PHOT 301 Summer (2024-2025) **Quiz 2** Date: 1 August 2025
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Student Name:

Grading: Each quiz counts for 7.5% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

The duration of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Ladder operators

Consider the following wave function $\psi(x)$ in a quantum harmonic oscillator.

$$\psi(x) = A\left(2\,\hat{a}_+\,\psi_0 + \hat{a}_-\,(\psi_1 + \psi_0)\right), \quad \text{where} \qquad \hat{a}_+\psi_n = \sqrt{n+1}\psi_{n+1}, \quad \hat{a}_-\psi_n = \sqrt{n}\psi_{n-1}$$

with A a normalization constant, $\hat{a}_\pm$ the ladder operators, and ψ_n the eigenstates.

(a) Simplify the expression for $\psi(x)$ by applying the ladder operators. The resulting expression shouldn't contain any ladder operators.
(b) Then calculate the normalization constant A of the wave function.

Question 2: Time evolution

A particle in a harmonic oscillator has following normalized wave function at time zero:

$$\Psi(x,0) = \frac{1}{\sqrt{2}}(\psi_0 + \psi_1), \quad \text{with} \qquad \begin{cases} \psi_0(x) = \pi^{-1/4}\sqrt{\beta}e^{-\beta^2 x^2/2} \\ \psi_1(x) = \sqrt{2}\pi^{-1/4}\beta^{3/2}xe^{-\beta^2 x^2/2} \end{cases}, \ \text{and} \ \ \beta = \sqrt{\frac{m\omega}{\hbar}}$$

(a) Write down the expression for the time-dependent wave function $\Psi(x,t)$.
(b) Calculate expectation value $\langle x \rangle$. Show that it oscillates around $x = 0$ in time. *Hint:* Only fill in the explicit functions for the eigenstates ψ_n in the end.

Question 3: Delta-function potential well

A particle with mass m in an delta-function potential well with strength α, that is $V(x) = -\alpha\,\delta(x)$ has the following normalized wave function (defined with $x \in \mathbb{R}$):

$$\psi(x) = \sqrt{\kappa}e^{-\kappa|x|}, \qquad \text{with} \quad \kappa = \frac{m\alpha}{\hbar^2}, \quad \text{and} \quad E = -\frac{m\alpha^2}{2\hbar^2}$$

(a) Calculate the probability P for the particle to be found inside interval $[-1/\kappa,\, 1/\kappa]$.
(b) Suppose the well is now three times as strong, that is $\alpha_{\text{new}} = 3\alpha$. Calculate the "new" energy and compare with the original one: $E_{\text{new}} \leftrightarrow E$?