PHOT 301: Quantum Photonics

Homework: Solving Equations in One Variable

Michaël Barbier, Summercourse (2024-2025)

Solving equations in one variable

While solving an equations in one variable we can be interested in either only real solutions or also allow for complex-valued solutions. In the following, we suggest to try to think about both cases.

Polynomial equations in one variable

Examples of polynomial equations in one variable:

Equation	(partial) solution
$4x - 3 = 5$	$x = 2$
$2x^2 - x + 2 = 0$	$x_{\pm} = \dfrac{1 \pm i\sqrt{15}}{4}$
$x^3 - 27 = 0$	$(x-3)(x^2+3x+9)$
$x^5 - x^3 = 0$	$x^3(x^2-1)$
$x^3 - x^2 + x - 1 = 0$	

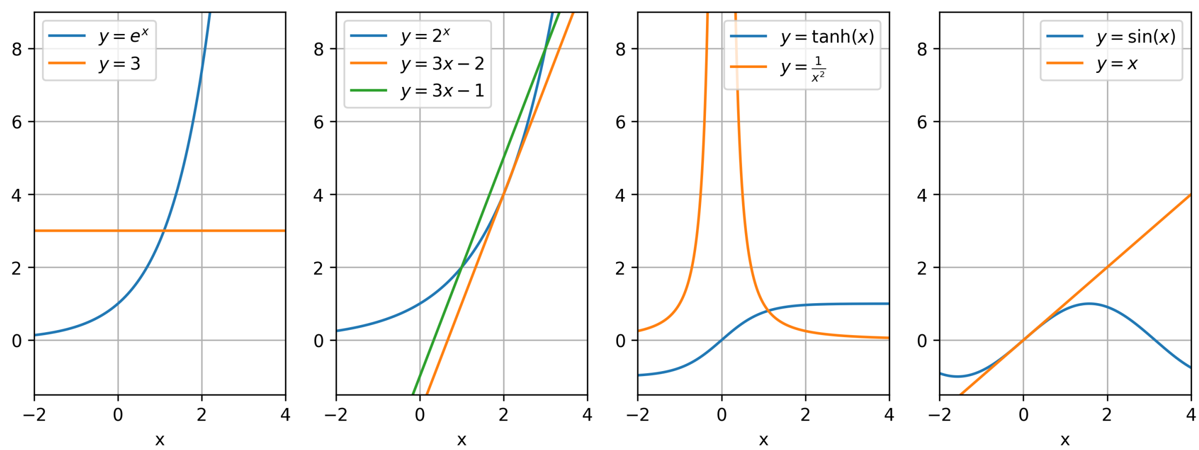
Exercises on polynomial equations: First solve the above equations with partial solutions. Then solve the following equations:

$$x^2 = 2$$
$$2x + 2 = 0$$
$$x^2 - x + 3 = 0$$
$$x^3 - 8 = 0$$
$$x^3 + x^2 - 3x = 0$$
$$x^3 - 2x^2 + 2x - 1 = 0$$

Transcendental equations

Transcendental equations contain transcendental functions, such as exponentials, trigoniometric, and hyperbolic functions (such functions cannot be written as a finite polynomial). Closed form solutions can be non-existent, graphical or numerical solving. Examples for such equations are:

Equation	(partial) solution
$e^x = 3$	$x = \ln 3$
$2^x = 3x - 2$	$x = 2$ guess and confirm only solution by sketch
$2^x = 3x - 1$	$x = 1, 3$ expect two solutions from sketch
$2 \ln x = x$	
$\sin(\pi x) = 0$	$x = \ldots, -2, -1, 0, 1, 2, \ldots$
$\tanh(x) = \dfrac{1}{x^2}$	Sketch suggests single real solution
$\sin(x) = x$	$x = 0$, but an infinite number of complex solutions



Exercises on transcendental equations

$\sin(x^2) = 0$	$\sin^2(5\pi x) = 1$
$\tan(x) = 1$	$\tan^2(x) + 1 = \cos^2(x)$
$\sqrt{x+1} = x^2$	$e^{i\pi x} = x$
$x^2 e^{-x^2} = 1$	$\tanh(x) = \dfrac{1}{x^2}$
$\cos(x) = \sqrt{x+1}$	$e^{i\pi x} = \cos(\pi x)$