PHOT 301 Summer (2024-2025) **Quiz 3** Date: 8 August 2025
Michaël Barbier Student No:
Student Name:

**Grading**: Each quiz counts for 7.5% of your total grade.

**Exam type**: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

**The duration** of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

## Question 1: Operators and Commutators

**(a)** Calculate the commutator $[1/x, \hat{p}]$.
**(b)** Show that $e^{-x^2}$ is an eigenstate of the operator $\hat{Q} = -\frac{1}{x}\frac{d}{dx}$. Calculate its eigenvalue.

## Question 2: Operators in finite dimensional space

Assume that the Hamiltonian operator $\hat{H}$ of a two-level system is represented by the following matrix:

$$H = \begin{pmatrix} 1 & i\sqrt{3} \\ -i\sqrt{3} & -1 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

**(a)** Solve the (time-independent) eigenvalue equation $H|\psi\rangle = E|\psi\rangle$ to obtain eigenenergies $E_n$ of the system.
**(b)** Then calculate the normalized eigenstates $|\psi_n\rangle$. *Hint:* The eigenstate s should be superpositions of the basis vectors $|1\rangle$ and $|2\rangle$.

## Question 3: Dirac notation

Consider the 3-dimensional space with orthonormal basis: $\{|1\rangle, |2\rangle, |3\rangle\}$. Further, kets $|\alpha\rangle$ and $|\beta\rangle$ are given by:

$$|\alpha\rangle = i|1\rangle + |2\rangle - |3\rangle, \qquad |\beta\rangle = |1\rangle + |2\rangle$$

**(a)** Show that $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^*$.
**(b)** Compute the operator $\hat{A} = |\alpha\rangle\langle\beta|$. Represent the operator as a $3 \times 3$ matrix in the $\{|1\rangle, |2\rangle, |3\rangle\}$ basis and calculate its elements.