PHOT 301 Fall (2025-2026) **Quiz 2** Date: 6 November 2025
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Student Name:

Grading: Each quiz counts for 15% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

The duration of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Harmonic oscillator

Consider the harmonic oscillator in the first excited state (already normalized) defined with $x \in \mathbb{R}$:

$$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{2m\omega}{\hbar}}\, x\, e^{-\frac{m\omega}{2\hbar}x^2} = \sqrt{2}\,\pi^{-1/4}\,\beta^{3/2} x\, e^{-\beta^2 x^2/2} \qquad \text{with } \beta = \sqrt{\frac{m\omega}{\hbar}}$$

(a) Calculate the expectation value for the position $\langle x \rangle$.
(b) Then calculate the expectation value $\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

Hint: Expectation value for a function $f(x)$: $\langle f(x) \rangle = \int_{-\infty}^{+\infty} f(x)\, |\psi|^2\, dx$.

Question 2: Ladder operators

Assume a particle in a 1D harmonic oscillator has following wave function:

$$\psi(x) = A\left[(\hat{a}_+^2 + \hat{a}_+\hat{a}_-)\psi_0 + \hat{a}_+\psi_3\right]$$

(a) Apply the ladder operators so you get an expression in only the eigenstates of the harmonic oscillator: $\psi_n(x)$. No ladder operators should appear.
(b) Derive the normalization constant A.

Hint: Eigenstates are orthonormal, and $\hat{a}_+\psi_n = \sqrt{n+1}\,\psi_{n+1}, \quad \hat{a}_-\psi_n = \sqrt{n}\,\psi_{n-1}$.

Question 3: Free particle: Gaussian wave packet

A free particle has following wave function at time zero:

$$\Psi(x,0) = \psi(x) = Ae^{-ax^2}$$

(a) Derive the normalization constant A.
(b) Apply the Fourier transform to obtain $\phi(k)$. This is: $\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x,0) e^{-ikx}\, dx$.

Hint: You can make use of the known integral: $\int_{-\infty}^{+\infty} e^{-(ax^2+bx)}\, dx = \sqrt{\frac{\pi}{a}} e^{b^2/4a}$ where "a" and "b" are constants which can be complex values.