PHOT 451: Quantum Photonics

Quiz 1: questions & solutions

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Exam questions

Grading: Each quiz counts for 15% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators are allowed but not necessary for the exam (numerical values will be fractions).

The duration of the quiz is 1 hour.

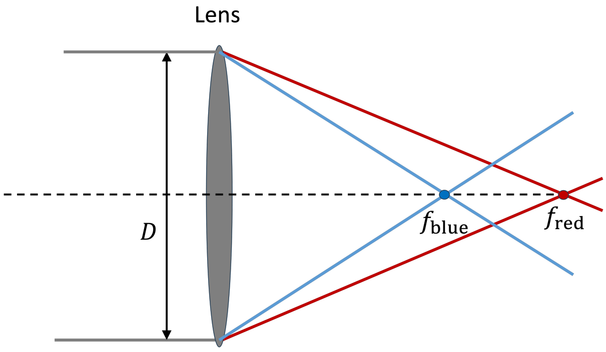
This document contains both the problems and their solutions.

Question 1: Dispersion in a thin lens

Assume a thin plastic lens with diameter $D = 5$ mm that focuses collimated red light. The plastic has refraction index $n_{\text{red}} = 1.5 = 3/2$ for red light, focal length $f_{\text{red}} = 10$ mm. The plastic is highly dispersive and the refraction index for blue light is $n_{\text{blue}} = 1.67 = 5/3$. The Lensmaker's formula is valid for the optical system:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(a) Derive the focal length for **blue** light: f_{blue}.
(b) Add a vertical line to the sketch in the figure below where to put the imaging plane as to minimize the spot size due to chromatic abberation?



Solution (Q1)

(a) From the Lensmaker's formula we find

$$f\,(n-1) = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1}$$

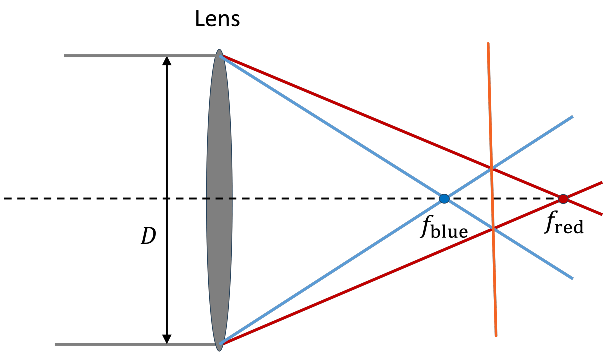
Since R_1 and R_2 are independent of the wavelength:

$$f_{\text{blue}}(n_{\text{blue}} - 1) = f_{\text{red}}(n_{\text{red}} - 1)$$

The focal length f_{blue} is then given by

$$f_{\text{blue}} = f_{\text{red}}\frac{(n_{\text{red}} - 1)}{(n_{\text{blue}} - 1)} = 10\frac{\frac{3}{2} - 1}{\frac{5}{3} - 1}\,\text{mm} = 7.5\,\text{mm}$$

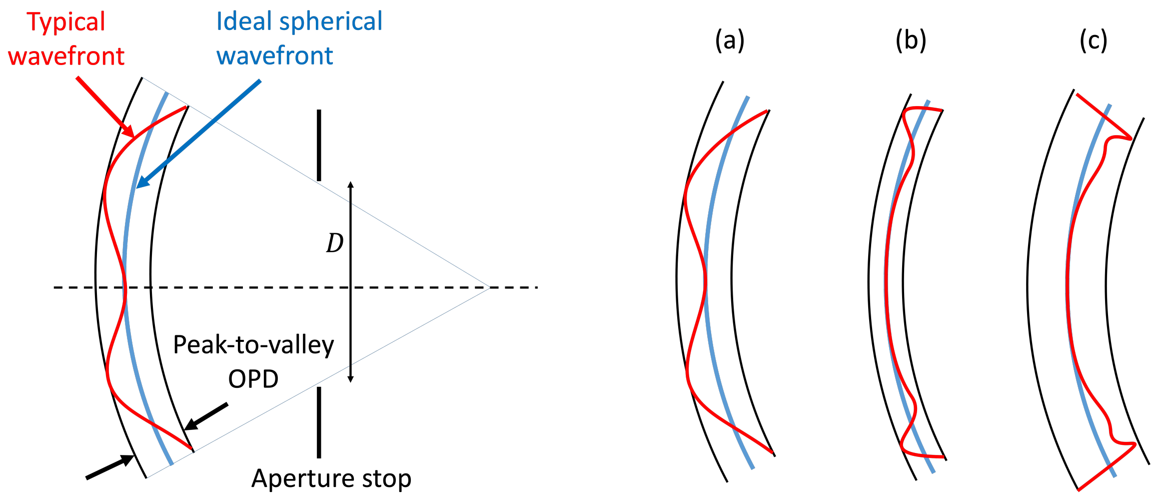
(b) See the added vertical orange line in the figure below, to minimize the spot size the position of the imaging plane should be between the two focal points where the marginal rays cross.



Question 2: Root Mean Square ODP

Three different optical systems lead to the wavefronts (a), (b) and (c) in below figure. The corresponding Optical Path Differences (ODP), can be quantified by RMS $= \sqrt{\langle \text{OPD}^2 \rangle}$. The aperture stop is located after the lens and has diameter D.

(a) Which wavefront has the largest peak-to-valley OPD, and which one has the largest RMS ODP value?
(b) If you decrease the aperture stop diameter to $D \longrightarrow D/2$, which wavefront will afterwards have the largest peak-to-valley OPD? Explain why.



Solution (Q2)

(a) Which wavefront has the largest peak-to-valley OPD and which has largest RMS OPD can be argued as follows:

- Wavefront "c" has the largest peak-to-valley OPD since the limiting spheres have largest difference in radius,
- Wavefront "a" has the largest RMS OPD. The average distance squared from the reference sphere is clearly larger than for the other wavefronts, wavefront "c" has a higher OPD for some small angle intervals but not enough to compensate.

(b) Wavefront "a" will have the largest peak-to-valley OPD afterwards.
Decreasing the aperture diameter from $D \longrightarrow D/2$ will reduce the solid angle (cone) of rays that arrive. If the aperture is not too far from the lens then this means that the wavefronts will be reduced and only the middle part will remain (roughly half of it). Wavefront "a" is the only one with large wavefront errors close to the optical axis.

Question 3: Diffraction limit

Consider an optical system existing out of a single thin lens with diameter $D = 10$ mm and focal length $f = 20$ mm. For green incident light ($\lambda = 520$ nm) the Airy disk diameter $d = 2.44\lambda f/D = 2.54$ micron.
For this question you can assume that the Lensmaker's formula is valid:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(a) What is the effect on the airy disk diameter d when scaling down the lens by decreasing R_1, R_2, and D by a factor 10?
(b) When only decreasing the lens diameter $D \longrightarrow D/10$, what is the resulting value of the Airy disk diameter?

Solution (Q3)

(a) First, denote the lens properties after downscaling by a tilde: $\tilde{f}$, $\tilde{D} = D/10$, $\tilde{R}_1 = R_1/10$, $\tilde{R}_2 = R_2/10$. Then we apply the Lensmaker's formula the find the new focal length diameter $\tilde{f}$:

$$\tilde{f} = (n-1)^{-1}\left(\frac{1}{\tilde{R}_1} - \frac{1}{\tilde{R}_2}\right)^{-1} = (n-1)^{-1}\left(\frac{10}{R_1} - \frac{10}{R_2}\right)^{-1} = \frac{f}{10}$$

Filling this in into the formula for the Airy disk diameter:

$$\tilde{d} = 2.44\lambda\tilde{f}/\tilde{D} = 2.44\lambda\,\frac{f/10}{D/10} = 2.44\lambda\frac{f}{D} = 2.54\,\text{micron}$$

Therefore the Airy disk diameter does not change: $\tilde{d} = d$

(b) Different from (a), if we only downscale $\tilde{D} = D/10$ then $\tilde{f} = f$, then the Airy disk diameter will increase by a factor 10:

$$\tilde{d} = 2.44\lambda\tilde{f}/\tilde{D} = 2.44\lambda\,\frac{f}{D/10} = 10\,d = 25.4\,\text{micron}$$