PHOT 451: Microscale optical design

Final exam: questions & solutions

Michaël Barbier, Fall (2025-2026)

Exam questions

Grading: The final exam counts for 30% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators are allowed but not necessary for the exam (numerical values will be fractions).

The duration of the final exam is 3 hours.

This document contains both the problems and their solutions.

Question 1: PSF of a thin lens

Consider a thin convex lens with $R_1 = 20$ micron, $R_2 = -60$ micron, lens diameter $D = 10$ micron, $n = 1.5$ and surrounded by air. Assume the Lensmaker's formula is valid:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(a) Calculate the Airy disk diameter $d = 2.44\lambda f/D$ for light with wavelength $\lambda = 500$ nm.
(b) How will the Airy disk diameter d change if you scale both $R_1' = \alpha R_1$ and $R_2' = \alpha R_2$ by $\alpha = 1/2$ (microscopic scaling)?

Solution (Q1)

(a) First the focal length f is derived from the Lensmaker's formula:

$$f = \left[(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]^{-1} = \left[\frac{1}{2}\left(\frac{1}{20} + \frac{1}{60}\right)\right]^{-1} \mu m = 30\,\mu m$$

The Airy disk diameter is then given by:

$$d = 2.44\lambda\frac{f}{D} = 2.44 \cdot 0.5 \cdot \frac{30}{10}\,\mu m = 3.66\,\mu m$$

(b) The curvature radii of the lens change, and thereby the focal length f:

$$f' = \left[(n-1)\left(\frac{1}{\alpha R_1} - \frac{1}{\alpha R_2}\right)\right]^{-1} = \alpha\left[(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]^{-1} = \alpha f$$

The diameter D is constant under microscopic scaling and thus the rescaled Airy disk diameter is given by:

$$d' = 2.44\lambda\frac{f'}{D} = 2.44\lambda\frac{\alpha f}{D} = \alpha d = \frac{1}{2}3.66\,\mu m = 1.83\,\mu m$$

Question 2: the Strehl ratio

The Marechal criterium states that good imaging quality has Strehl ratio $S \geq 0.82$.

(a) Extract the Strehl ratio S from the PSF profiles in figure 1, is Marechal's condition fulfilled?

(b) Give an expression for the RMS wavefront error σ_w by using $S \approx 1 - \left(\frac{2\pi}{\lambda}\sigma_w\right)^2$.
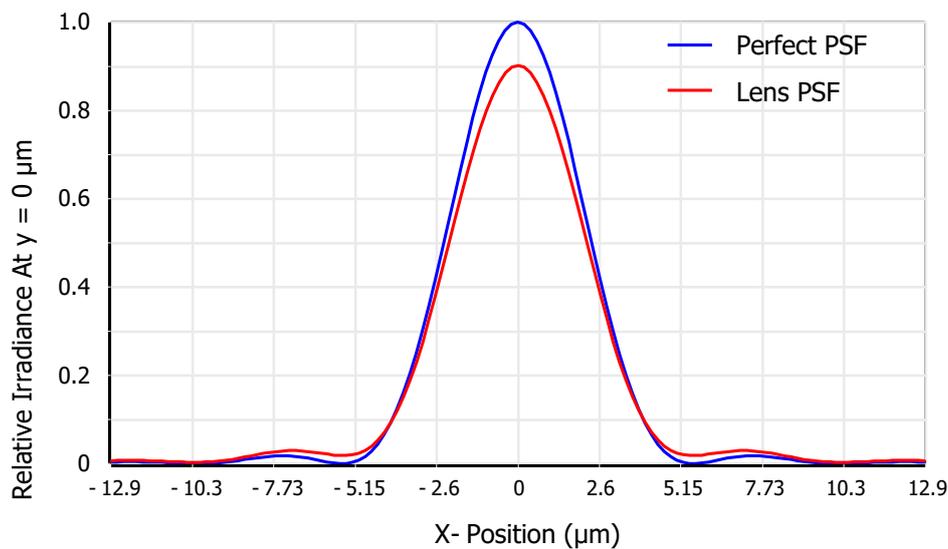

Figure 1: Point spread functions for a "perfect" paraxial lens and a lens with aberrations.

Solution (Q2)

(a) From figure 1 one can see that $S = \frac{I_{\text{peak, aberrations}}}{I_{\text{peak, perfect}}} = \frac{0.9}{1} = 0.9$. Therefore the Marechal criterium is fulfilled: $S = 0.9 \geq 0.82$.

(b)

$$S \approx 1 - (\frac{2\pi}{\lambda}\sigma_w)^2$$

$$\Rightarrow \quad (\frac{2\pi}{\lambda}\sigma_w)^2 = 1 - S$$

$$\Rightarrow \quad (\frac{2\pi}{\lambda}\sigma_w) = \sqrt{1 - S}$$

$$\Rightarrow \quad \sigma_w = \frac{\lambda}{2\pi}\sqrt{1 - S}$$

Since $S = 0.9$ the first approximation formula was valid and the resulting expression for σ_w is a good approximation.

Question 3: Seidel aberrations

Distortion is one of the 5 Seidel aberrations.

(a) Name the other 4 Seidel aberrations.
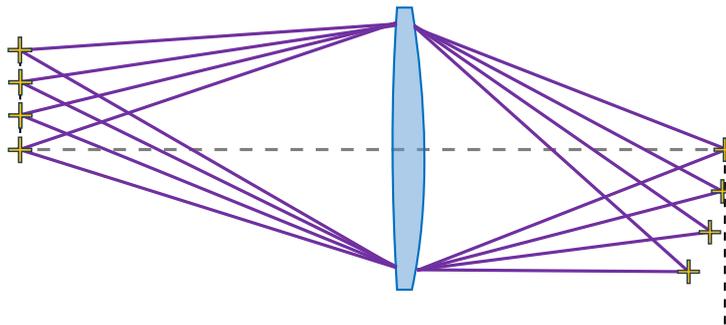**(b)** Illustrate distortion by making a sketch in the style of the below sketch for field curvature aberration.

Figure 2: Schematic illustration of field curvature.

Solution (Q3)

(a) Beyond distortion, the other 4 Seidel aberrations:

- Spherical aberration,
- Coma,
- Astigmatism,
- Field curvature.

(b) The sketch below shows that equal distances $A_o = B_o$ in the object plane are not equal in the image plane $(A_i \neq B_i)$.
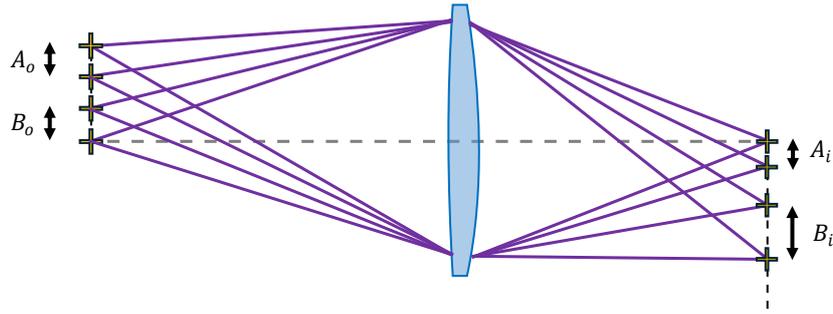
Figure 3: Sketch illustrating distortion, relative distances in the object plane are not conserved in the image plane.

Question 4: Blazed Fresnel lenses

(a) Which type of aberration is minimal for Fresnel lenses? Which one is often problematic for Fresnel lenses?
(b) What is the theoretical minimum thickness of a blazed Fresnel lens (variable pitch) having $n = 1.5$ and surrounded by air, for light with wavelength $\lambda = 600$ nm? Assume that the minimal thickness is given by the depth of the grooves (height of the teeth).

Solution (Q4)

(a) Fresnel lenses are thinner and flat, can increase the numerical aperture, and allow simpler aspheric implementation. Thereby they can in principle reduce many of the standard aberrations:

- Spherical aberration if using aspheric Fresnel lenses, which are easy to fabricate (relatively to regular Fresnel lenses).
- Decreasing the diffraction spot by increasing the lens diameter (numerical aperture).
- Reduces the ray displacement of chief rays, a typical problem in ticker lenses.

Typical problematic aberrations are:

- Chromatic aberration: hard to correct for in a Fresnel lens and the more complex Fresnel surface shape limits the options of achromatic materials.
- Loss of light and background glare: Light under an angle can scatter at and interfere with the walls of the grooves.
- Fabrication errors (imperfect groove profile, surface roughness) can lead to increased loss of light and background glare (randomly scattered light).

For this question it is sufficient to explain one typically minimal aberration and one problematic aberration when using Fresnel lenses.

(b) The phase difference is given by $\Delta\phi = \frac{2\pi}{\lambda}h(n-1)$ with h the groove depth (or heigth of the teeth). If we assume the thickness can be approximated by h (in theory), then:

$$h = \Delta\phi\frac{\lambda}{2\pi(n-1)} = \frac{\lambda}{(n-1)} = 1.2\,\text{micron},$$

when $\Delta\phi = 2\pi$ (smallest sensible phase difference).

Question 5: Waveguides

The number of modes $m = 1, 2, 3, \ldots, m_{\text{max}}$ in a waveguide of width d with perfect mirroring walls is given by $m_{\text{max}} < \frac{2d}{\lambda}$.

(a) If $d = 500$ nm and wavelength $\lambda = 488$ nm, what is the number of modes?
(b) What is the maximum λ to have at least one mode $(m = 1 = m_{\text{max}})$?

Solution (Q5)

(a) If $d = 500$ nm and wavelength $\lambda = 488$ nm, $m_{\text{max}} = 2$, therefore the number of modes is 2.
(b) The maximum $\lambda = 2d = 1000$ nm.