In the following formulas parameters n, m are integers, and $0 < a \in \mathbb{R}$ and $b \in \mathbb{R}_0$:

Trigonometric identities

Here α and β are real numbers.

$$\sin 30^\circ = \frac{1}{2}, \quad \sin 45^\circ = \frac{\sqrt{2}}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$
$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$
$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \mp \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$
$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}, \qquad \cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)}$$
$$\sec(\alpha) = \frac{1}{\cos(\alpha)}, \qquad \csc(\alpha) = \frac{1}{\sin(\alpha)}$$

Complex numbers

Assume $z = x + iy \in \mathbb{C}$:

$$i = \sqrt{-1}, \quad i^2 = -1$$
$$|z|^2 = z^* z = x^2 + y^2 = \Re\{z\}^2 + \Im\{z\}^2$$
$$z = x + i\,y = re^{i\theta} = r\,(\cos\theta + i\,\sin\theta)$$
$$z^* = x - i\,y = re^{-i\theta} = r\,(\cos\theta - i\,\sin\theta)$$
$$|z|^2 = z^* z = x^2 + y^2 = \Re\{z\}^2 + \Im\{z\}^2$$
$$\theta = -i\ln(z/|z|) = \arctan(y/x)$$

Operations:

$$z_1 + z_2 = (x_1 + x_2) + i\,(y_1 + y_2)$$
$$z_1\,z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$\cos(\theta) + i\sin(\theta) = e^{i\theta}$$

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}, \qquad \sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$
$$\cosh(\theta) = \frac{e^{\theta} + e^{-\theta}}{2}, \qquad \sinh(\theta) = \frac{e^{\theta} - e^{-\theta}}{2}$$

Derivative: rules

Sum rule	$(f+g)' = f' + g'$
Scalar mult.	$(af)' = af'$
Product rule	$(fg)' = f'g + fg'$
Quotient rule	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
Chain rule	$(f(g(x)))' = f'(g(x)) \cdot g'(x)$

Standard derivatives

$$x^b = bx^{b-1}$$
$$\ln|x| = \frac{1}{x}$$
$$e^x = e^x$$
$$a^x = a^x \cdot \ln a$$
$$\log_b x = \frac{1}{x \cdot \ln b}$$
$$\sin x = \cos x$$
$$\cos x = -\sin x$$
$$\tan x = \sec^2 x$$
$$\sec x = \sec x \cdot \tan x$$
$$\csc x = -\csc x \cdot \cot x$$
$$\sinh x = \cosh x$$
$$\cosh x = \sinh x$$
$$\tanh x = (\operatorname{sech} x)^2$$
$$\arcsin x = \frac{1}{\sqrt{1-x^2}}$$
$$\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\arctan x = \frac{1}{1+x^2}$$

Anti-derivatives: rules

Fund. law $f(x) = \left(\int_b^x f(t)\,dt \right)'$

Partial int. $\int f\,dg = f\,g - \int g\,df$

$$\int f(x)g'(x)\,dx = f(x)\,g(x) - \int g(x)f'(x)\,dx$$

Leibniz $\frac{d}{d\nu}\left[\int f(x;\nu)\,dx\right] = \int \frac{\partial f(x;\nu)}{\partial \nu}\,dx$

Standard anti-derivatives

Rational functions:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$
$$\int \frac{1}{x}\,dx = \ln|x|$$
$$\int (x+b)^n\,dx = \frac{(x+b)^{n+1}}{n+1}$$
$$\int x\,(x+b)^n\,dx = \frac{(x+b)^{n+1}}{n+1}\frac{x(n+1)-b}{n+2}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\ \arctan(x/a)$$
$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$
$$\int \frac{x^2}{x^2+a^2}\,dx = x - a\ \arctan(x/a)$$
$$\int \frac{x^3}{x^2+a^2}\,dx = \frac{1}{2}\left[x^2 - a^2\ln(x^2+a^2)\right]$$

$$\int \frac{1}{(x^2+1)^2}\,dx = \frac{1}{2}\left(\arctan(x) + \frac{x}{x^2+1}\right)$$
$$\int \frac{x}{(x^2+1)^2}\,dx = -\frac{1}{2}\frac{1}{x^2+1}$$
$$\int \frac{x^2}{(x^2+1)^2}\,dx = \frac{1}{2}\left(\arctan(x) - \frac{x}{x^2+1}\right)$$
$$\int \frac{x^3}{(x^2+1)^2}\,dx = \frac{1}{2}\left(\frac{1}{x^2+1} + \ln(x^2+1)\right)$$

$$\int \frac{1}{(x+a)(x+b)}\,dx = \frac{1}{b-a}\left[\ln(x+a) - \ln(x+b)\right]$$
$$\int \frac{x}{(x+a)^2}\,dx = \frac{a}{x+a} + \ln(x+a)$$

Trigonometric:

$$\int \cos(x)\,dx = \sin(x)$$
$$\int \sin x\,dx = -\cos x$$
$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$
$$\int \sin^3 x\,dx = -\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$$
$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$
$$\int \cos^3 x\,dx = \frac{3}{4}\sin x + \frac{1}{12}\sin 3x$$
$$\int \sin x\cos x\,dx = -\frac{1}{2}\cos^2 x$$
$$\int \cos^2(x)\sin^2(x)\,dx = \frac{x}{8} - \frac{1}{32}\ \sin 4x$$
$$\int \cos^n(ax)\sin(ax)\,dx = -\frac{1}{a(n+1)}\ \cos^{n+1}(ax)$$
$$\int \cos(ax)\sin^n(ax)\,dx = \frac{1}{a(n+1)}\ \sin^{n+1}(ax)$$

Exponentials:

$$\int e^{bx}\,dx = \frac{1}{b}\,e^{bx}$$
$$\int xe^{bx}\,dx = \left(\frac{x}{b} - \frac{x}{b^2}\right)\ e^{bx}$$
$$\int x^2e^{bx}\,dx = \left(\frac{x^2}{b} - \frac{2x}{b^2} + \frac{2}{b^3}\right)\ e^{bx}$$
$$\int x^3e^x\,dx = \left(x^3 - 3x^2 + 6x - 6\right)\ e^{bx}$$

Definite integrals

$$\int_0^1 \sin(m\pi x)\sin(n\pi x)\,dx = \frac{1}{2}\delta_{mn}$$

$$\int_0^1 x\sin^2(m\pi x)\,dx = \frac{1}{4}$$

$$\int_0^1 x^2\sin^2(m\pi x)\,dx = \frac{1}{6} - \frac{1}{4\pi^2 m^2}$$

$$\int_0^1 x\sin(\pi x)\sin(3\pi x)\,dx = 0$$

$$\int_0^1 x^2\sin(\pi x)\sin(3\pi x)\,dx = \frac{3}{16\pi^2}$$

$$\int_0^1 x^3\sin(\pi x)\sin(3\pi x)\,dx = \frac{9}{32\pi^2}$$

Exponentials $x \in [0, +\infty[$

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{2\sqrt{a}}$$

$$\int_0^\infty x e^{-ax^2}\,dx = \frac{1}{2a}$$

$$\int_0^\infty x^2 e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{4a^{3/2}}$$

$$\int_0^\infty x^3 e^{-ax^2}\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^4 e^{-ax^2}\,dx = \frac{3\sqrt{\pi}}{8a^{5/2}}$$

Exponentials $x \in]-\infty, +\infty[$

$$\int_{-\infty}^\infty x^n e^{-ax^2}\,dx = 0 \quad \text{for n odd}$$

$$\int_{-\infty}^\infty e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{\sqrt{a}}$$

$$\int_{-\infty}^\infty x^2 e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{2a^{3/2}}$$

$$\int_{-\infty}^\infty x^4 e^{-ax^2}\,dx = \frac{3\sqrt{\pi}}{4a^{5/2}}$$

Integration in spherical coordinates

$$\int_{-\infty}^\infty dx \int_{-\infty}^\infty dy \int_{-\infty}^\infty dz\, f(x,y,z) = \int_0^\infty d\rho \int_0^\pi d\theta \int_0^{2\pi} d\phi\, \rho^2 \sin\theta\; F(\rho,\theta,\phi)$$

where volume element $\; dx\,dy\,dz = \rho^2 \sin\theta\, d\theta\, d\phi\, d\rho$

$x = \rho\sin(\theta)\cos(\phi),\;\; y = \rho\sin(\theta)\sin(\phi),\;\; z = \rho\cos(\theta)$