

**Grading:** Each quiz counts for 7.5% of your total grade.

**Exam type:** Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

**The duration** of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

### Question 1: Wave functions

Consider the wave function  $\psi(x)$  defined with  $x \in \mathbb{R}$ :

$$\psi(x) = A x e^{-x^2/2}$$

with  $A$  a normalization constant.

- (a) First calculate the normalization constant  $A$  of the wave function.
- (b) Then calculate the expectation value  $\langle x^2 \rangle$ .

### Question 2: Coefficient expansion

Assume a particle in an infinite well of width  $L$  has following wave function at time zero:

$$\Psi(x, 0) = \psi(x) = A \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi x}{L}\right)$$

- (a) Calculate the normalization constant  $A$ .
- (b) The wave function at time zero  $\psi(x)$  can be expanded in the stationary states of the infinite well given by  $\psi_n(x) = \sqrt{2/L} \sin(n\pi x/L)$ . Prove that only one coefficient in the expansion  $\psi(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$  is nonzero (and calculate its value).

### Question 3: Infinite well

Assume a particle in an infinite well is in a superposition state  $\psi(x) = c_2 \psi_2(x) + c_3 \psi_3(x)$  (at time equal to zero) and has energy  $E = 6E_1^\infty$ , where  $E_1^\infty = \frac{\hbar^2 \pi^2}{2mL^2}$ .

- (a) Express the energy values  $E_2$  and  $E_3$  in energy units of  $E_1^\infty$ .
- (b) Calculate the values of the coefficients  $c_2$  and  $c_3$ .

**Hints for questions 2 and 3:** The expansion  $\psi(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$  has coefficients:

$$c_n = \int_0^L \psi_n^* \psi dx, \quad \text{where} \quad \sum_{n=1}^{\infty} |c_n|^2 = 1 \quad \text{and} \quad E = \sum_{n=1}^{\infty} |c_n|^2 E_n$$