Grading: Each quiz counts for 15% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

The duration of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Delta-function Potential

Consider a system with delta-function potential barrier $V(x) = \alpha\delta(x)$ with $\alpha > 0 \in \mathbb{R}$.

(a) Derive an expression for the reflection coefficient R (not only the end result, please write down the derivation).
(b) If you increase the barrier strength fourfold: $\alpha \longrightarrow 4\alpha$, by what factor β should you increase $E \longrightarrow \beta E$ to retain the same reflection coefficient? Explain.

Question 2: Operator and Commutators

(a) Calculate the commutator $[e^{ix^2}, \frac{1}{x}\frac{\partial}{\partial x}]$.
(b) Functions $f(x) = e^{-2x}$ and $g(x) = \sin(3x)$ are eigenfunctions of the operator $\hat{Q} = \frac{d^2}{dx^2}$. What are the corresponding eigenvalues?

Question 3: Matrix Representation

Consider a three-state system where an operator $\hat{Q}$ and a state $|\alpha\rangle$ are represented in matrix notation by:

$$Q = \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad |\alpha\rangle = \frac{1}{\sqrt{5}}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

(a) What are the eigenvalues of Q^2?
(b) Calculate the expectation value $\langle\alpha|\hat{Q}|\alpha\rangle$.