# PHOT 301: Quantum Photonics

# LECTURE 04

Michaël Barbier, Summer (2024-2025)

# OVERVIEW

| week | Topic | Reading |
|---|---|---|
| Week 1 | Introduction & Required Mathematical Methods.<br>Waves and Schrödinger's equation, Probability, Uncertainty and Time evolution.<br>Infinite square well. | |
| Week 2 | The harmonic oscillator, Creation and annihilation operators.<br>Free particle, 1D Bound states & Scattering/Transmission, Finite well | **Ch. 2**<br>**(from Harmonic oscillator)** |
| Week 3 | Quantum mechanics formalism: Functions and operators, uncertainty.<br>Approximation methods. | |
| Week 4 | Angular momentum and the Hydrogen atom, Spin<br>Magnetic fields, The Pauli equation, Minimal Coupling, Aharonov Bohm<br>Perturbation: Fine Structure of Hydrogen, The Zeeman Effect | |
| Week 5 | Identical particles, Periodic table, Molecular bonds, Periodic structures, Band structure, Bloch functions<br>Time-dependent perturbation: Absorption, spontaneous emission, and stimulated emission | |
| Week 6 | Final exam | |

# FOR NEXT WEEK

- Textbook Chapter 2: 2.11, 2.13, 2.14, 2.17, 2.18, 2.25, 2.31, 2.34, 2.41, 2.53
- Homework documents:
  - phot301_homework_matrices.pdf
  - phot301_homework_system_of_equations.pdf
  - phot301_homework_eigenvalue_equations.pdf
- Reading (by Thursday 31 July 2025): Chapter 3 of Griffiths

# SUMMARY

So far we looked at bound states

- Infinite well
- Linear potential well (Electrical field, not seen yet)
- Harmonic oscillator

Different well potentials lead to different allowed energy levels

Narrower wells $\longrightarrow$ less energy levels (more spread)

Discrete spectrum of energy-levels

# FREE PARTICLES

# FREE PARTICLE: PROPAGATING WAVES

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x), \qquad \text{as } V(x) = 0$$

$$\Rightarrow \qquad \frac{d^2\psi(x)}{dx^2} = -k^2\psi(x), \qquad \text{with } k = \frac{\sqrt{2mE}}{\hbar}$$

- Solutions are unconstrained: all energy values
- Similar to a very wide well (with infinite walls)

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

# FREE PARTICLE SOLUTIONS

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

- Problem 1: Not normalizable
- Problem 2: Velocity is half of classical velocity

# PROBLEM 1: NOT NORMALIZABLE

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$\begin{aligned}
|\psi(x)|^2 &= \psi(x)^* \, \psi(x) \\
&= (A^* e^{-ikx} + B^* e^{ikx})(Ae^{ikx} + Be^{-ikx}) \\
&= |A|^2 + |B|^2 + (AB^* e^{i2kx} + A^* Be^{-i2kx}) \\
&= |A|^2 + |B|^2 + \Re\left(AB^* e^{i2kx}\right)
\end{aligned}$$

$$\Rightarrow \int_{-\infty}^{+\infty} |\psi(x)|^2 dx = (|A|^2 + |B|^2)\,\infty + \int_{-\infty}^{+\infty} \Re\left(AB^* e^{i2kx}\right) dx$$

The last integral is bounded (oscillating between finite values)

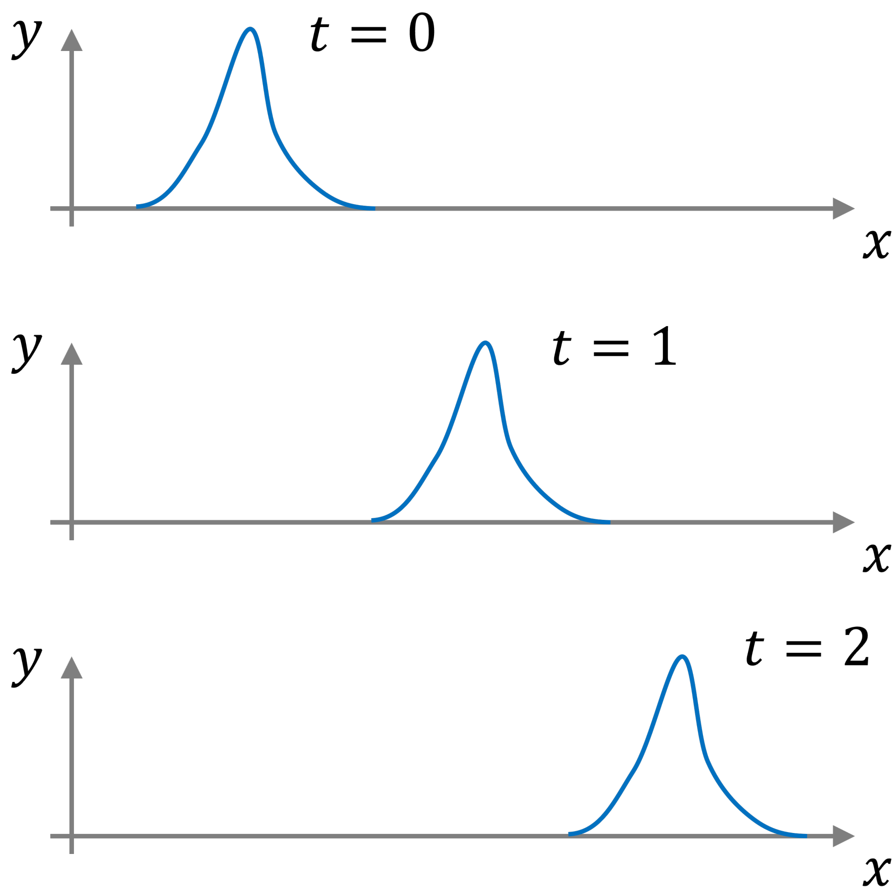
The total integral $\int |\psi|^2 dx \longrightarrow +\infty$, and therefore doesn't exist.

# PROBLEM 2: VELOCITY TOO SLOW

We can rewrite $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ with $E = \frac{\hbar^2 k^2}{2m}$:

$$\begin{aligned}\Psi(x,t) &= Ae^{ikx - iEt/\hbar} + Be^{-ikx - iEt/\hbar} \\ &= Ae^{ikx - i\hbar k^2 t/2m} + Be^{-ikx - i\hbar k^2 t/2m} \\ &= Ae^{ik(x - \frac{\hbar k}{2m}t)} + Be^{-ik(x - \frac{\hbar k}{2m}t)}\end{aligned}$$

- Both exponentials are a function of $x \pm vt$, a "wave" moving with velocity $v$



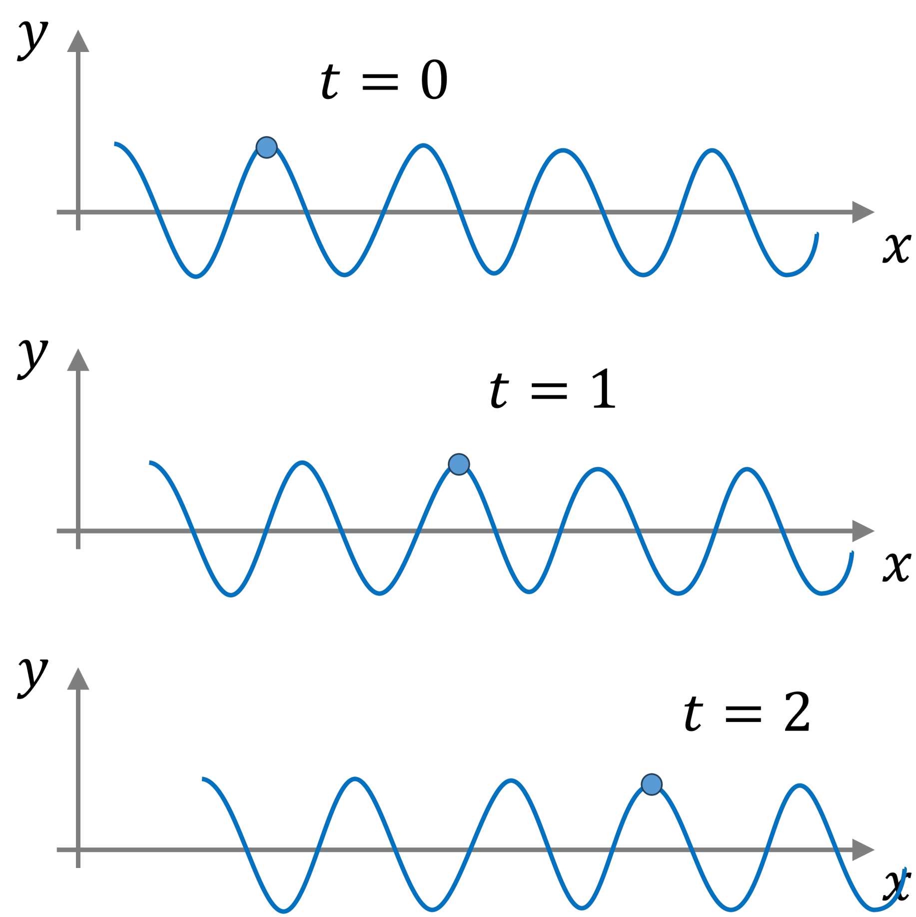

Example of a propagating "wave". It does not change its shape in time.

# PROBLEM 2: VELOCITY TOO SLOW

We can rewrite $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ with $E = \frac{\hbar^2 k^2}{2m}$:

$$\begin{aligned}\Psi(x,t) &= Ae^{ikx - iEt/\hbar} + Be^{-ikx - iEt/\hbar} \\ &= Ae^{ikx - i\hbar k^2 t/2m} + Be^{-ikx - i\hbar k^2 t/2m} \\ &= Ae^{ik(x - \frac{\hbar k}{2m}t)} + Be^{-ik(x - \frac{\hbar k}{2m}t)}\end{aligned}$$

- Both exponentials are a function of $x \pm vt$, a "wave" moving with velocity $v$




Real part of a propagating wave $e^{ik(x - \frac{\hbar k}{2m}t)}$ (Propagating to the right).

# PROBLEM 2: VELOCITY TOO SLOW

We can rewrite $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ with $E = \frac{\hbar^2 k^2}{2m}$:

$$\begin{aligned}\Psi(x,t) &= Ae^{ikx-iEt/\hbar} + Be^{-ikx-iEt/\hbar}\\ &= Ae^{ik(x-\frac{\hbar k}{2m}t)} + Be^{-ik(x-\frac{\hbar k}{2m}t)}\end{aligned}$$

- This is a function of $x \pm vt$, a “wave” moving with velocity $v$
- The velocity is $v_{\text{quantum}} = \frac{\hbar k}{2m} = \sqrt{\frac{E}{2m}}$
- Classically the velocity $v_{\text{classical}} = \sqrt{\frac{2E}{m}}$ because $E = \frac{1}{2}mv^2$
- The classical velocity is twice the one according to quantum mechanics!

# WAVE PACKETS

- Superposition of propagating waves is normalizable
- Solves the velocity problem as well!
- Is consistent with uncertainty principle
- Fourier's trick to find coefficients

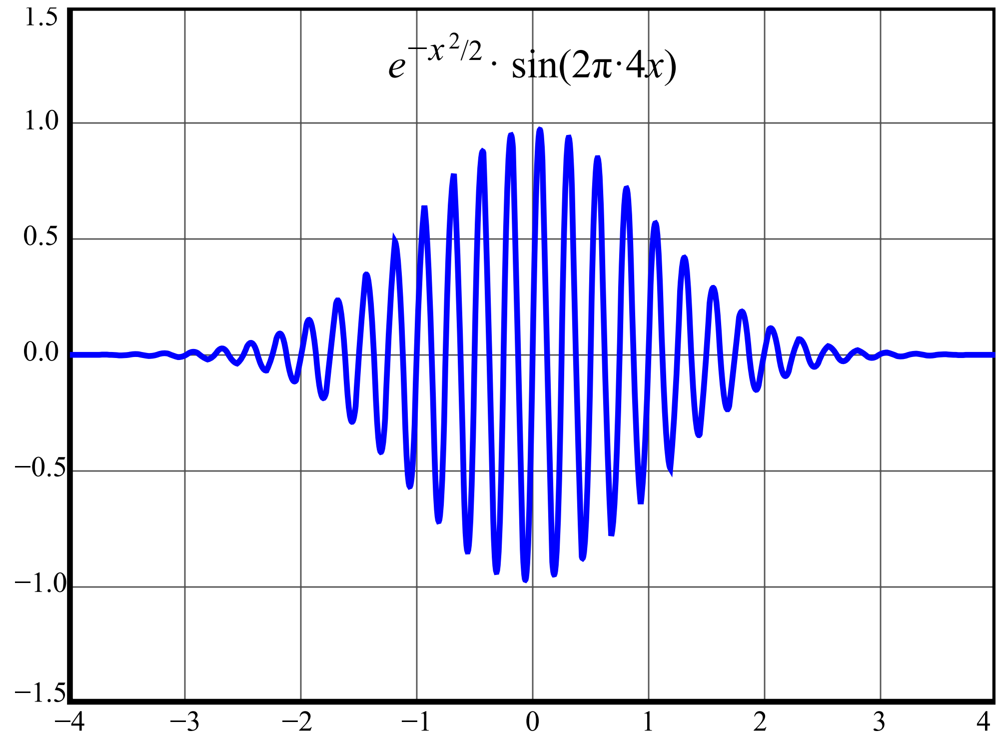
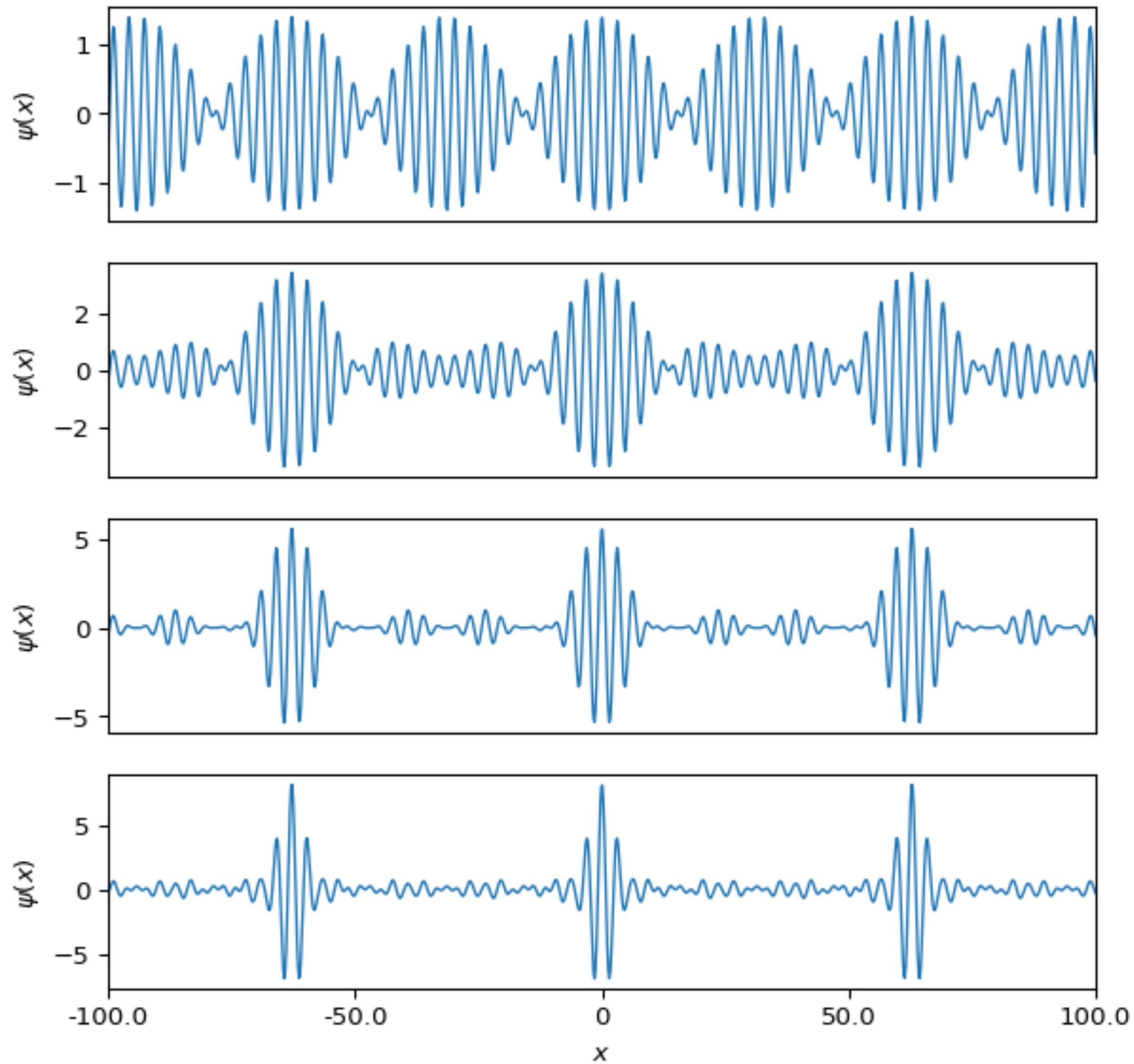



Image from Wikipedia

# LOCALIZED WAVES USING MORE K-VALUES

- Superposition of waves
- Momentum around main $\sum_{n=-N}^{N} k_0 \pm n\delta k$
- Plots from top to bottom $N = 1, 2, 4, 8$

$$\psi(x) = \sum_{n=-N}^{N} e^{i(k_0 + n\delta k)x}$$



# GROUP VELOCITY OF THE WAVE PACKET

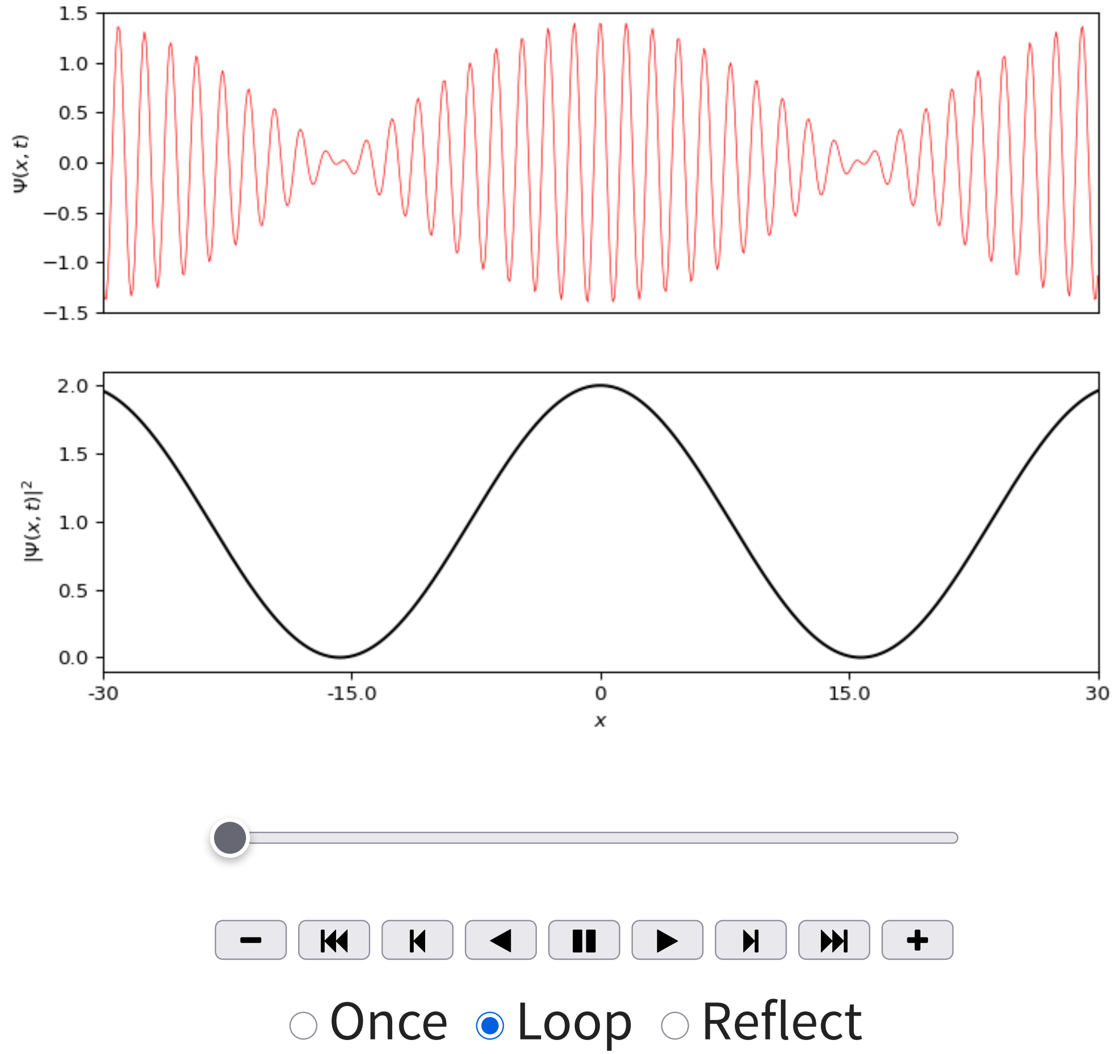
Phase velocity $v$

$$v = \frac{\omega}{k}$$

Group velocity:

$$v_g = \frac{d\omega}{dk}$$

$$\Psi(x,t) = \sum_{n=-N}^{N} e^{i(k_0 + n\delta k)x}$$



# PHASE AND GROUP VELOCITY

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int \phi(k) e^{i(kx-\omega t)}$$

Assume $\phi(k)$ around $k_0$

$$\begin{aligned}\Psi(x,t) &\approx \frac{1}{\sqrt{2\pi}} \int \phi(k_0+s) e^{i((k_0+s)x-(\omega_0+s\omega_0')t)} \\ &\approx \frac{1}{\sqrt{2\pi}} e^{i(k_0 x-\omega_0 t)} \int \phi(k_0+s) e^{is(x-\omega_0' t)}\end{aligned}$$

# GENERAL WAVE PACKETS: FOURIER TRANSFORMS

- Wave packets of **any shape**
- Includes all possible k-values (energies)

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i\left(kx - \frac{\hbar k^2}{2m} t\right)}\, dk$$

- Time-independent $\Psi(x,0) = \psi(x)$:

$$\Psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{ikx}\, dk$$

Fourier transform

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ikx}\, dx,$$

Inverse Fourier transform

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{ikx}\, dk$$

# WAVE PACKET TIME EVOLUTION

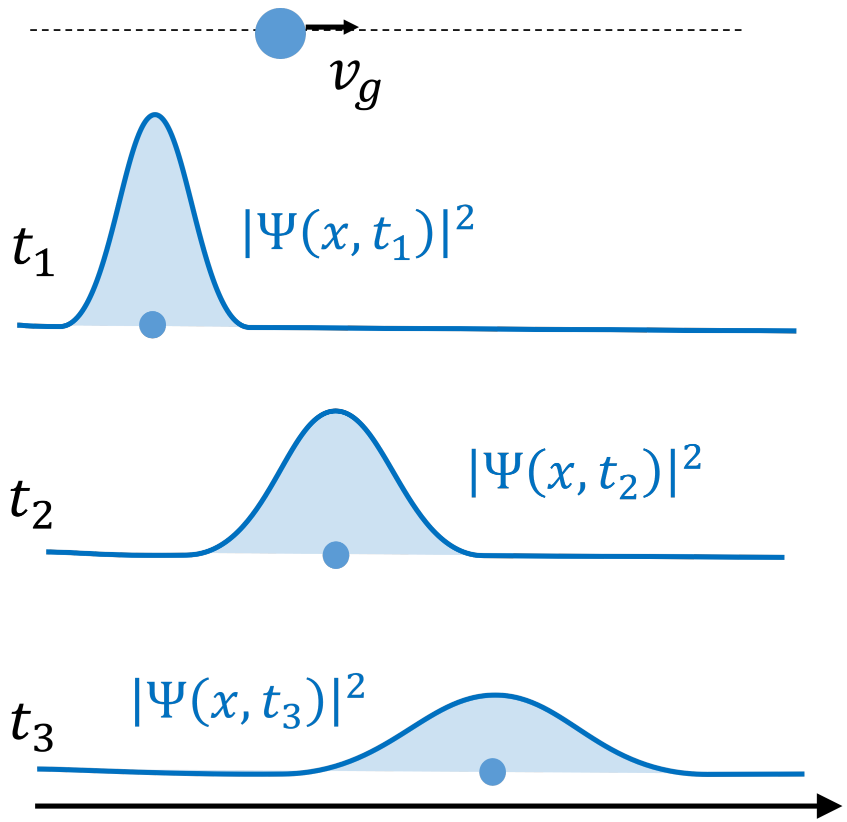
- We know $\Psi(x, 0)$ and

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) e^{i(kx-\omega(k)t)}\, dk,$$

$$\omega(k) = \frac{\hbar k^2}{2m}$$

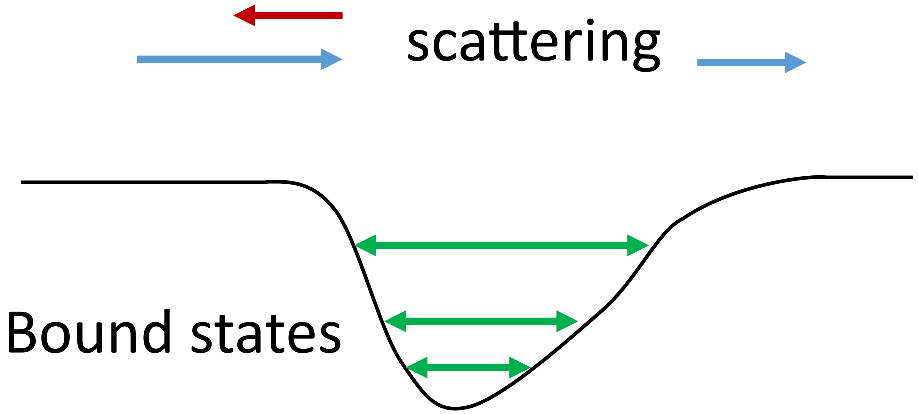
- Phase velocity of $k < \langle k \rangle$ slower
- Phase velocity of $k > \langle k \rangle$ faster

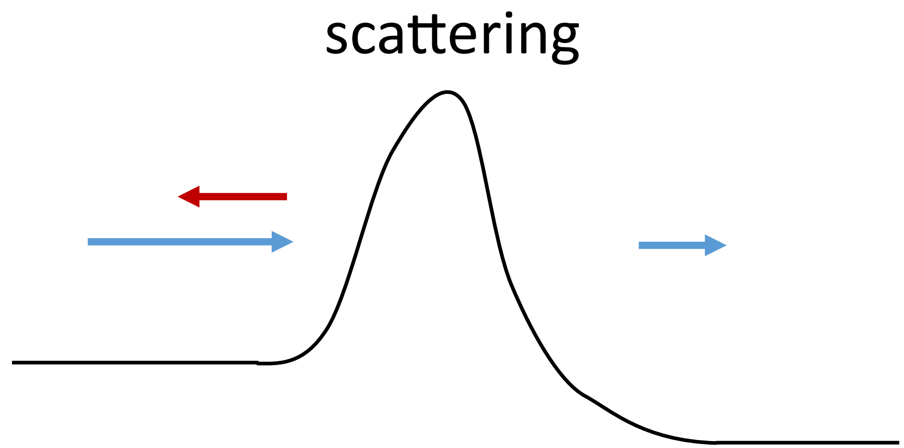
–> Wave packets broadens in time



# BOUND STATES AND SCATTERING

# BOUND STATES AND FREE PARTICLES



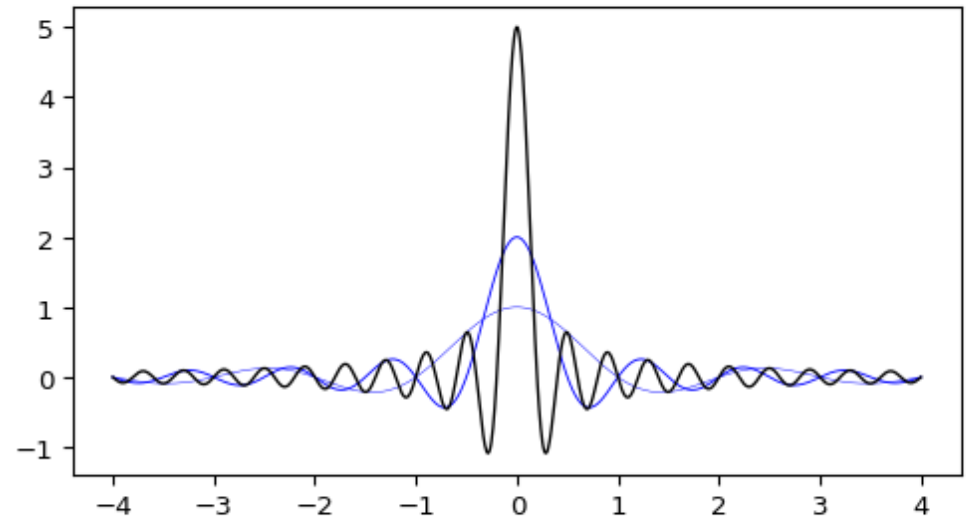

- Bound states have a bounded domain
- Particle is stuck in a potential valley
- Free particles can go everywhere
- **Scattering** of particles with $E > V_\infty$
- Scattering also called **Tunneling** when $E < V_{\max}$

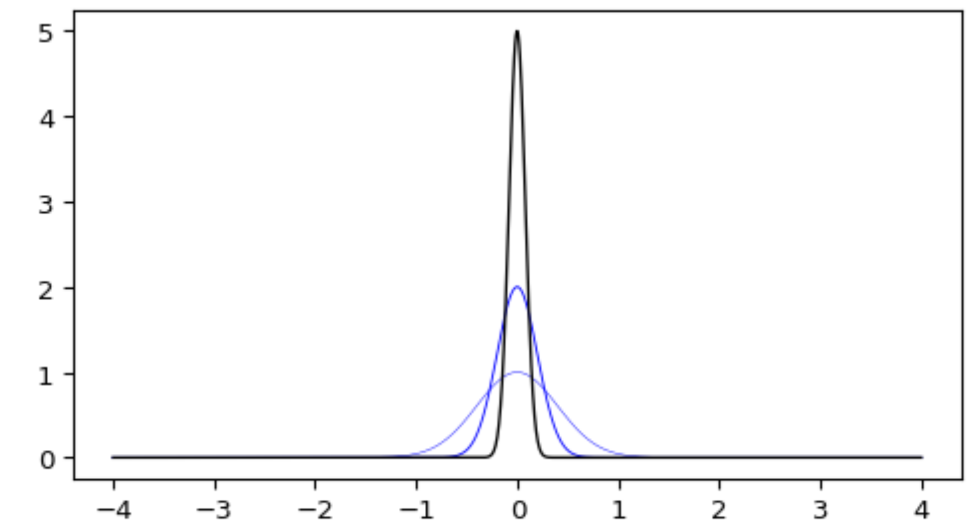
# INTERMEZZO: THE DIRAC DELTA FUNCTION

Dirac delta distribution:

$$\begin{cases} \delta(x \neq 0) = 0 \\ \delta(x = 0) = +\infty \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(x) = 1$$




Limit of series of functions:

- peaked such as sinc(x) or Gaussian
- limit to infinitely *thin* and *high*
- Area kept normalized

**Filters** out single point:

$f(a) = \int_{-\infty}^{+\infty} f(x)\, \delta(x-a)\, dx$

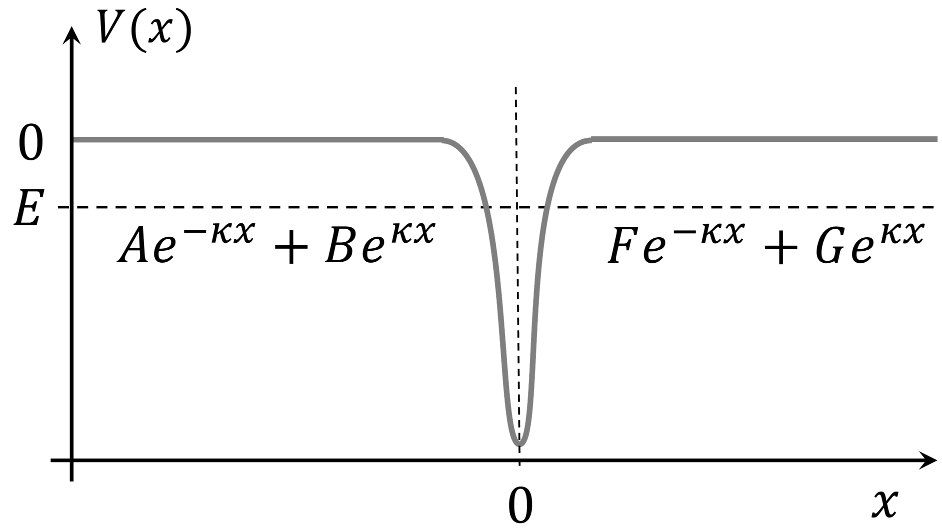
# DELTA FUNCTION POTENTIAL WELL

The potential energy function:

$$V(x) = -\alpha\delta(x \neq 0)$$

$$\begin{cases} V(x \neq 0) = -\alpha\delta(x \neq 0) = 0 \\ V(x \neq 0) = -\alpha\delta(x \neq 0) = -\alpha\infty \end{cases}$$

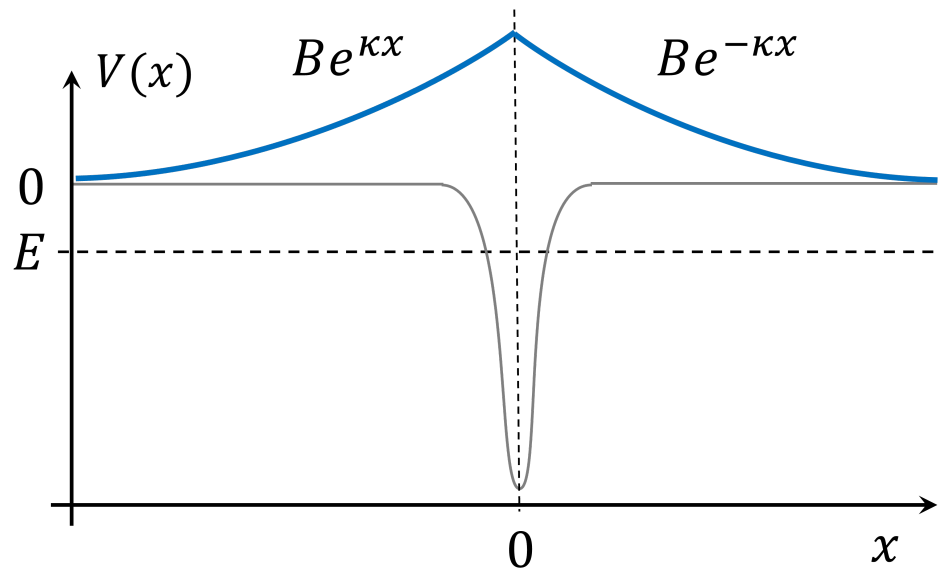
$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} - \alpha\delta(x)\psi = E\psi$$




Bound states have $E < 0$. Outside the well:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi$$

where $E < 0$ and thus exponential solutions



# DELTA FUNCTION POTENTIAL WELL

Bound states have $E < 0$. Outside the well:

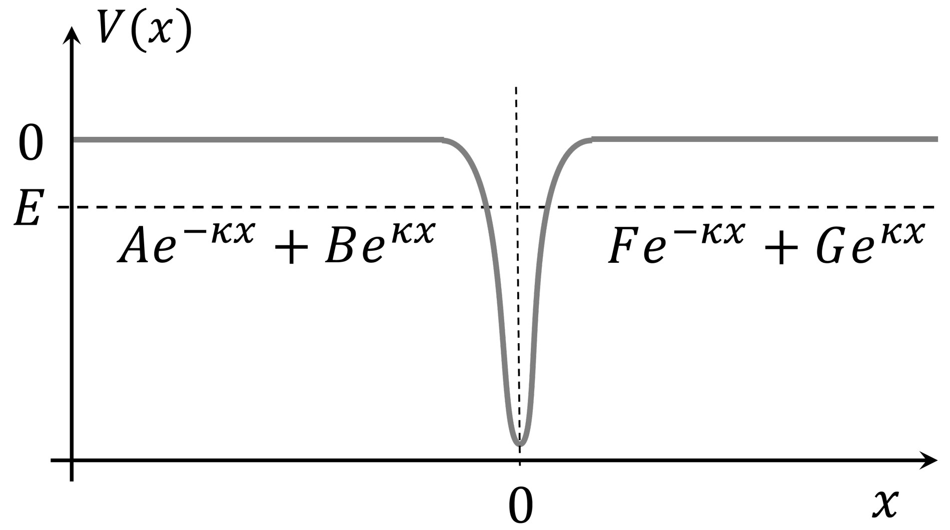
$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi$$

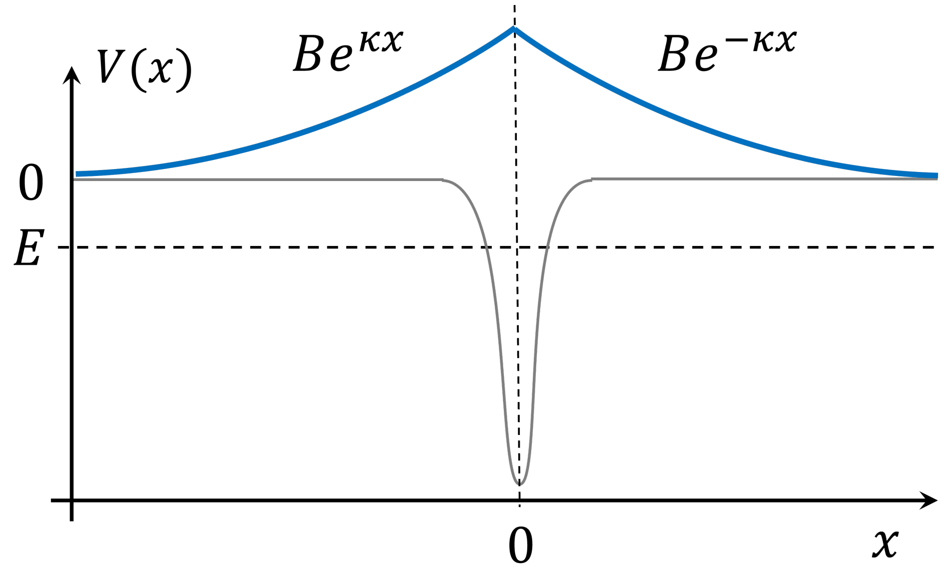
Exponential solutions:

$$\psi(x) \propto e^{\pm\kappa x}, \qquad \text{with} \quad \kappa = \frac{\sqrt{-2mE}}{\hbar}$$

Wave function must be continuous:

$$\psi(x<0) = \psi(x>0)$$



# DELTA FUNCTION POTENTIAL WELL

Discontinuity: integrate the Schrodinger equation

$$-\frac{\hbar^2}{2m}\int_{-\epsilon}^{+\epsilon}\frac{d^2\psi(x)}{dx^2}\,dx-\alpha\int_{-\epsilon}^{+\epsilon}\delta(x)\psi\,dx=E\int_{-\epsilon}^{+\epsilon}\psi\,dx$$

$$-\frac{\hbar^2}{2m}\frac{d\psi(x)}{dx}\Big|_{-\epsilon}^{+\epsilon}-\alpha\psi(0)=0$$

Then take the limit for $\epsilon\longrightarrow 0$

$$\Delta\left(\frac{d\psi(x)}{dx}\right)=-\frac{2m}{\hbar^2}\alpha\psi(0)$$

We know the derivatives in $x=\pm 0$:

$$-B\kappa-(B\kappa)\quad\Rightarrow\quad\Delta\left(\frac{d\psi(x)}{dx}\right)=-2\kappa B$$

# DELTA FUNCTION POTENTIAL WELL

$$\begin{cases} \Delta\left(\dfrac{d\psi(x)}{dx}\right) = -\dfrac{2m}{\hbar^2}\alpha\psi(0) \\ \Delta\left(\dfrac{d\psi(x)}{dx}\right) = -2\kappa B \end{cases}$$

Make use of $\psi(0) = B$

$$-\frac{2m}{\hbar^2}\alpha B = -2\kappa B$$

$$\Rightarrow \quad \kappa = \frac{m\alpha}{\hbar^2}$$

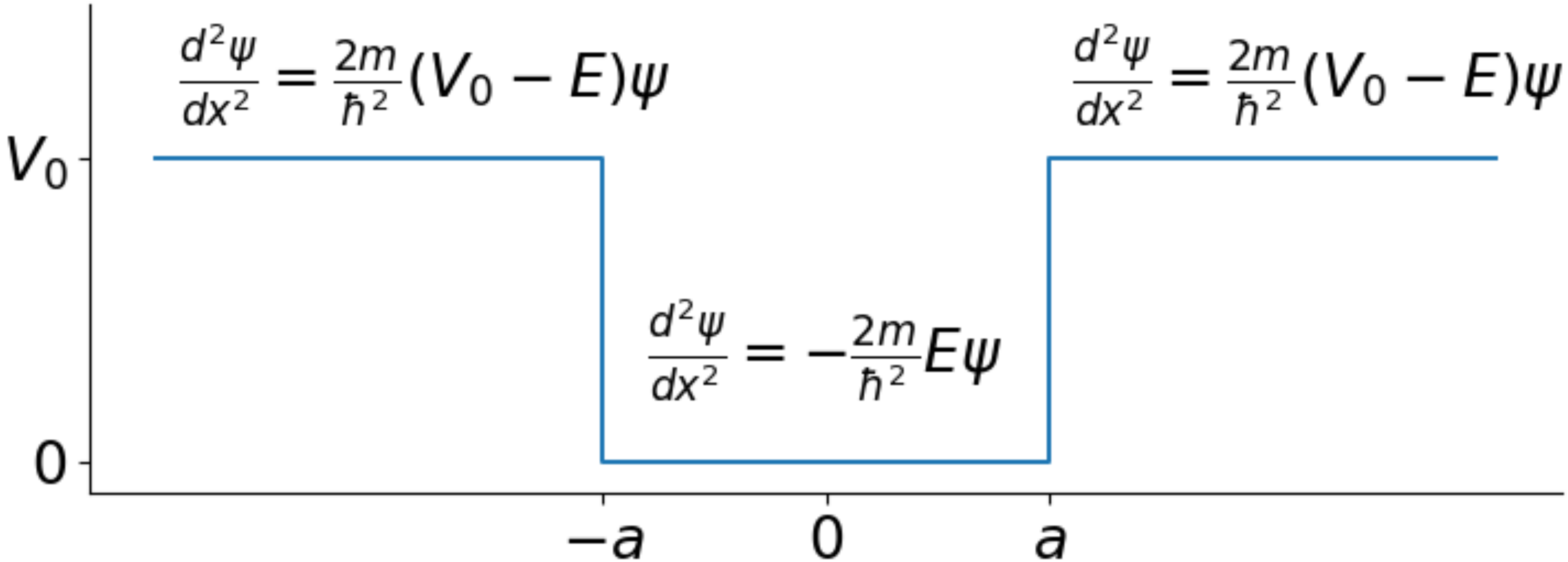
$$\Rightarrow \quad E = -\frac{\hbar^2\kappa^2}{2m} = -\frac{m\alpha^2}{2\hbar^2}$$

The eigenstate with energy $E$ becomes after normalization:

$$\psi = Be^{-\kappa|x|} = \sqrt{\kappa}e^{-\kappa|x|}$$

# FINITE POTENTIAL BARRIERS/ STEPS/WELLS

# FINITE SQUARE WELL

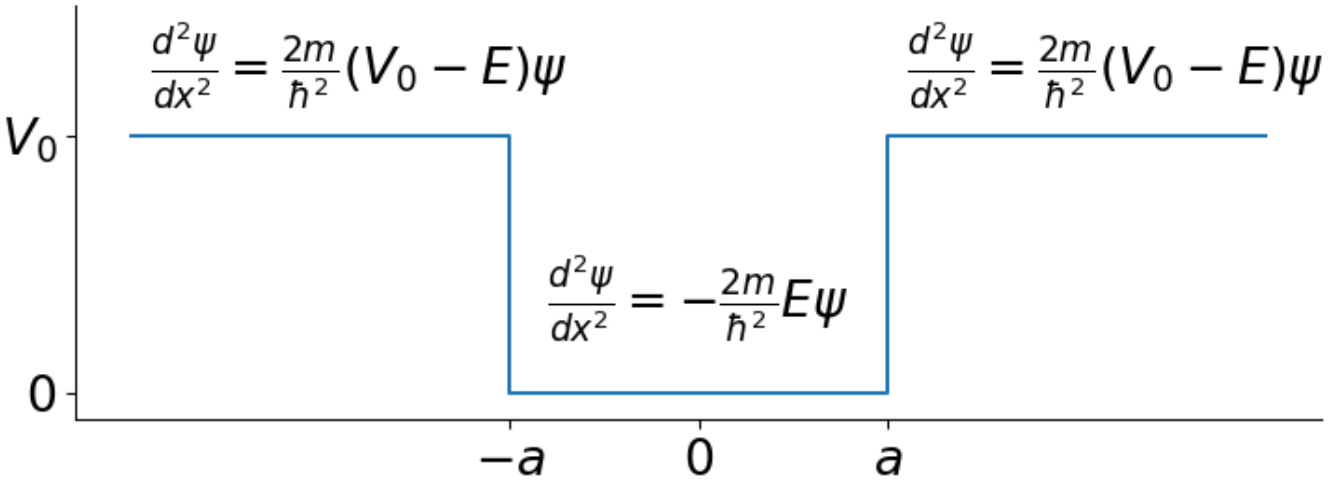



- Finite square potential well
- Bound states $0 < E < V_0$
- Scattering states $E > V_0$

Schrodinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = (E - V(x))\;\psi(x)$$

# FINITE WELL: BOUND STATES

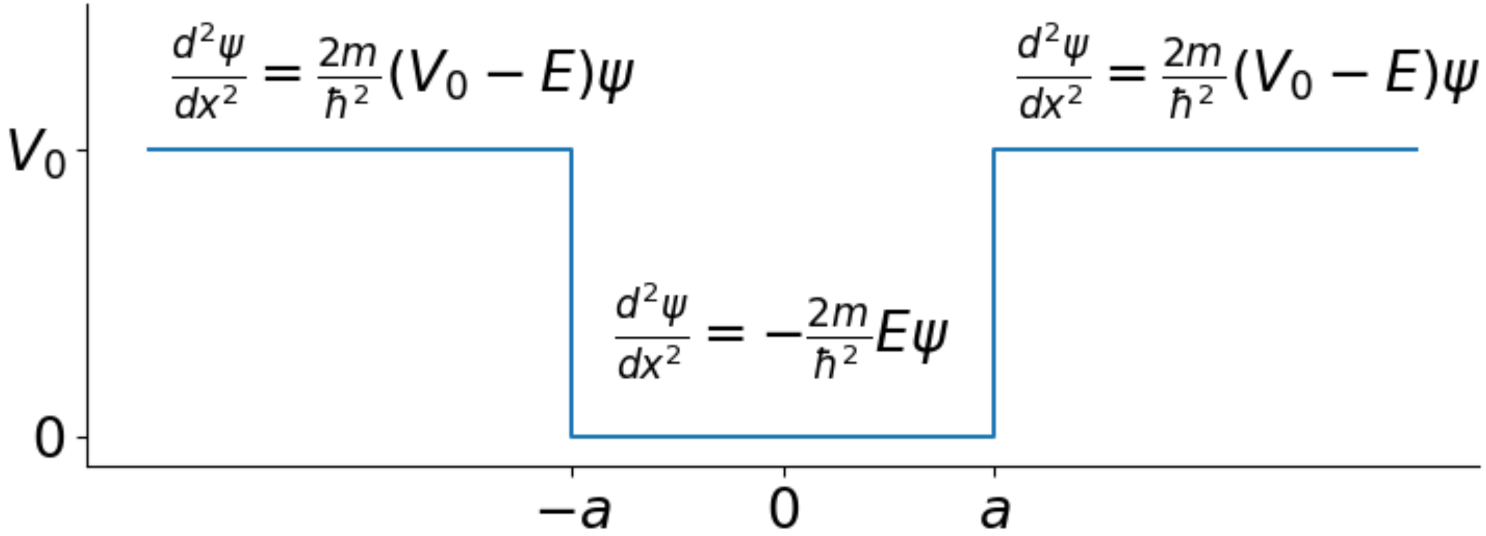



For **bound states** $0 < E < V_0$

$$\text{inside:} \quad \psi(-a < x < a) = C\sin(\lambda x) + D\cos(\lambda x) \qquad \lambda = \frac{\sqrt{2mE}}{\hbar}$$

$$\text{outside:} \quad \begin{aligned} \psi(x < -a) &= Ae^{-\kappa x} + Be^{\kappa x} = Be^{\kappa x} \\ \psi(x > a) &= Fe^{-\kappa x} + Ge^{\kappa x} = Fe^{-\kappa x} \end{aligned} \qquad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

# FINITE WELL: BOUND STATES




Since $\psi(x)$ is continuous in $x = -a$ and $x = a$:

$$\begin{aligned} Be^{-\kappa a} &= -C\sin(\lambda a) + D\cos(\lambda a) \\ Fe^{-\kappa a} &= C\sin(\lambda a) + D\cos(\lambda a) \end{aligned} \qquad \psi(x) \text{ continuous in } x = -a, a$$

$$\begin{aligned} B\kappa e^{-\kappa a} &= \lambda C\cos(\lambda a) + D\sin(\lambda a) \\ -F\kappa e^{-\kappa a} &= \lambda C\cos(\lambda a) - \lambda D\sin(\lambda a) \end{aligned} \qquad \frac{d\psi(x)}{dx} \text{ continuous in } x = -a, a$$

# FINITE WELL: BOUND STATES

$$\begin{aligned} Be^{-\kappa a} &= -C\sin(\lambda a) + D\cos(\lambda a) \\ Fe^{-\kappa a} &= C\sin(\lambda a) + D\cos(\lambda a) \end{aligned} \qquad \psi(x) \text{ continuous in } x = -a, a$$

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Add the first two equations and subtract the others:

$$\begin{aligned} (B+F)e^{-\kappa a} &= 2D\cos\lambda a \\ (B+F)\frac{\kappa}{\lambda}e^{-\kappa a} &= 2D\sin\lambda a \end{aligned}$$

And divide them $\Leftrightarrow B \neq -F$

$$\frac{\kappa}{\lambda} = \tan\lambda a$$

# FINITE WELL: BOUND STATES

$$\begin{aligned} Be^{-\kappa a} &= -C\sin(\lambda a) + D\cos(\lambda a) \\ Fe^{-\kappa a} &= C\sin(\lambda a) + D\cos(\lambda a) \end{aligned} \qquad \psi(x) \text{ continuous in } x = -a, a$$

$$\begin{aligned} B\kappa e^{-\kappa a} &= \lambda C\cos(\lambda a) + \lambda D\sin(\lambda a) \\ -F\kappa e^{-\kappa a} &= \lambda C\cos(\lambda a) - \lambda D\sin(\lambda a) \end{aligned} \qquad \frac{d\psi(x)}{dx} \text{ continuous in } x = -a, a$$

Now subtract the first two equations and add the others:

$$\begin{aligned} (B-F)e^{-\kappa a} &= -2C\sin\lambda a \\ (B-F)\frac{\kappa}{\lambda}e^{-\kappa a} &= 2C\cos\lambda a \end{aligned}$$

And divide them $\Leftrightarrow B \neq F$

$$\begin{aligned} \frac{\kappa}{\lambda} &= -\cot\lambda a \qquad && \text{if } B \neq F \\ \frac{\kappa}{\lambda} &= \tan\lambda a \qquad && \text{if } B \neq -F \text{ from before} \end{aligned}$$

# FINITE WELL: BOUND STATES

$$\frac{\kappa}{\lambda} = -\cot\lambda a \qquad \text{if } B \neq F$$
$$\frac{\kappa}{\lambda} = \tan\lambda a \qquad \text{if } B \neq -F \text{ from before}$$

- The relations are inconsistent!
- Only possible if one of them is invalid: $B = F$ or $B = -F$

$$B = F \quad \Rightarrow \quad \begin{aligned} (B-F)e^{-\kappa a} &= -2C\sin\lambda a \\ (B-F)\frac{\kappa}{\lambda}e^{-\kappa a} &= 2C\cos\lambda a \end{aligned} \quad \Rightarrow C = 0, \psi(x) = D\cos(\lambda x)$$

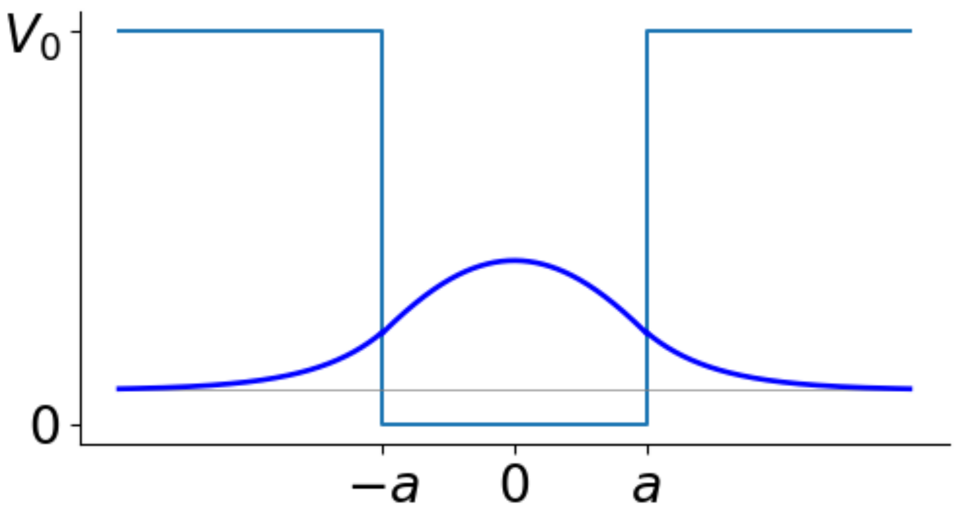
$$B = -F \quad \Rightarrow \quad \begin{aligned} (B+F)e^{-\kappa a} &= 2D\cos\lambda a \\ (B+F)\frac{\kappa}{\lambda}e^{-\kappa a} &= 2D\sin\lambda a \end{aligned} \quad \Rightarrow D = 0, \psi(x) = C\sin(\lambda x)$$

# FINITE WELL: BOUND STATES (WAVE FUNCTION)

**Symmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = D\cos(\lambda x) \\ \psi(x > a) = Be^{-\kappa x} \end{cases}$$

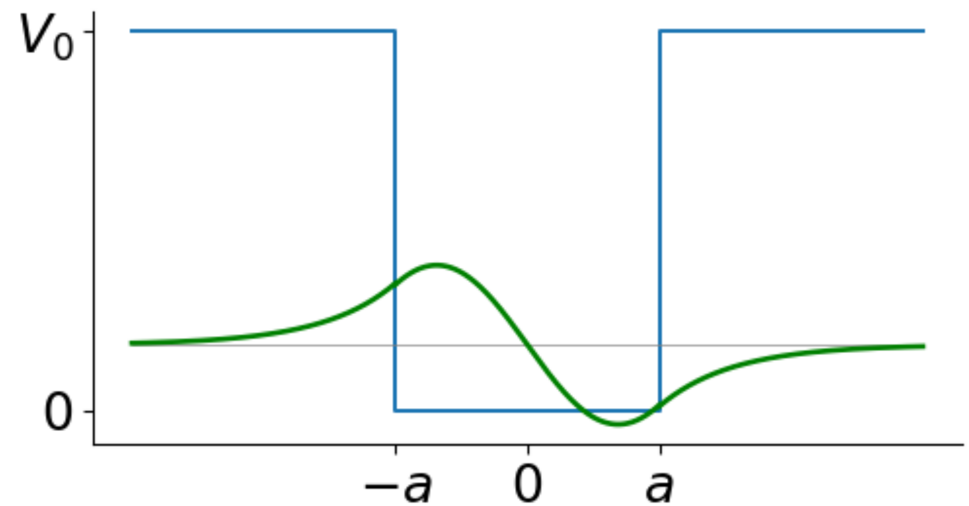
with $F = B \quad \& \quad \kappa = \lambda \tan \lambda a$




**Asymmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = C\sin(\lambda x) \\ \psi(x > a) = -Be^{-\kappa x} \end{cases}$$

with $F = -B \quad \& \quad \kappa = -\lambda \cot \lambda a$



# FINITE WELL: BOUND STATES

**Symmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = D\cos(\lambda x) \\ \psi(x > a) = Be^{-\kappa x} \end{cases}$$

with $F = B \quad \& \quad \kappa = \lambda \tan \lambda a$

**Asymmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = C\sin(\lambda x) \\ \psi(x > a) = -Be^{-\kappa x} \end{cases}$$

with $F = -B \quad \& \quad \kappa = -\lambda \cot \lambda a$

Let's first obtain the energy from $\kappa$ and $\lambda$:

$$\kappa^2 = \frac{2m}{\hbar^2}(V_0 - E), \qquad \lambda^2 = \frac{2m}{\hbar^2}(E)$$

Define $z = \lambda\, a$ and $z_0 = a\,\frac{\sqrt{2mV_0}}{\hbar}$

$$\implies \quad \kappa^2 + \lambda^2 = \frac{2m}{\hbar^2}(V_0) = z_0^2/a^2 \quad \implies \quad \kappa^2 a^2 = z_0^2 - z^2$$

# FINITE WELL: BOUND STATES

**Symmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = D\cos(\lambda x) \\ \psi(x > a) = Be^{-\kappa x} \end{cases}$$

with $F = B \quad \& \quad \kappa = \lambda \tan \lambda a$

$$\implies \tan z = \sqrt{\frac{z_0^2}{z^2} - 1}$$

**Asymmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = C\sin(\lambda x) \\ \psi(x > a) = -Be^{-\kappa x} \end{cases}$$

with $F = -B \quad \& \quad \kappa = -\lambda \cot \lambda a$

$$\implies -\cot z = \sqrt{\frac{z_0^2}{z^2} - 1}$$

where we made use of:

$$\kappa^2 a^2 = z_0^2 - z^2 \implies \kappa/\lambda = \sqrt{z_0^2/z^2 - 1}$$

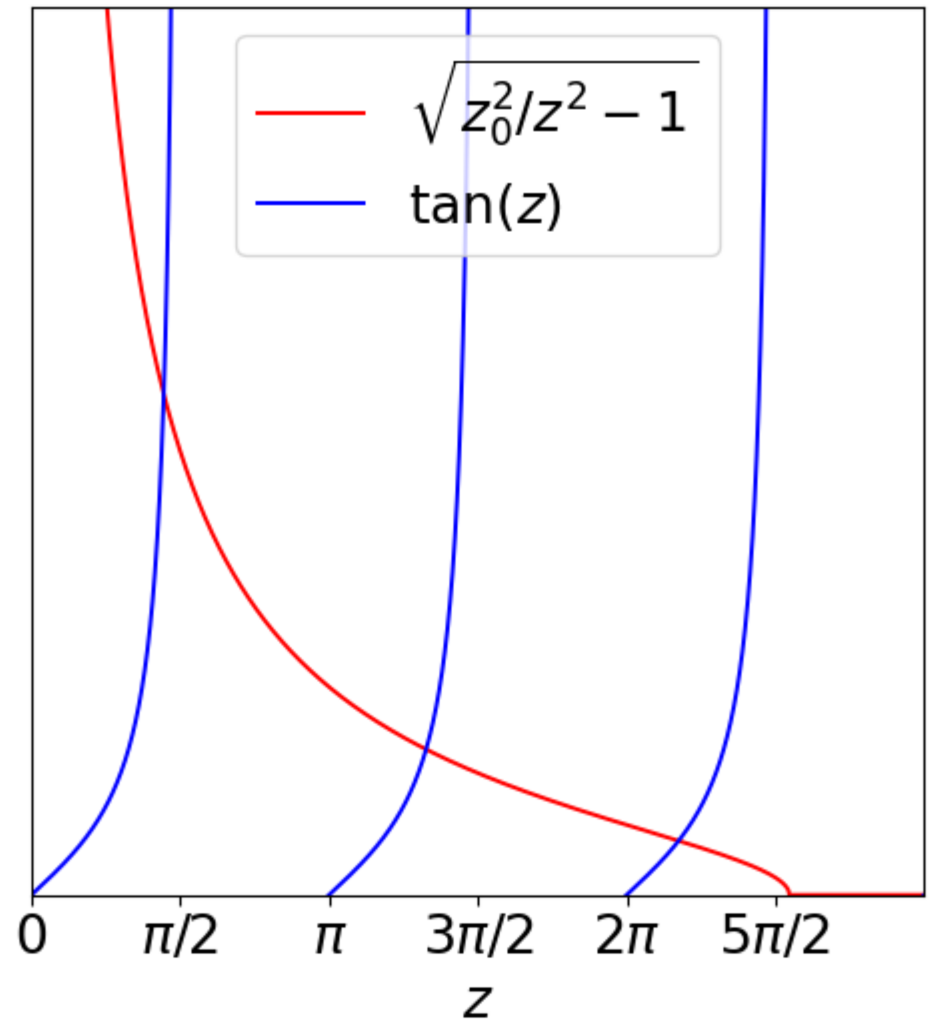
# FINITE WELL: E FOR SYMMETRIC BOUND STATES

**Symmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = D\cos(\lambda x) \\ \psi(x > a) = Be^{-\kappa x} \end{cases}$$

with $F = B \quad \& \quad \kappa = \lambda \tan \lambda a$

$$\implies \tan z = \sqrt{\frac{z_0^2}{z^2} - 1}$$



# FINITE WELL: E FOR ASYMMETRIC BOUND STATES

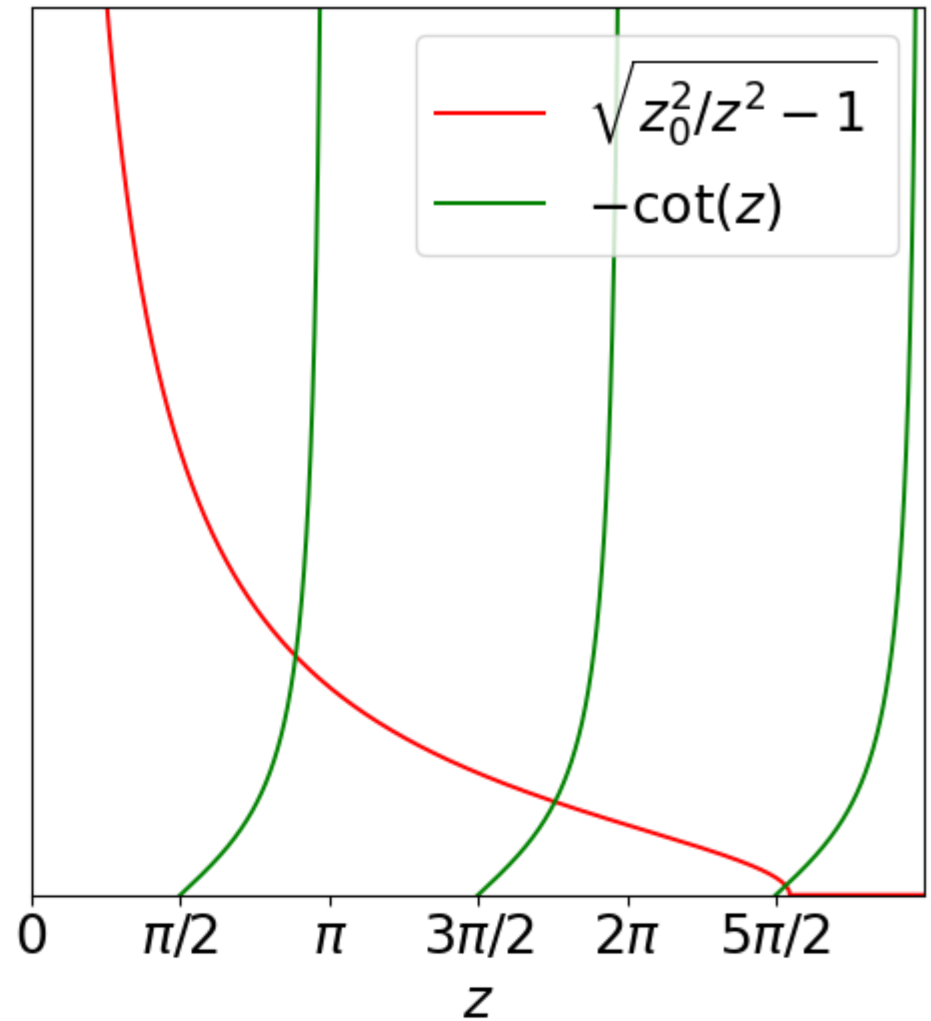
**Asymmetric solutions**

$$\begin{cases} \psi(x < -a) = Be^{\kappa x} \\ \psi(-a < x < a) = C\sin(\lambda x) \\ \psi(x > a) = -Be^{-\kappa x} \end{cases}$$

with $F = -B \quad \& \quad \kappa = -\lambda \cot \lambda a$

$$\implies -\cot z = \sqrt{\frac{z_0^2}{z^2} - 1}$$

Asymmetric energies approximately "shifted" to the right

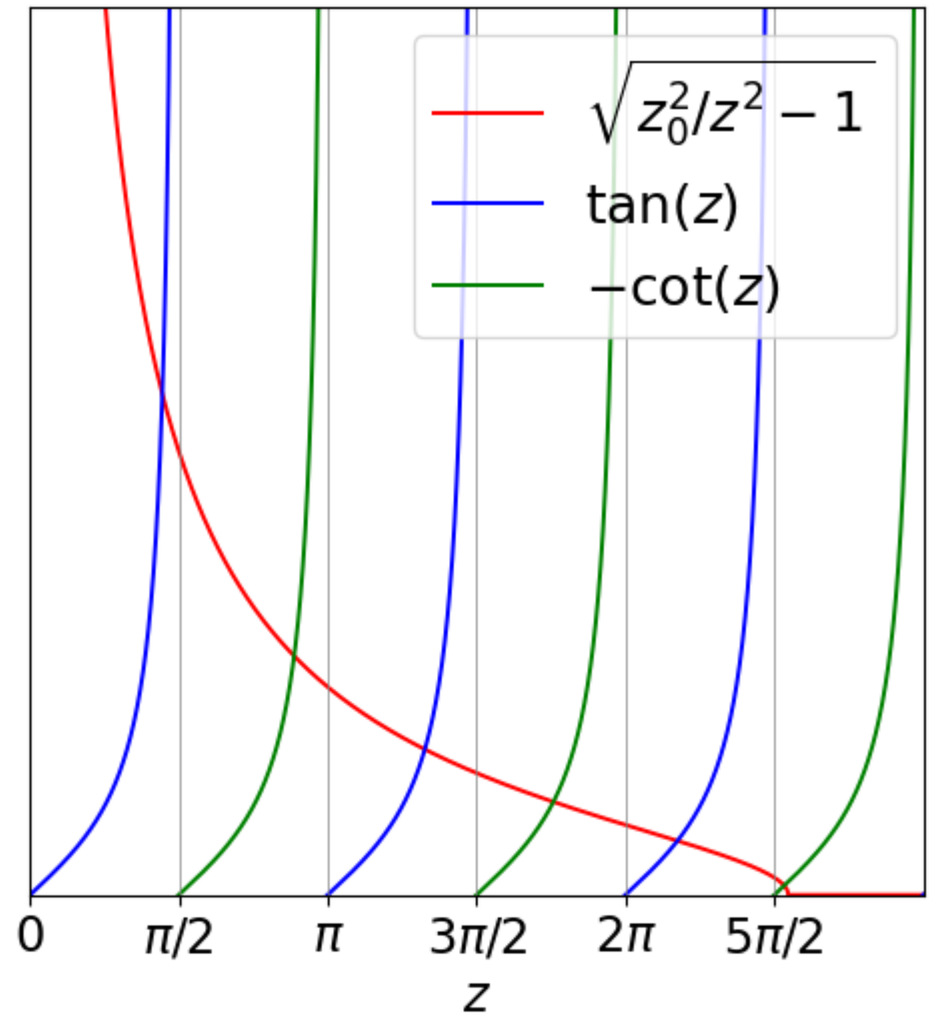


# FINITE WELL: GRAPHICAL ENERGY-LEVELS

- Symmetric and asymmetric solutions
- Energy levels at intersections of red curve with blue/green curves.
- Energies close to infinite well energies (vertical asymptotes):

$$E_n = \frac{\hbar^2 z^2}{2ma^2} \lessapprox E_n^\infty = \frac{\hbar^2}{2m} \frac{n^2 \pi^2}{(2a)^2}$$

- $z_0$ is intersection of red curve with x-axis
- Increasing $z_0 \implies$ more energy-levels
- Decreasing $z_0 \implies$ less energy-levels
- $z_0 = \frac{\sqrt{2mV_0}}{\hbar} a \propto \sqrt{V_0} a$



# FINITE WELL: NUMERICAL SOLUTIONS

Numerically solving for $z$:

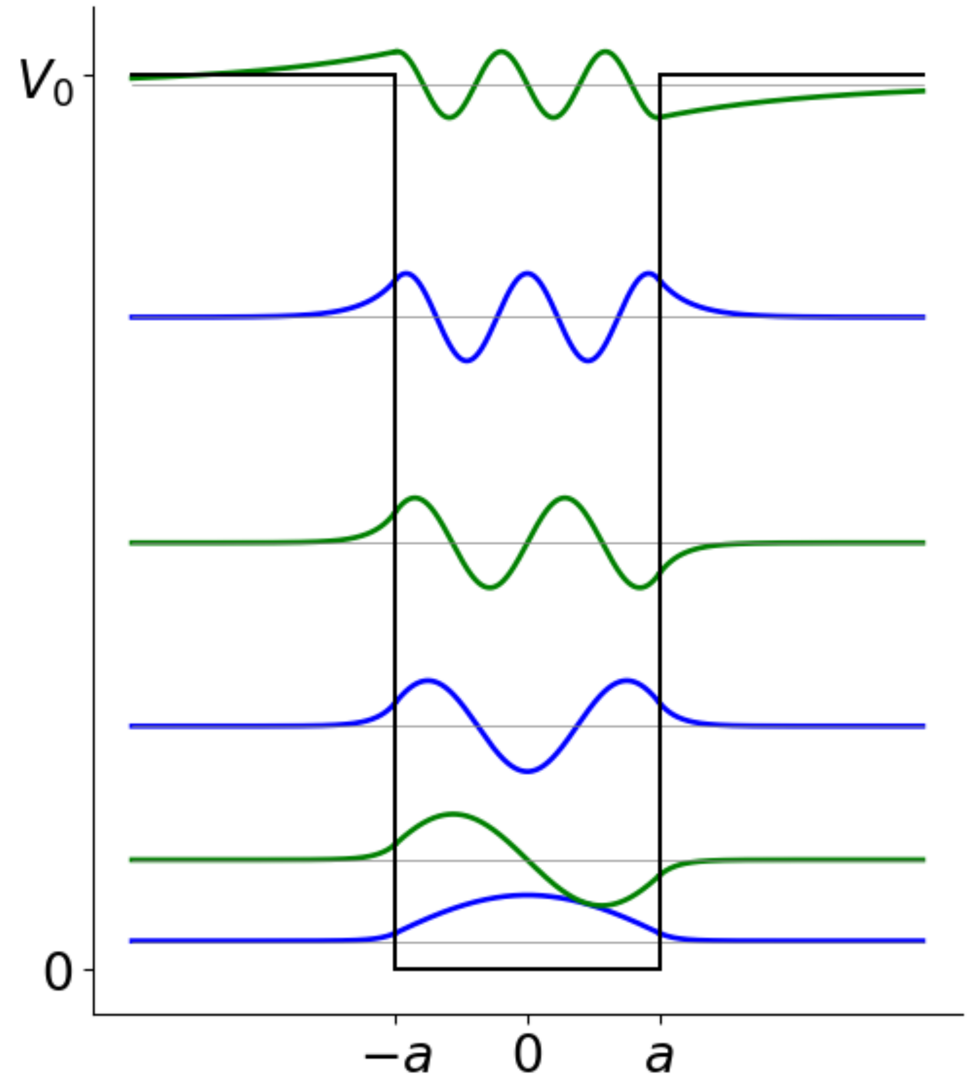
$$\tan z = \sqrt{z_0^2/z^2 - 1},$$
$$-\cot z = \sqrt{z_0^2/z^2 - 1}$$

Gives us:

- Energy levels $E_n = \frac{\hbar^2\lambda^2}{2m} = \frac{\hbar^2 z^2}{2ma^2}$
- Values for $C$ or $D$ as function of $B$:

$$B = D\cos(\lambda a)e^{\kappa a}$$
$$B = -C\sin(\lambda a)e^{\kappa a}$$

- Normalization $\longrightarrow$ final unknown $B$



# SCATTERING

# PROBABILITY DENSITY CURRENT

- Probability $\rho(x,t) = |\Psi(x,t)|^2$
- Continuity equation

$$\frac{\partial \rho(x,t)}{\partial t} =$$

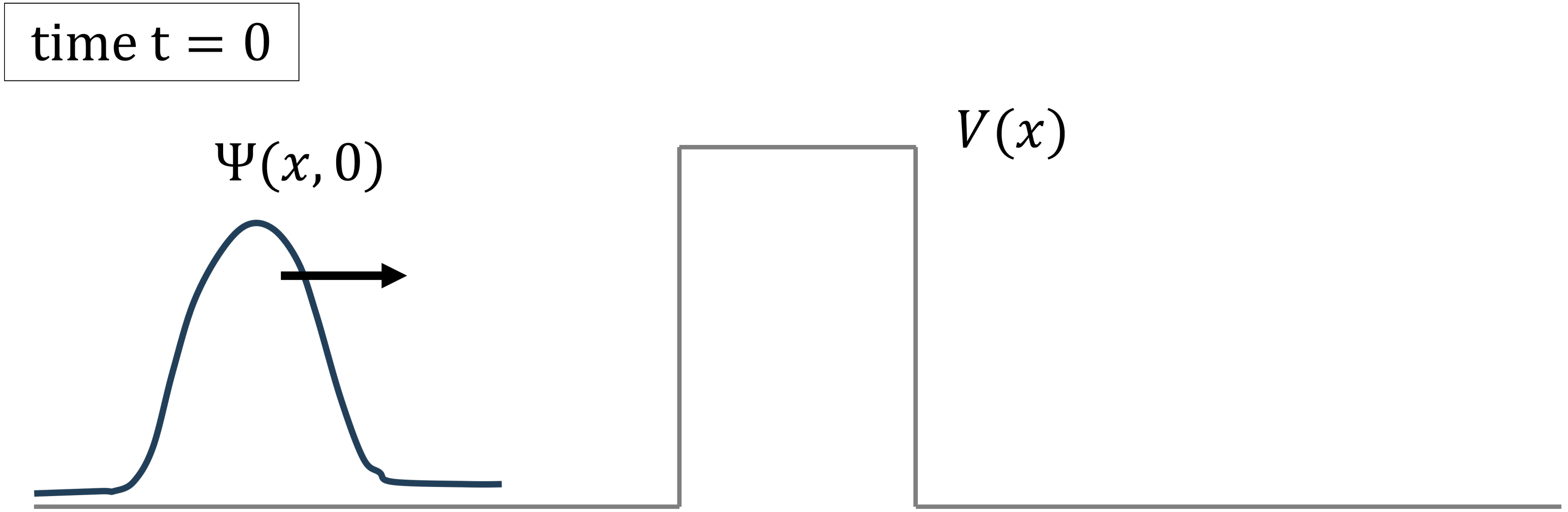
$$\frac{\partial \rho(x)}{\partial x}$$

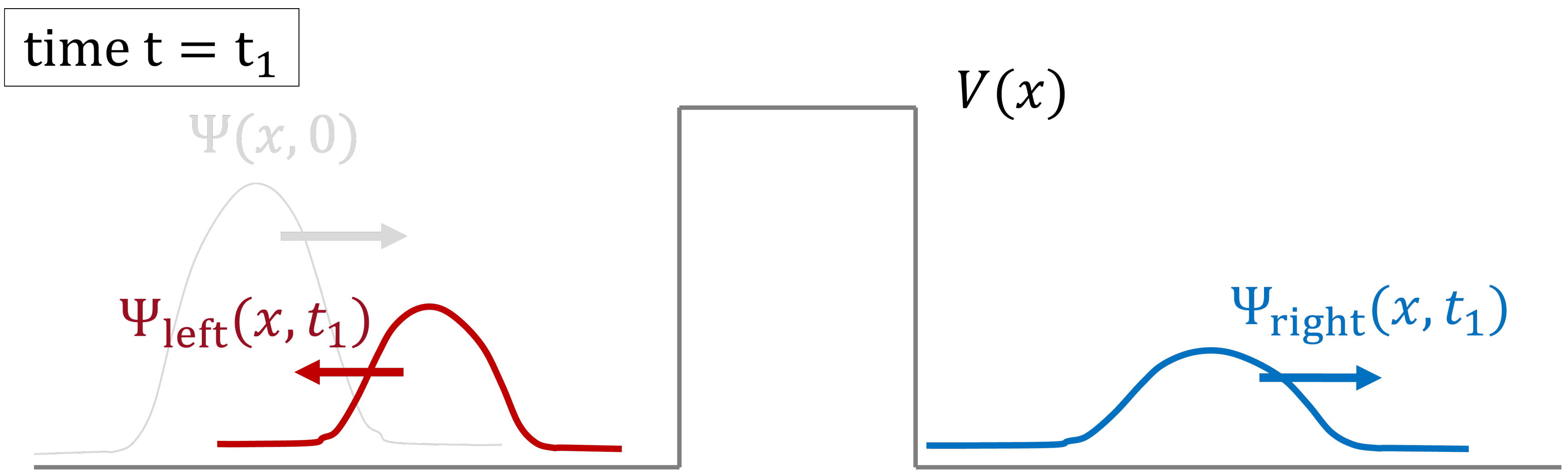
- Change in $\rho(x,t)$ leads to **probability current**

$$\frac{\partial \rho(x)}{\partial x}$$

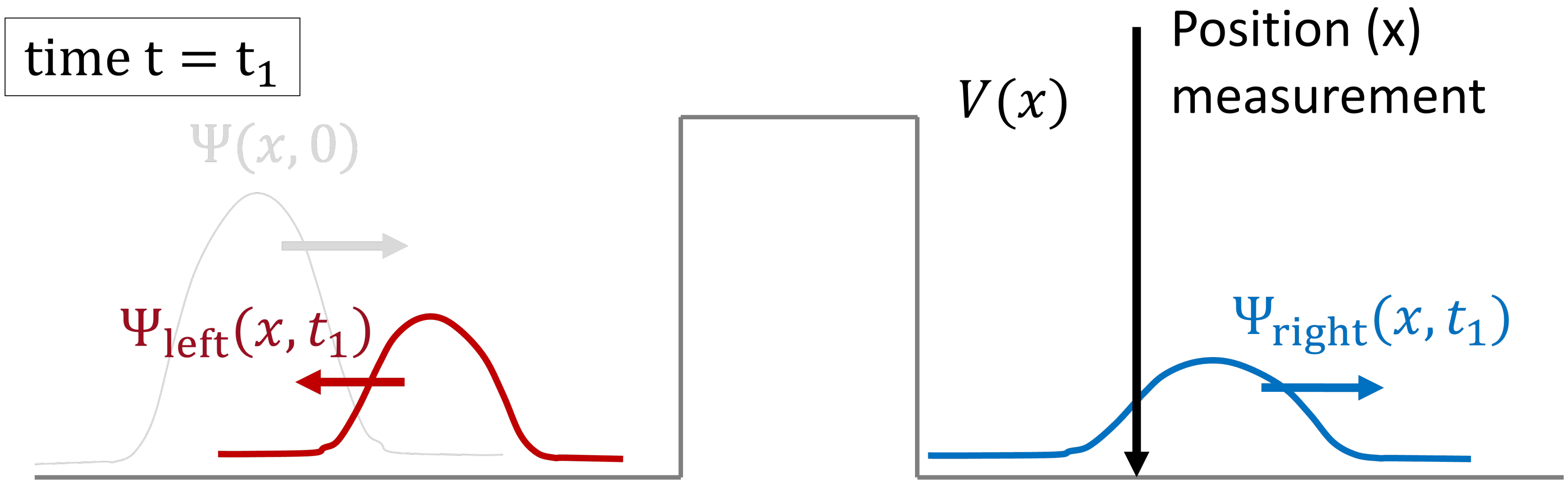
$$\frac{\partial \rho(x)}{\partial x}$$

# SCATTERING AT A BARRIER

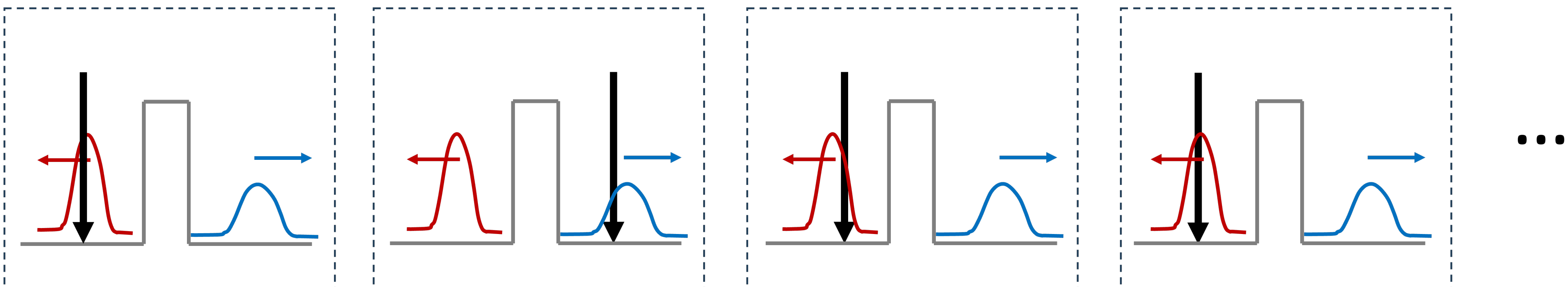


# SCATTERING AT A BARRIER




Every measurement probabilistic BUT average position $\langle x \rangle \propto \int_0^L P(x)\, x\, dx$



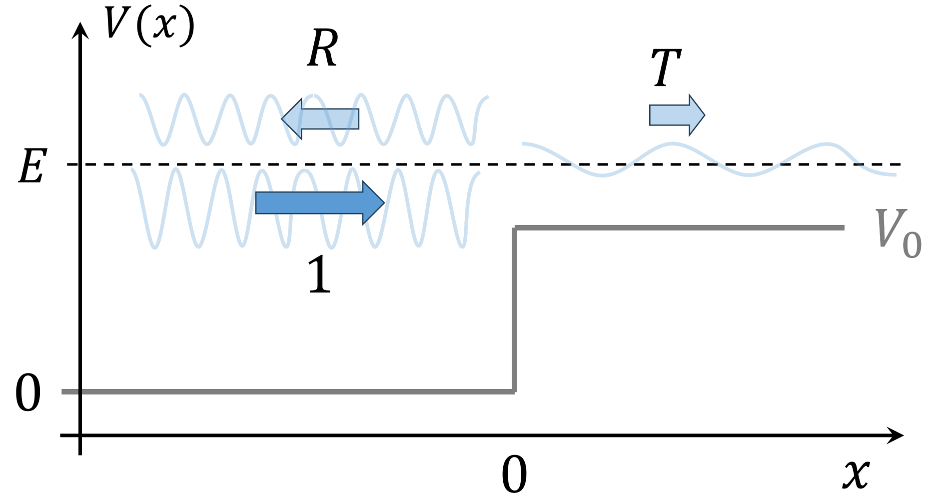
# SCATTERING AT A SINGLE POTENTIAL STEP

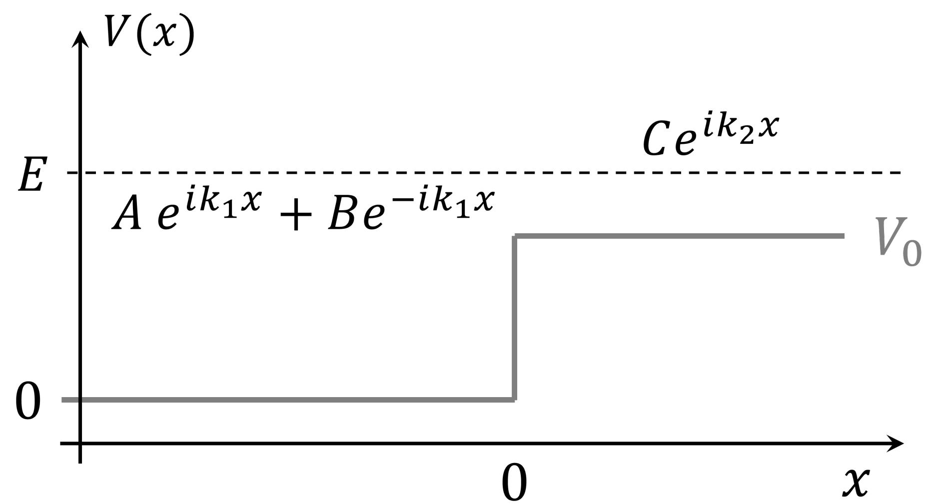
- Scattering states $E > V_0$
- Classically no reflection
- $R$: Reflection coefficient, probability to reflect
- $T$: Transmission probability/coefficient
- Total probability one:

$$T + R = 1$$

Schrodinger equation (TISE):

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = (E - V(x))\ \psi(x)$$



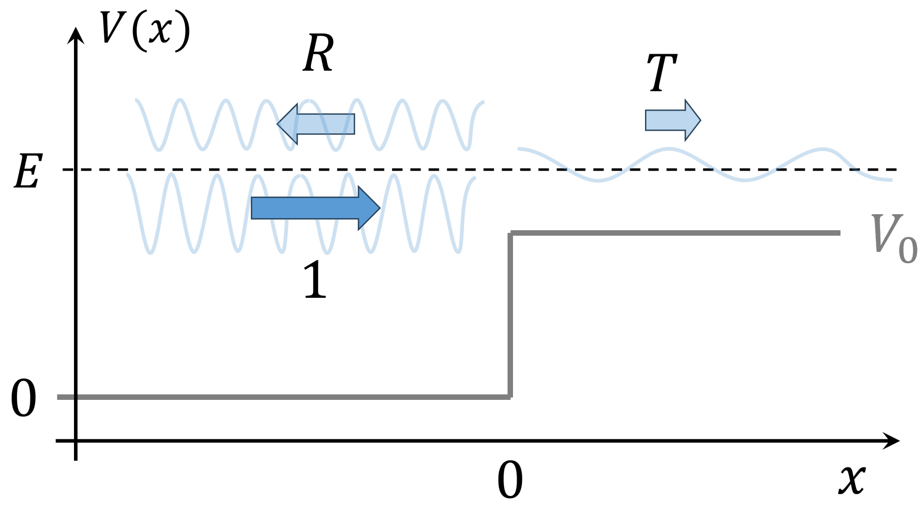



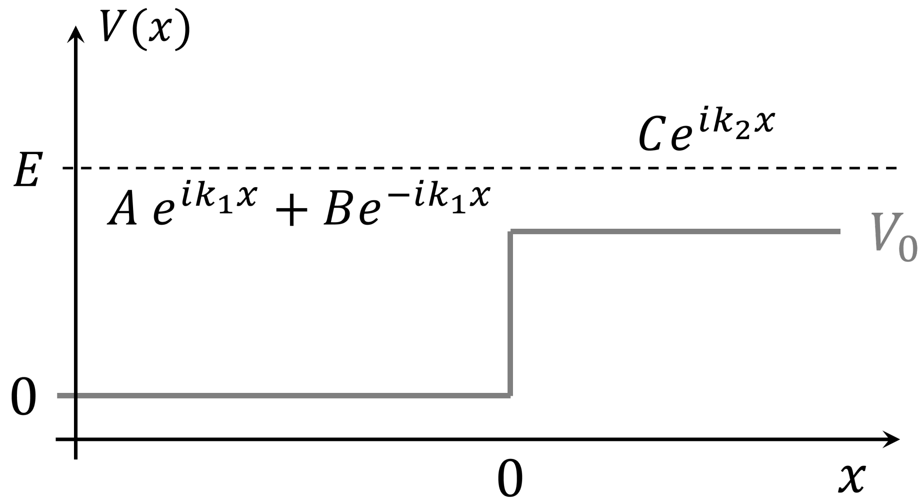
# SCATTERING AT A SINGLE POTENTIAL STEP

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = (E - V(x))\,\psi(x)$$

$$\begin{cases} \psi(x<0) = Ae^{ik_1x} + Be^{-ik_1x} \\ \psi(x<0) = Ce^{ik_2x} \end{cases}$$

$$k_1 = \frac{\sqrt{2mE}}{\hbar}, \qquad k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$



# SCATTERING AT A SINGLE POTENTIAL STEP

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Continuity of $\psi(x)$ and $\psi'(x)$

$$\begin{cases} Ae^{ik_1x} + Be^{-ik_1x} = Ce^{ik_2x} \\ ik_1Ae^{ik_1x} - ik_1Be^{-ik_1x} = ik_2Ce^{ik_2x} \end{cases}$$