PHOT 222: Quantum Photonics
LECTURE 09

Michaël Barbier, Spring semester (2024-2025)

OVERVIEW OF THE COURSE

week	topic	Serway 9th	Young
Week 1	Relativity	Ch. 39	Ch. 37
Week 2	Waves and Particles	Ch. 40	Ch. 38-39
Week 3	Wave packets and Uncertainty	Ch. 40	Ch. 38-39
Week 4	The Schrödinger equation and Probability	Ch. 41	Ch. 39
Week 5	**Midterm exam 1**		
Week 6	Quantum particles in a potential	Ch. 41	Ch. 40
Week 7	Bayram		
Week 8	**Harmonic oscillator**	**Ch. 41**	**Ch. 40**
Week 9	**Tunneling through a potential barrier**	**Ch. 41**	**Ch. 40**
Week 10	**Midterm exam 2**		
Week 11	The hydrogen atom, absorption/emission spectra		
Week 12	Many-electron atoms & Pauli-exclusion principle		
Week 13	Atomic bonds and molecules		
Week 14	Crystalline materials and energy band structure		

Harmonic potential

CLASSICAL HARMONIC POTENTIAL WELL

Various models:

(1) Weight connected to spring

- Potential:

$$V(x) = \frac{1}{2}k'x^2$$

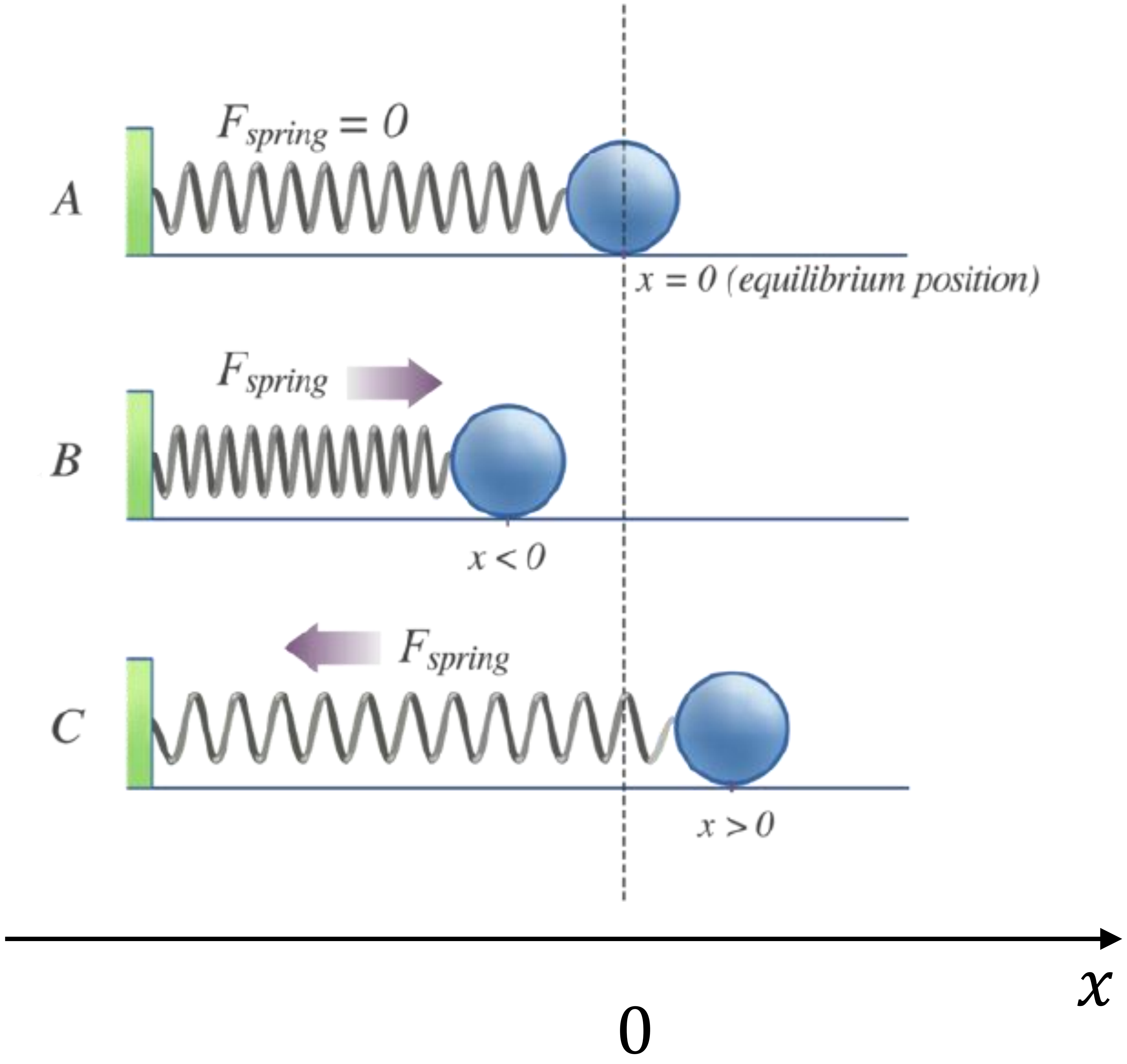
With Hooke's constant: k'

- Restoring force of spring:

$$F = -\frac{dV(x)}{dx} = -k'x$$

- Turning point distance: $x_{\max}$

$$\frac{1}{2}k'x_{\max}^2 = \frac{1}{2}mv_0^2$$



CLASSICAL HARMONIC POTENTIAL WELL

Various models:

(1) Weight connected to spring

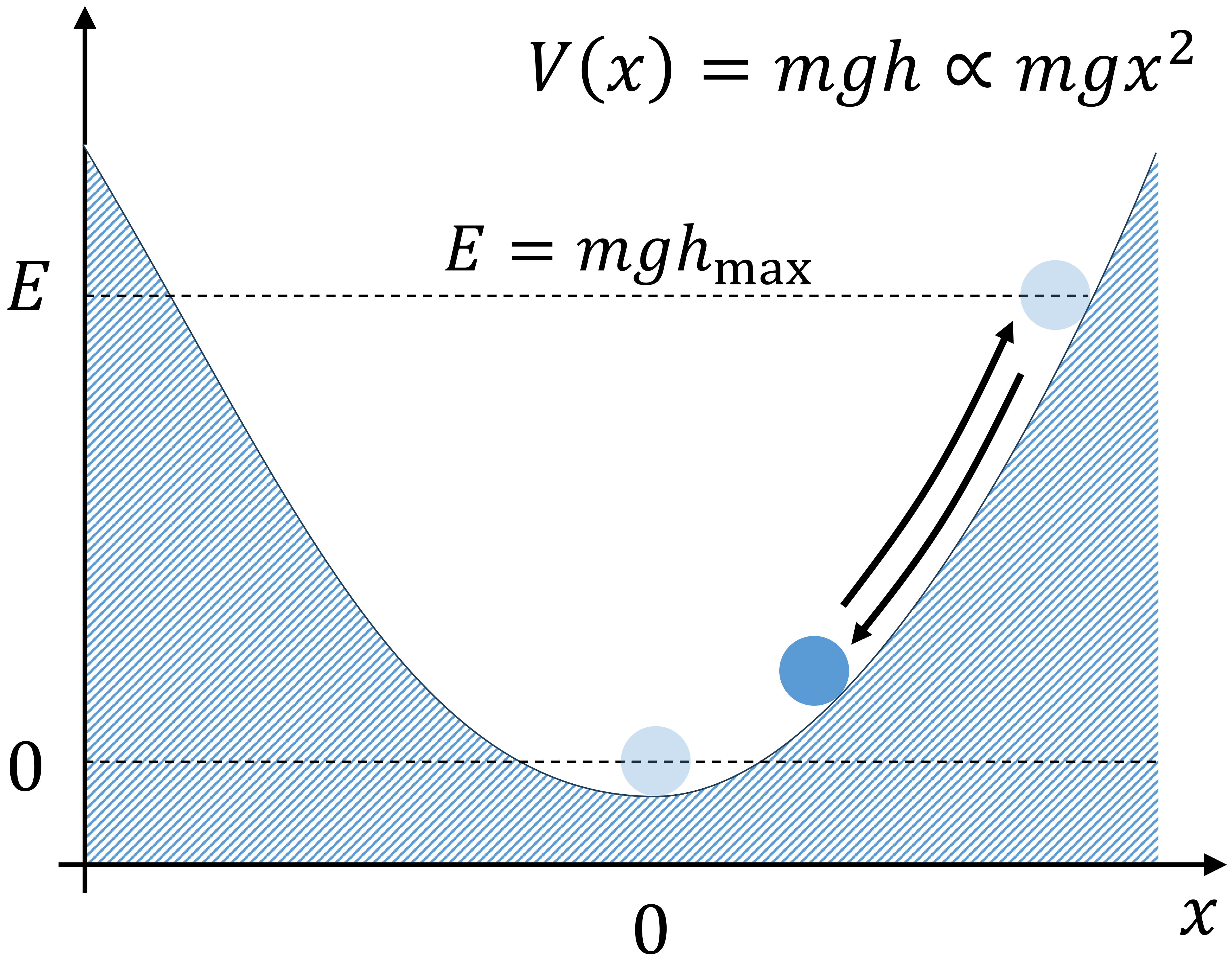
$$V(x) = \frac{1}{2}k'x^2$$

With Hooke's constant: k'

(2) Ball/car in a parabolic well

$$V(x) = mgh \propto mgx^2$$

(3) Pendulum or swing



THE HARMONIC POTENTIAL WELL

- The time-independent Schrodinger equation (TISE):

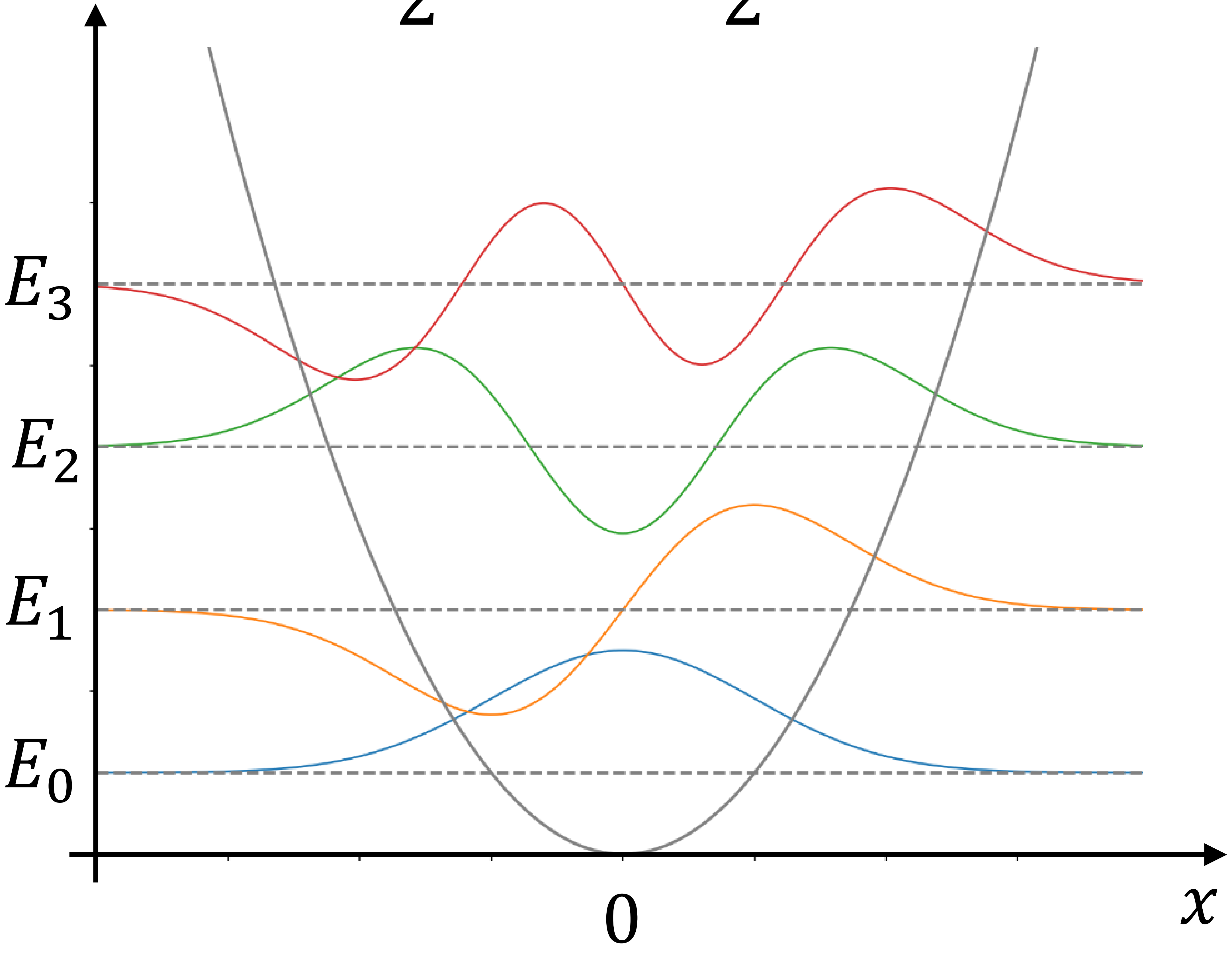
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}-\frac{1}{2}m\omega^2x^2\psi = E\psi$$

Solutions $\psi_n, \;\; E_n$:

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega, \mathrm{n} = 0, 1, 2, \ldots$$

$$\psi_n = A_n\, H_n\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$V(x) = \frac{1}{2}k'x^2 = \frac{1}{2}m\omega^2x^2$$



THE HARMONIC POTENTIAL WELL

$$E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega, \quad \mathrm{n} = 0, 1, 2, \ldots$$

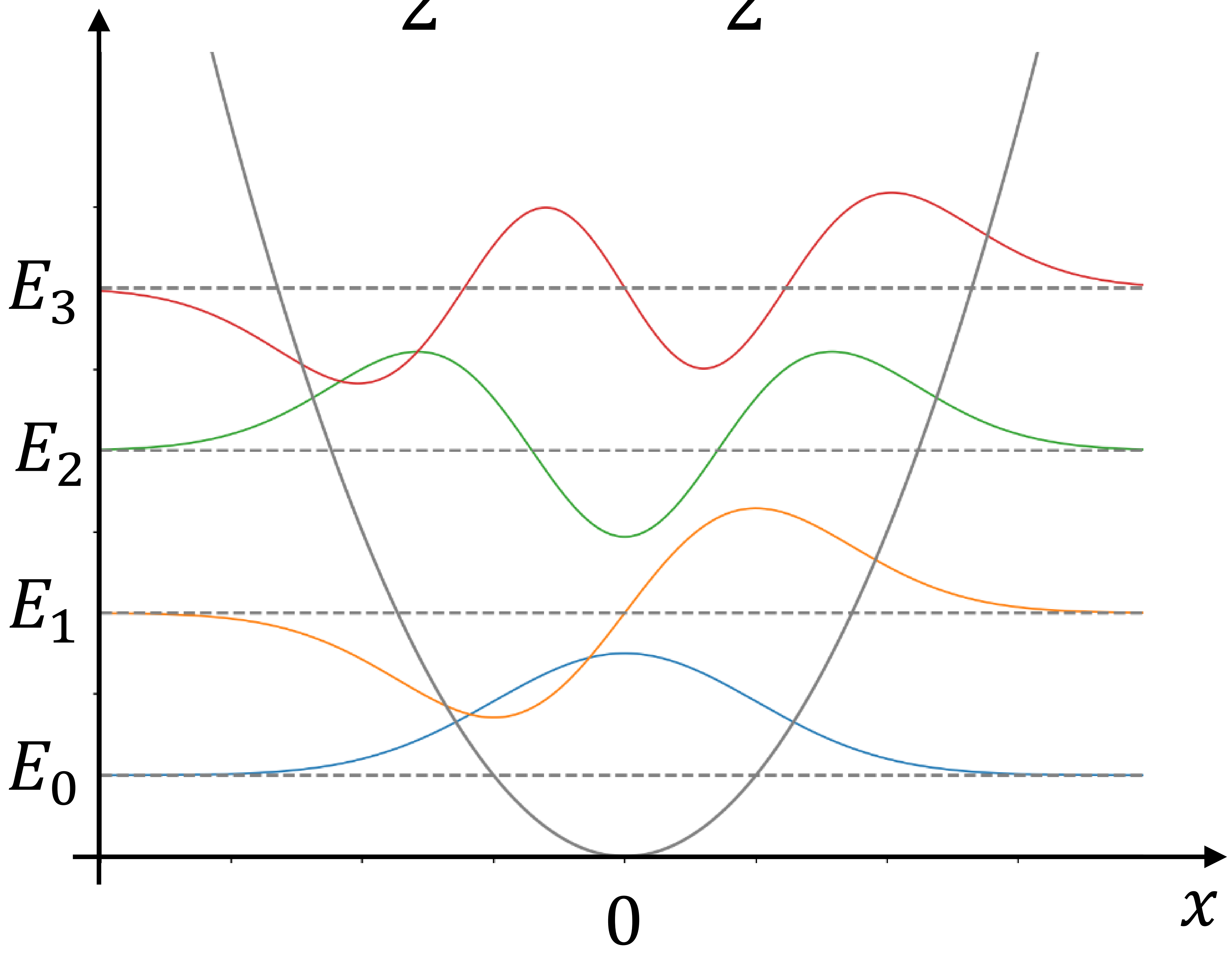
- Energy levels are evenly spaced
- Difference between levels depends on frequency as Max Planck predicted:

$$n\hbar\omega = nhf$$

- There is a zero-point energy

$$E_0 = \frac{1}{2}\hbar\omega \neq 0$$

$$V(x) = \frac{1}{2}k'x^2 = \frac{1}{2}m\omega^2x^2$$



THE HARMONIC POTENTIAL WELL

- Solutions for the wave function:

$$\psi_n = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-\frac{m\omega}{2\hbar}x^2}$$

- Gaussian
- Normalization
- Hermite polynomials $H_n(z)$

$H_0(z) = 1$

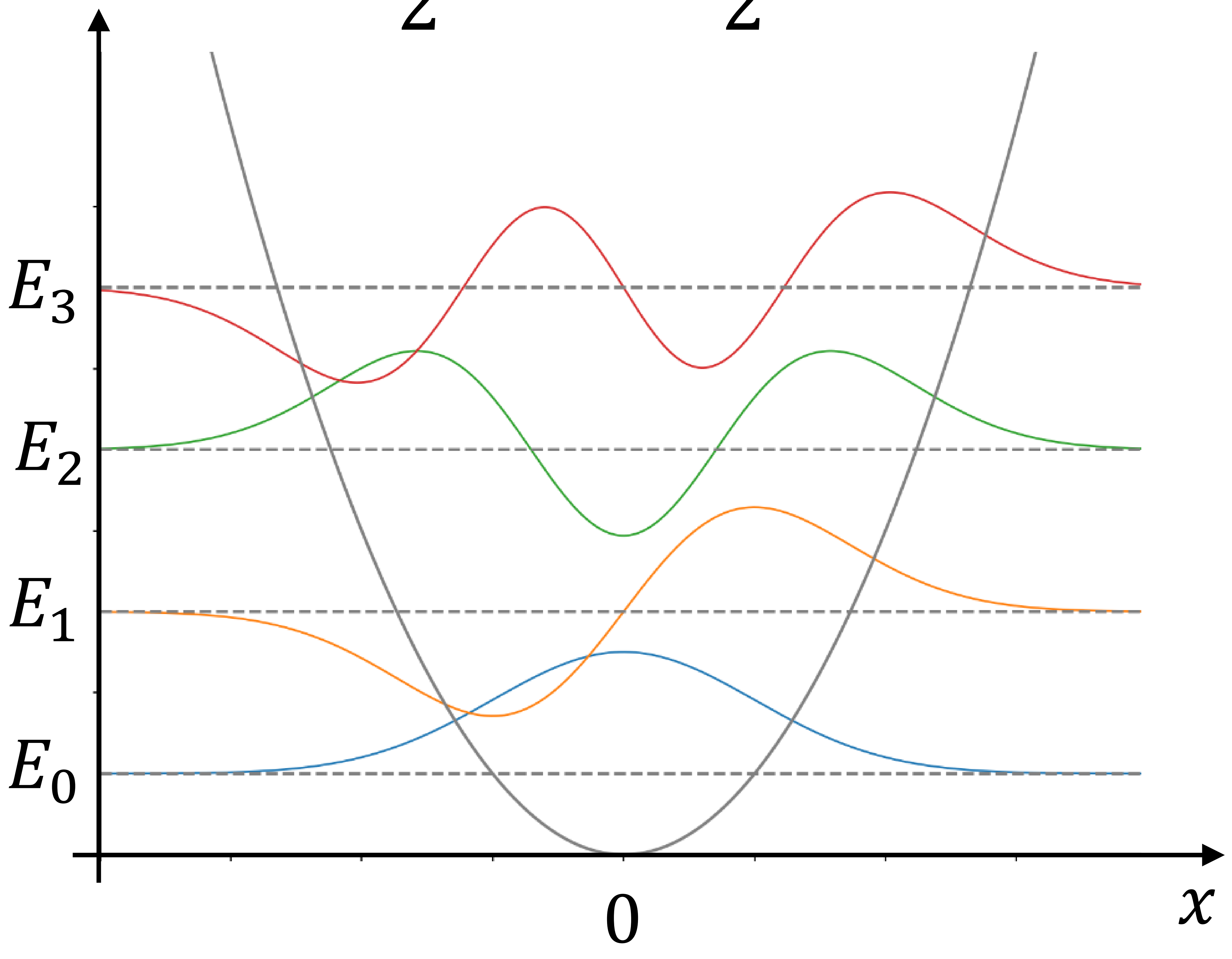
$H_1(z) = 2z$

$H_2(z) = -2 + 4z^2$

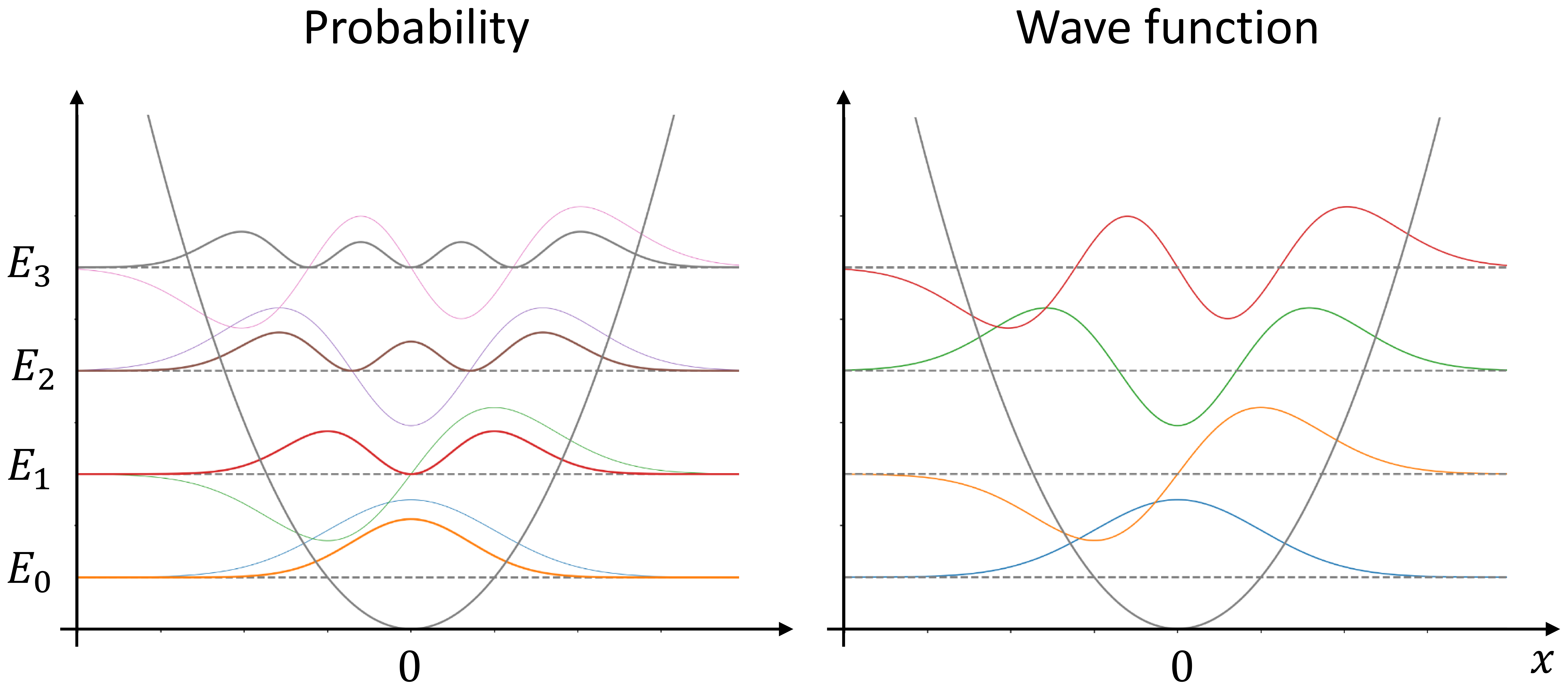
$H_3(z) = -12z + 8z^3$

...

$$V(x) = \frac{1}{2}k'x^2 = \frac{1}{2}m\omega^2x^2$$

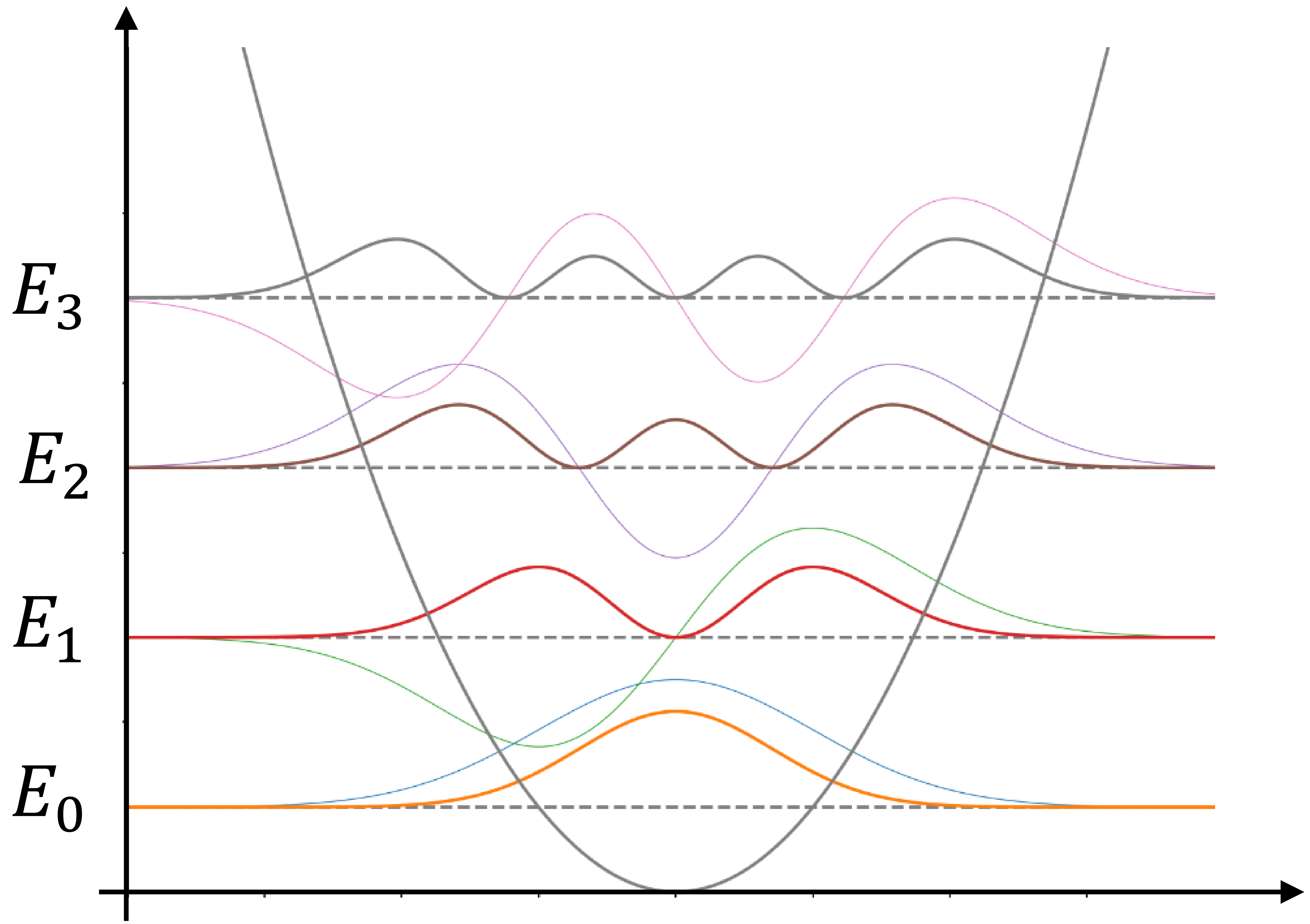


THE HARMONIC POTENTIAL WELL



THE HARMONIC POTENTIAL WELL

Probability



- Probability $|\psi_n(x)|^2$ gathers at boundaries for large n
- Similar as the classical harmonic oscillator which spends more time at the turning points
- Unlike classical case minimum $E_0 = \frac{1}{2}\hbar\omega \neq 0$