

Grading: The final exam counts for 30% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators are allowed but not necessary for the exam (numerical values will be fractions).

The duration of the final exam is 3 hours.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Beam expander

Consider a beam expander consisting of two thin lenses with $n = 1.5$. Lens 1 is a biconcave lens with curvature radii $R_1^{(1)} = -10$, $R_2^{(1)} = 10$ micron, clear diameter $D_1 = 5$ micron, and lens 2 is a biconvex lens with $R_1^{(2)} = 100$, $R_2^{(2)} = -100$ micron. See the figure below, and assume the Lensmaker's formula is valid (use ray optics):

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

- (a) What are the focal lengths of the lenses and what is the distance d between them?
- (b) What is the width w' of the beam after the lenses (assume D_2 is large enough)?

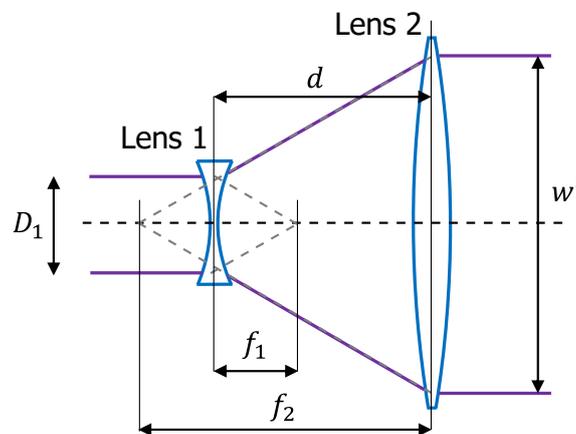


Figure 1: Optical setup for a beam expander.

Question 2: Chromatic aberration

Consider three thin lenses with same size ($R_1 = 50$, $R_2 = -10$, $D = 5$ micron), but made from three different materials with refraction indices $n_1(\lambda)$, $n_2(\lambda)$, and $n_3(\lambda)$. In below figure the refraction indices are plot as function of the wavelength λ .

- (a) Which lens will have the smallest focal length for red light? And which lens will have the smallest focal length for blue light?
- (b) Which lens will have the largest Abbe V-number?

Hint: The Abbe V-number can be calculated by: $V = \frac{n(588)-1}{n(486)-n(656)}$

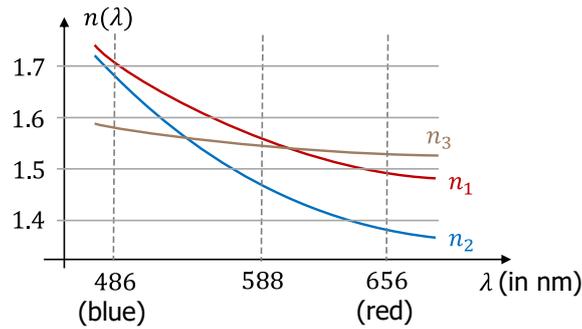


Figure 2: The refractive index as function of wavelength for three (artificial) materials.

Question 3: Strehl ratio

Consider a lens with $f = 5$ micron, diameter $D = 10$ micron, and $\sigma_w = \lambda/1000$.

- Approximate the Strehl ratio by $S \approx 1 - (4\pi^2/\lambda^2)\sigma_w^2$. Is this a good approximation?
- Calculate the Airy disk diameter $d = 2.44\lambda f/D$ for $\lambda = 1000$ nm. What can you say about the geometrical spot size versus the diffraction spot size?

Question 4: Discrete two-level (binary) lenses

Consider a binary version of a Fresnel convex lens with phase differences 0 and π .

- Sketch a convex Fresnel lens and binary approximation of it.
- If you want to have a diverging lens, what should you change? Sketch this for both the Fresnel lens and the binary lens.

Question 5: Waveguide directional coupler

Consider a directional coupler existing of two identical single-mode waveguides with coupling length L and coupling coefficient C . If light enters waveguide 1 with optical power $P_{1,in} = P_1(0)$ then the optical power at the exits of waveguide 1 and 2: $P_{1,out} = P_1(L)$ and $P_{2,out} = P_2(L)$ are given by:

$$P_1(L) = P_1(0) \cos^2(CL)$$

$$P_2(L) = P_1(0) \sin^2(CL)$$

- Suppose that $L = \pi/4C$, what are the optical powers $P_{1,out}$ and $P_{2,out}$ at the exits?
- Suppose that again $L = \pi/4C$, but then increase the coupling coefficient $C' = 2C$ (while L stays constant), how much optical power is transferred to waveguide 2: $P_{2,out}$?

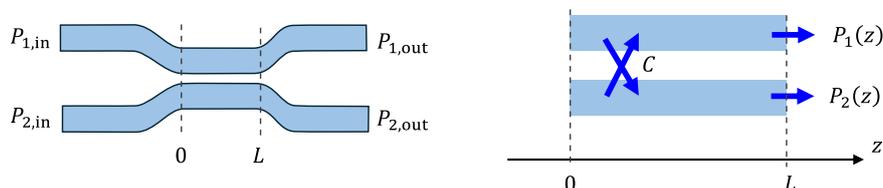


Figure 3: Schematic of a directional coupler.