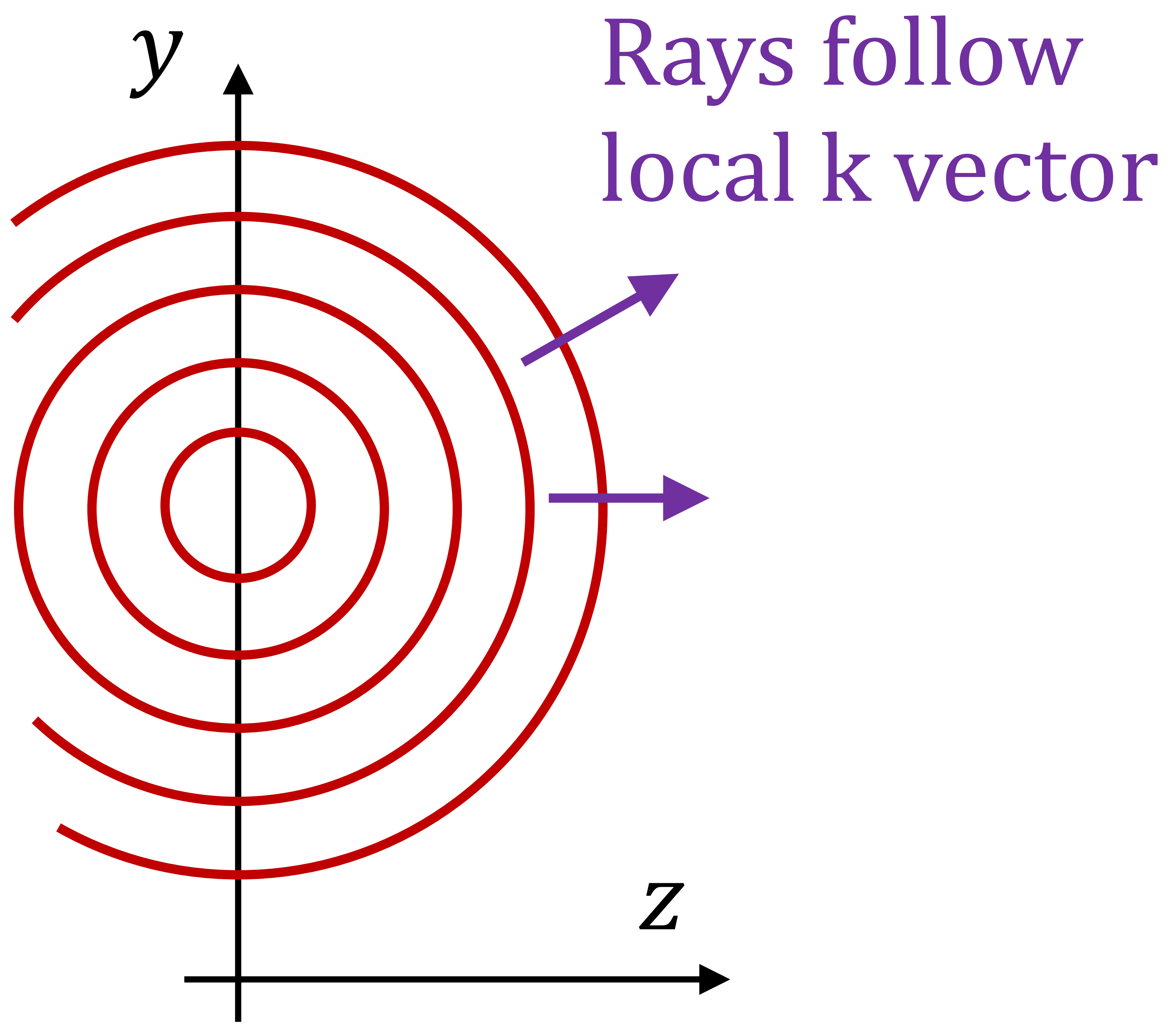
$$\boldsymbol{\varphi(r) = k \cdot r}$$

- Wavefront: line constant phase

- Light rays perpendicular: requires isotropic medium

- Ray-tracing: find light paths

- Require: isotropic medium



Paraxial Approximation

PARAXIAL APPROXIMATION: SPHERICAL WAVES

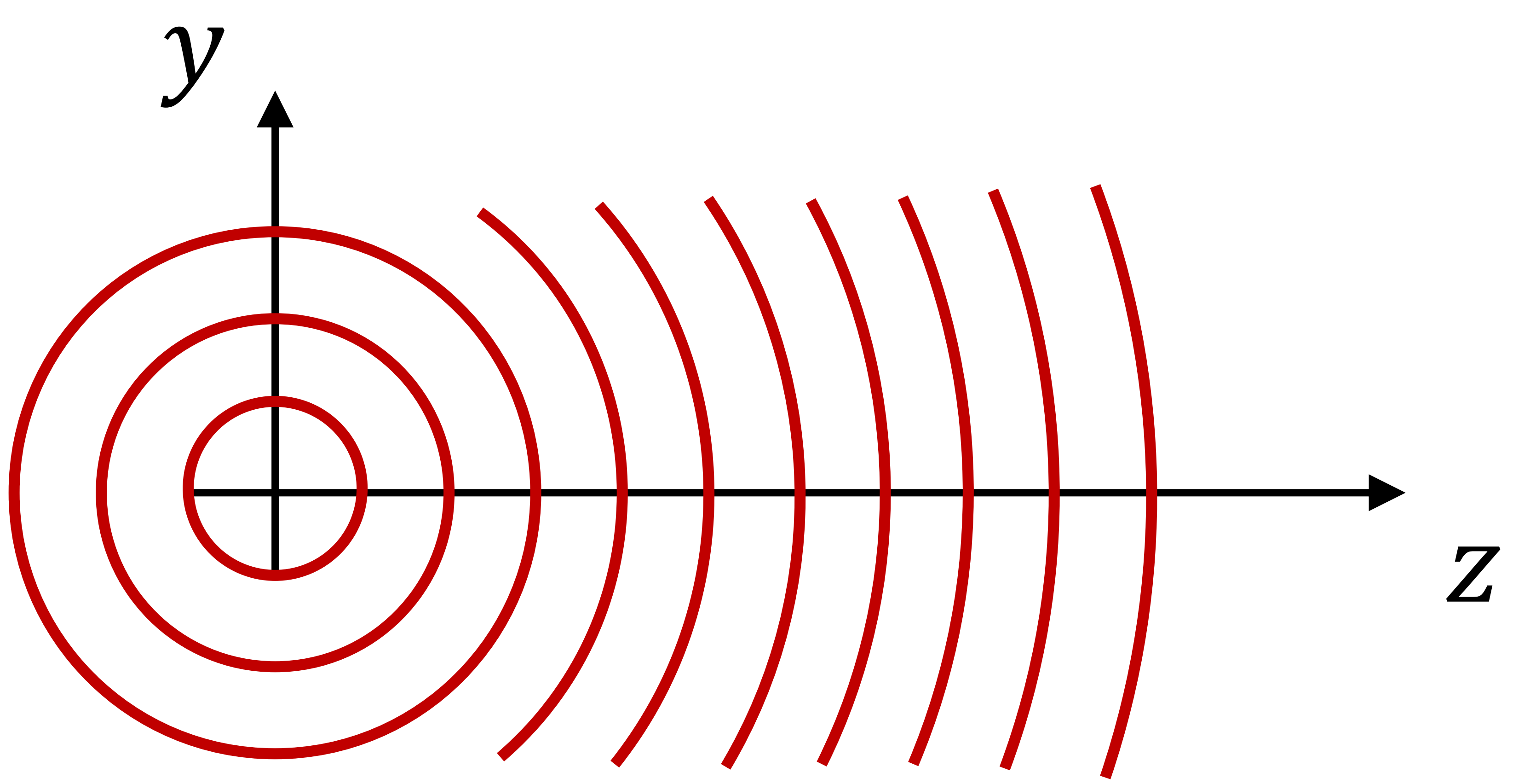
$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \frac{e^{ik(r\pm ct)}}{r}$$

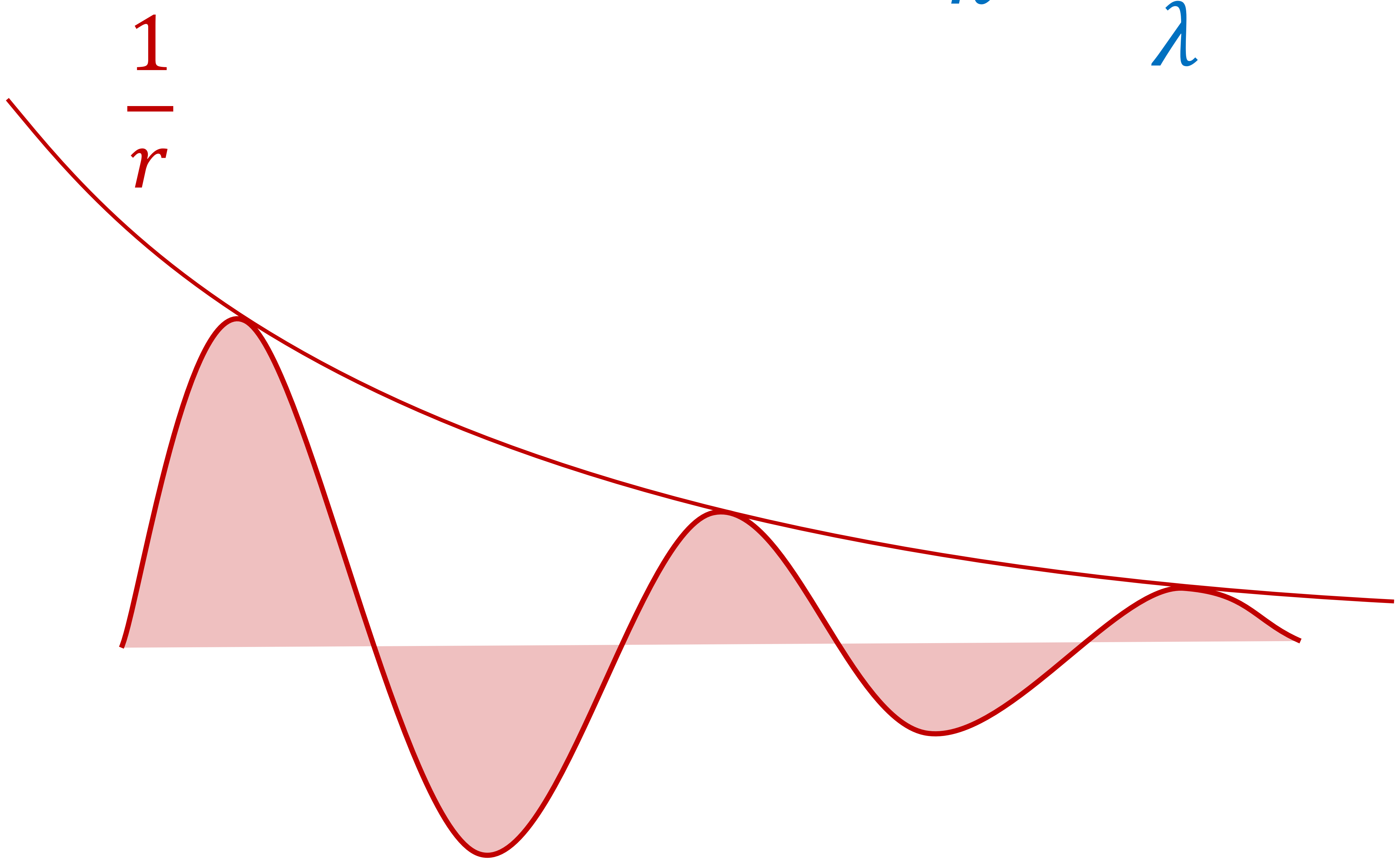
$$\approx e^{ik\sqrt{x^2+y^2+z^2}}\, e^{-i\omega t}$$

$$\approx e^{ik\frac{x^2+y^2}{2z}}\, e^{i(kz-\omega t)}$$

$$\omega = ck$$

$$k = \frac{2\pi}{\lambda}$$





PARAXIAL APPROXIMATION: SPHERICAL WAVES

- Light stays close to the propagation axis z
- Distance $z \gg x$ and $z \gg y$
- Decay $\frac{1}{r}$ is small for z large