

Grading: Each quiz counts for 15% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

The duration of the quiz is 1 hour.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Wave functions

Consider the time-independent wave function $\psi(x)$ defined with $x \in \mathbb{R}$:

$$\psi(x) = A \frac{1}{\sqrt{x^2 + 9}}$$

- (a) First calculate the normalization constant A of the wave function.
- (b) Then calculate the probability to find the particle in interval $[-3, 3]$.

Question 2: Time evolution

Assume a particle in the groundstate of an infinite well of width L has following wave function at time zero:

$$\Psi(x, 0) = \psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

- (a) Write down both the expression for the time-dependent wave function $\Psi(x, t)$ and the probability density function $|\Psi(x, t)|^2$.
- (b) The time-dependent wave function is a periodic function: $\Psi(x, t) = \Psi(x, t + T)$ with period T , derive an expression for T .

Question 3: Infinite well

A particle in an infinite well with width L is in a superposition state at time $t = 0$:

$$\Psi(x, 0) = \psi(x) = \frac{1}{\sqrt{5}} (\psi_2(x) - 2\psi_{10}(x))$$

- (a) What is the difference in energy $E_{10} - E_2$ between the eigenstates?
- (b) What is the expectation value of the Hamiltonian $\langle \hat{H} \rangle$.

Hints for questions 2 and 3: The eigenstates of the infinite well $\psi_n(x)$ have eigenenergies $E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$. The expansion $\psi(x) = \sum_{n=1}^{\infty} c_n \psi_n(x)$ has coefficients:

$$c_n = \int_0^L \psi_n^* \psi dx, \quad \text{where} \quad \sum_{n=1}^{\infty} |c_n|^2 = 1 \quad \text{and} \quad \langle \hat{H} \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n$$