PHOT 222: Quantum Photonics
LECTURE 02

Michaël Barbier, Spring semester (2024-2025)

OVERVIEW OF THE COURSE

week	topic	Serway	Young
Week 1	Relativity	Ch. 39	Ch. 37
Week 2	**Waves and Particles**	**Ch. 40**	**Ch. 38**
Week 3	Wave packets and Uncertainty		
Week 4	The Schrödinger equation and Probability		
Week 5	**Midterm exam 1**		
Week 6	Quantum particles in a potential		
Week 7	Harmonic oscillator		
Week 8	Tunneling through a potential barrier		
Week 9	The hydrogen atom, absorption/emission spectra		
Week 10	**Midterm exam 2**		
Week 11	Many-electron atoms		
Week 12	Pauli-exclusion principle		
Week 13	Atomic bonds and molecules		
Week 14	Crystalline materials and energy band structure		

Blackbody Radiation & the Ultraviolet Catastrophe

BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

- Any **object at nonzero temperature** emits **thermal radiation** from its surface
- **Black body**: Perfect absorber/emitter
- Its Radiation: **blackbody radiation**

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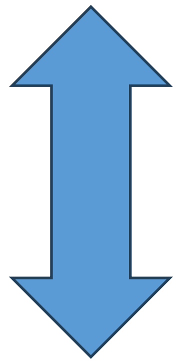

- Color of daily objects: **Reflection** of light

BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

- Any **object at nonzero temperature** emits **thermal radiation** from its surface
- **Black body**: Perfect absorber/emitter
- Its Radiation: **blackbody radiation**
- **Stefan's law**: Total power of emitted radiation grows with temperature as T^4

$$P = \sigma A e T^4$$

Stefan-Boltzmann: $\sigma = 5.7 \times 10^{-8} \frac{W}{m^2} K^4$

Surface area: A, emissivity: e

BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

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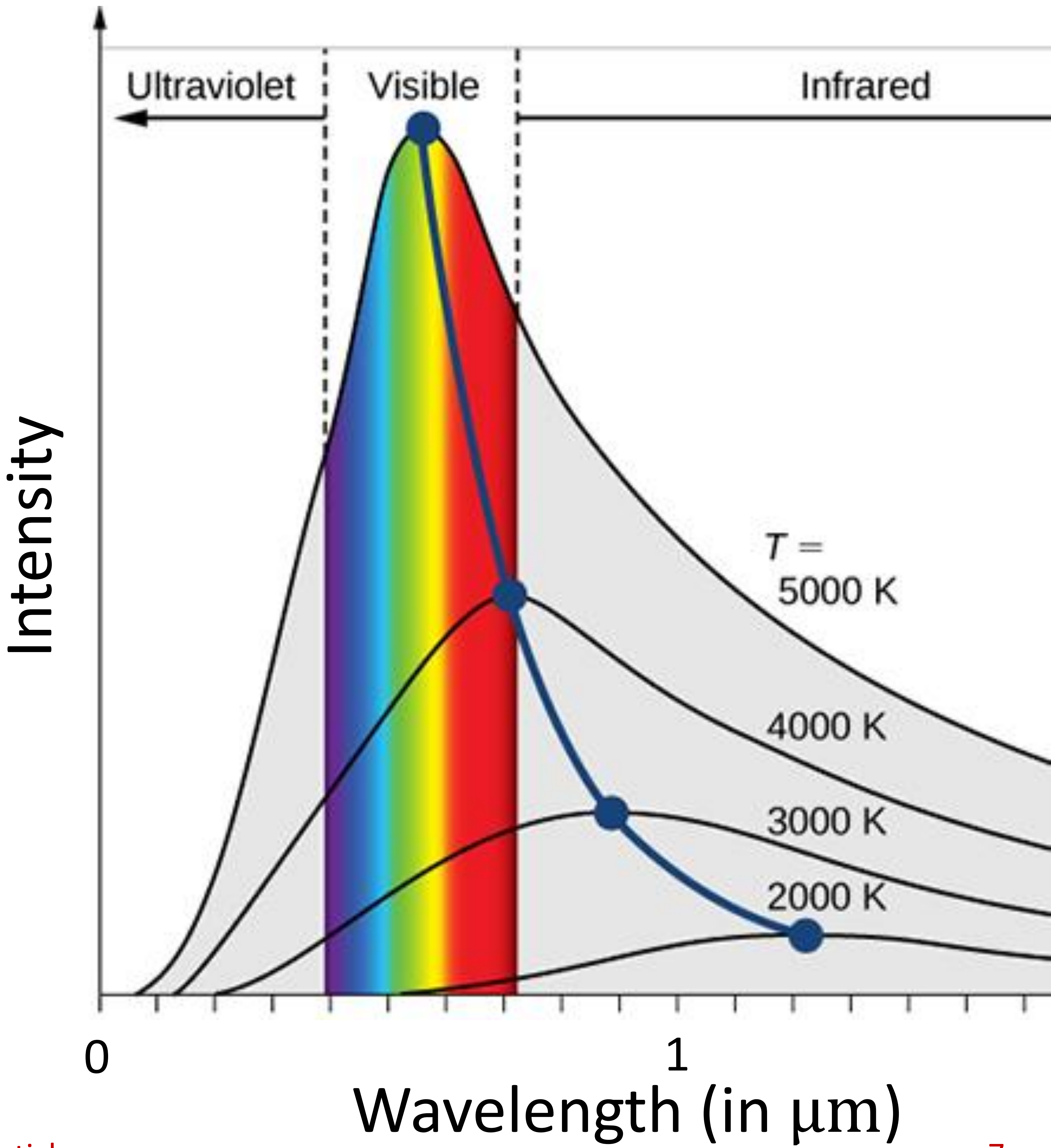
Surface area: A, emissivity: $0 \leq e \leq 1$

- **Wien's displacement law**: Spectrum shifts to shorter wavelength at higher temperature

$$\lambda_{\max} T = 2.898 \times 10^{-3}\, m \cdot K$$



BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

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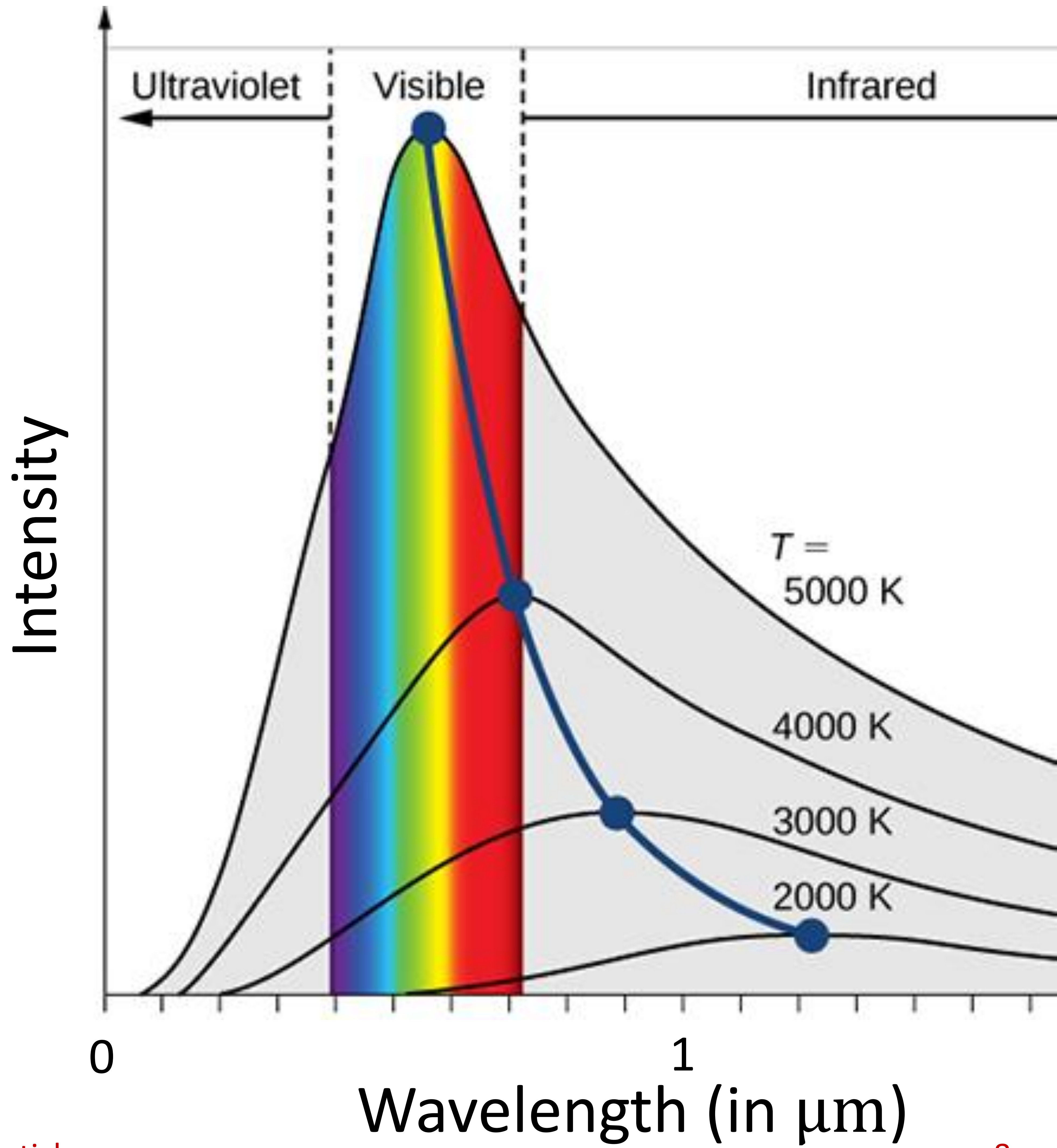
- Temperature determines color, i.e. dominating wavelength



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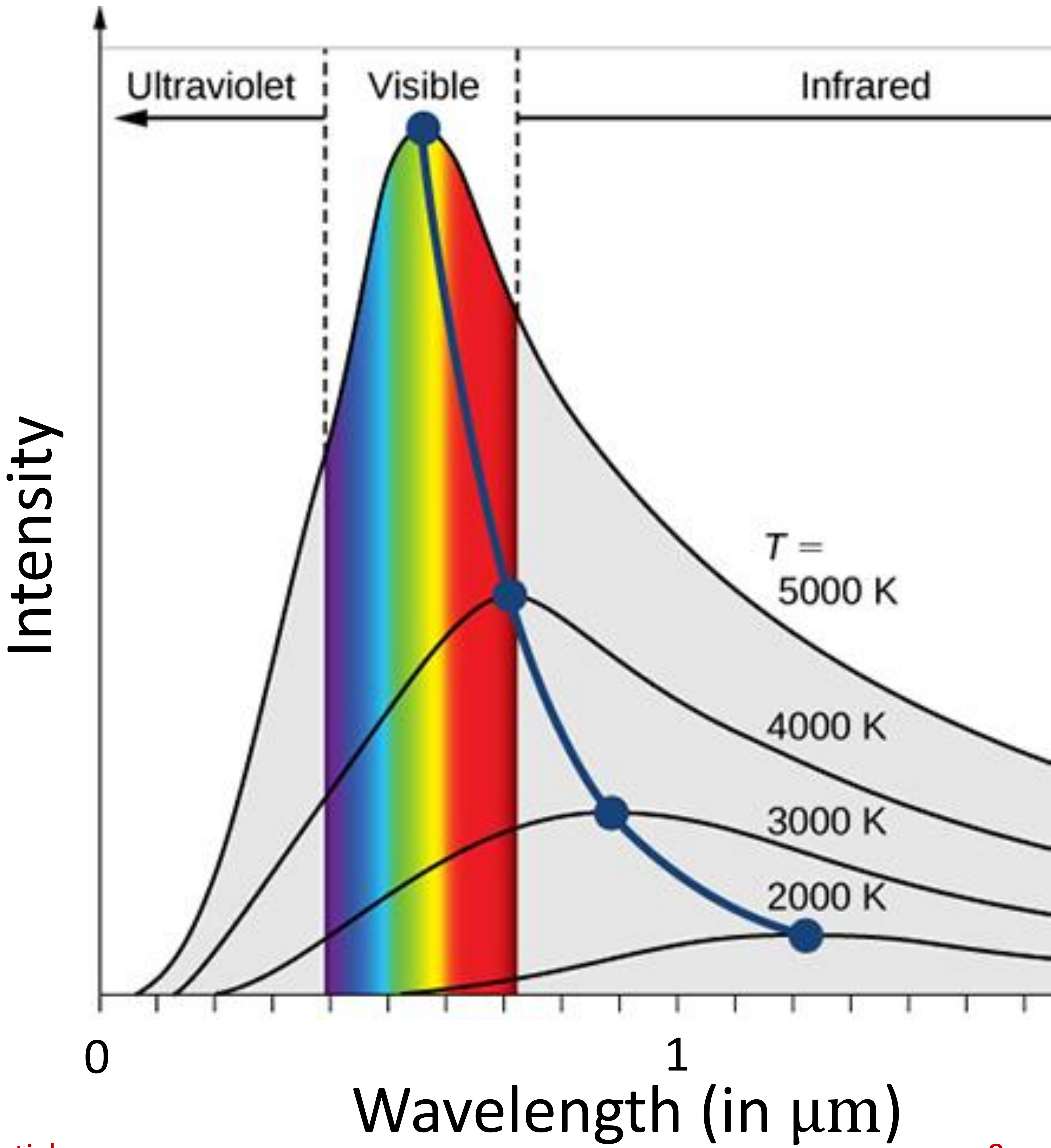
- Theory: **Rayleigh-Jeans law**

$$I(\lambda, T) = \frac{8\pi c k_B T}{\lambda^4}$$

Intensity increases for shorter wavelengths, goes to infinity



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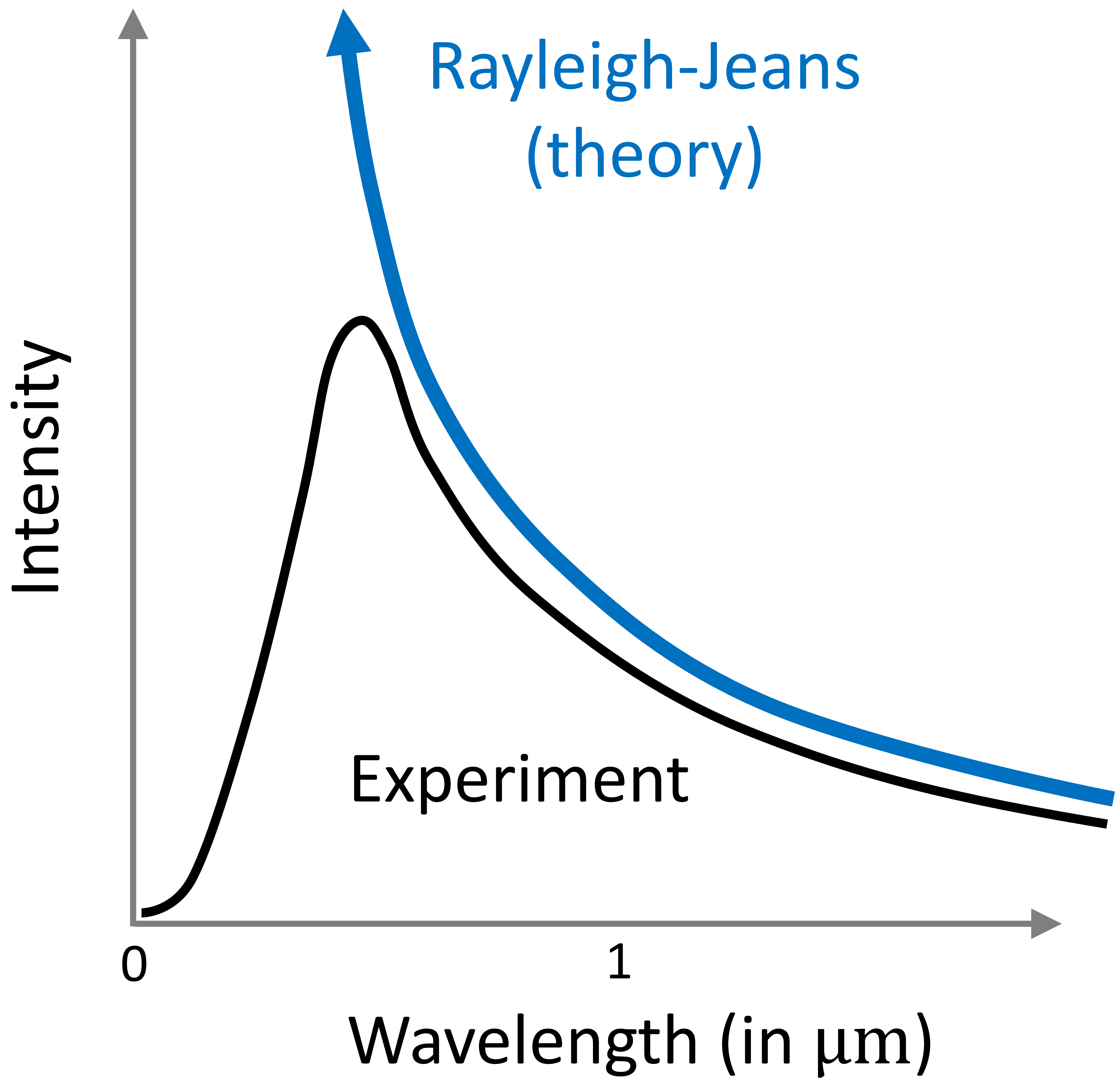
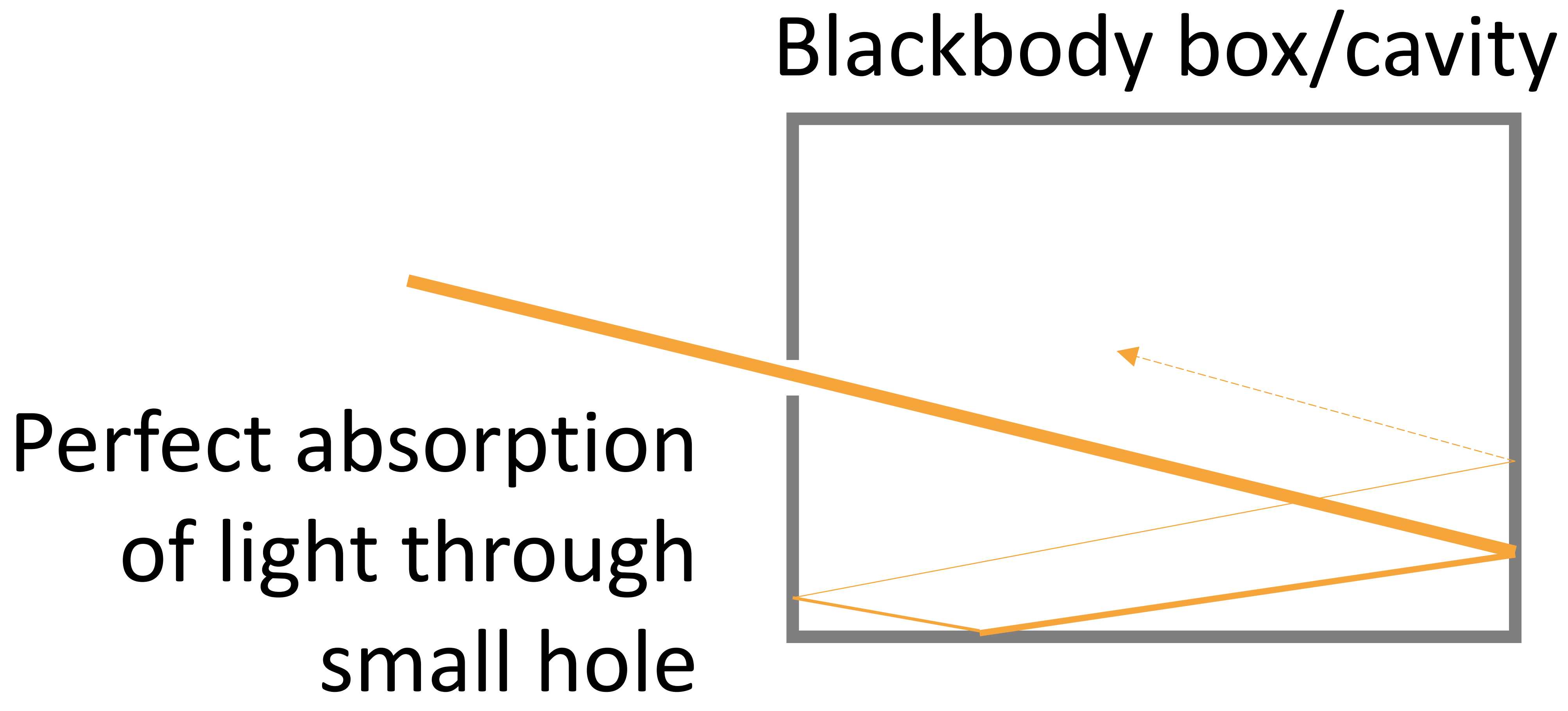


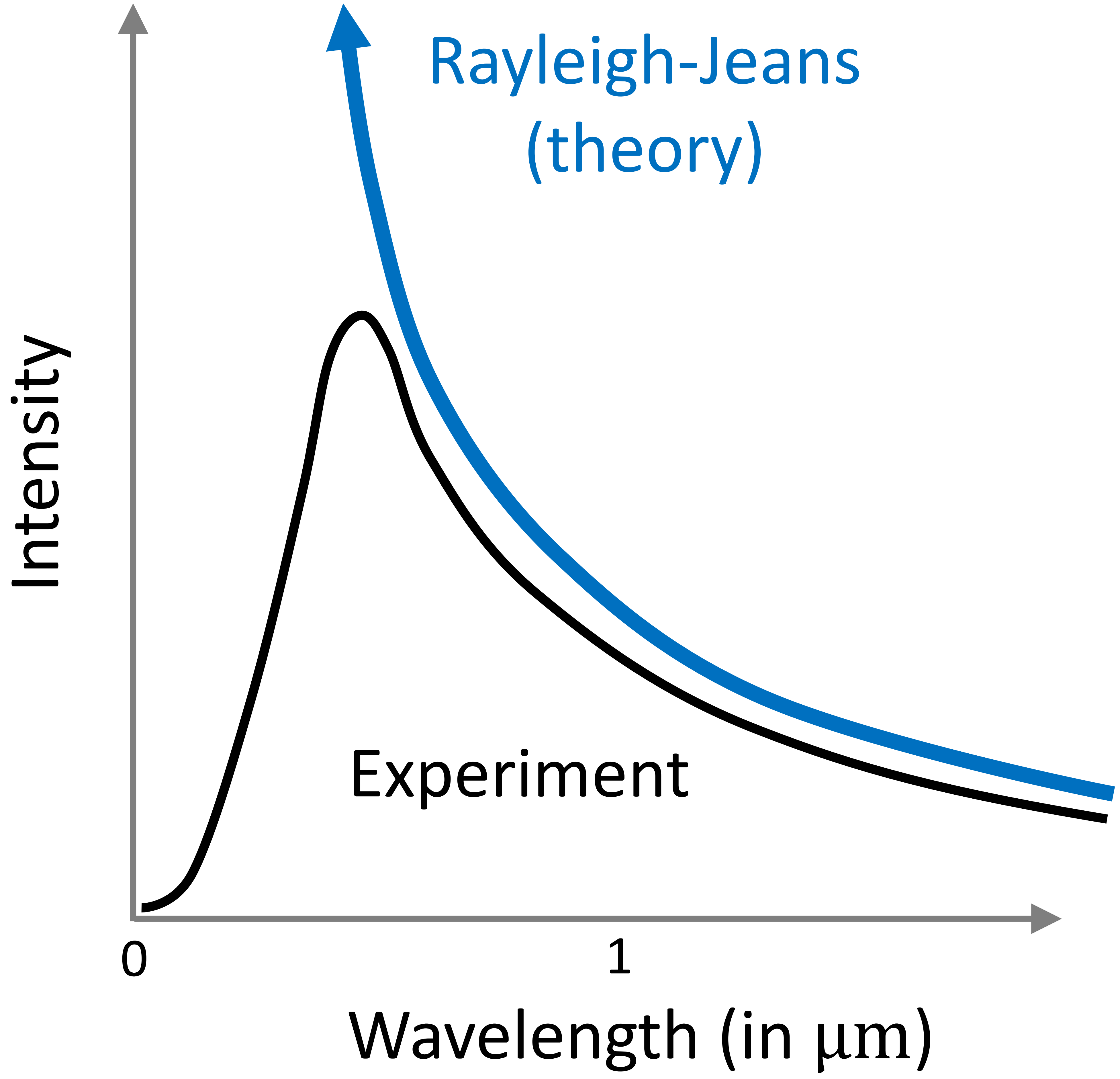
BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

- Theory: **Rayleigh-Jeans law**

$$I(\lambda, T) = \frac{8\pi c k_B T}{\lambda^4}$$

- Theoretical model





Kirchoff's law: absorption = emission

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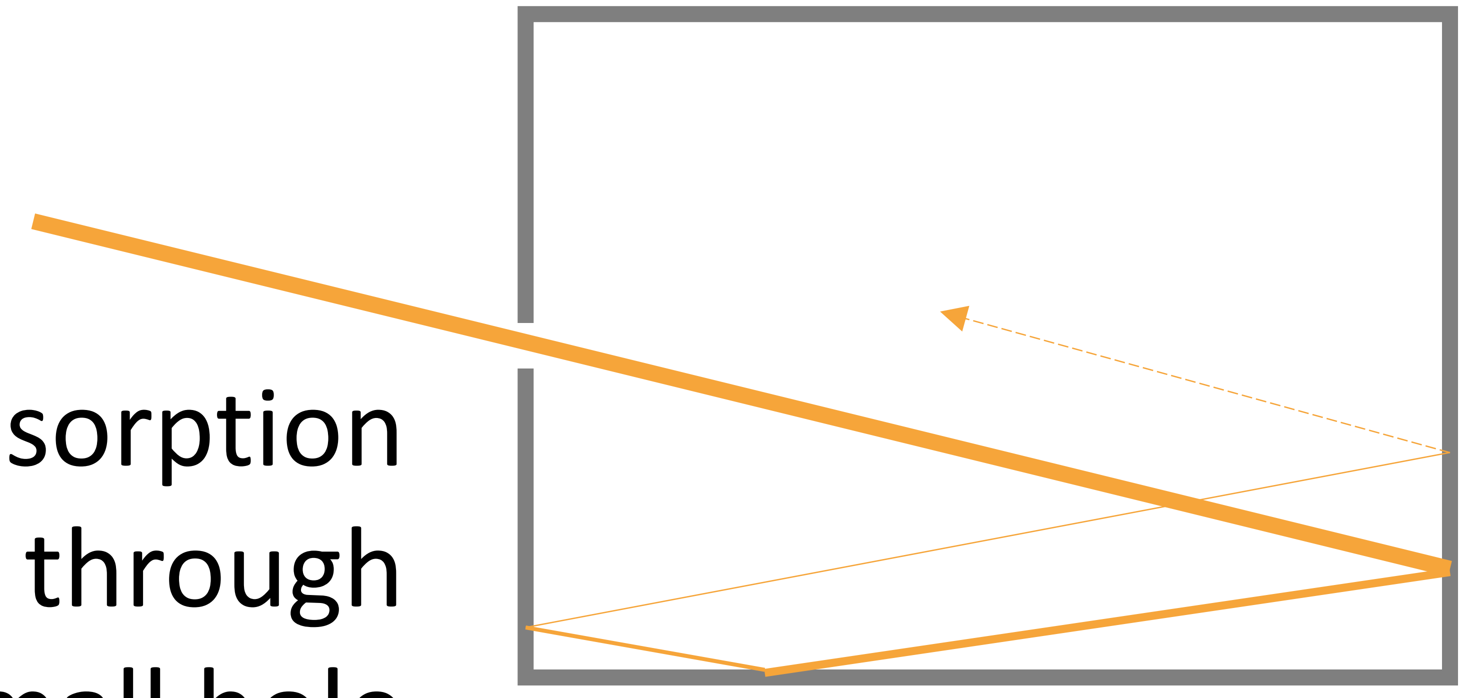
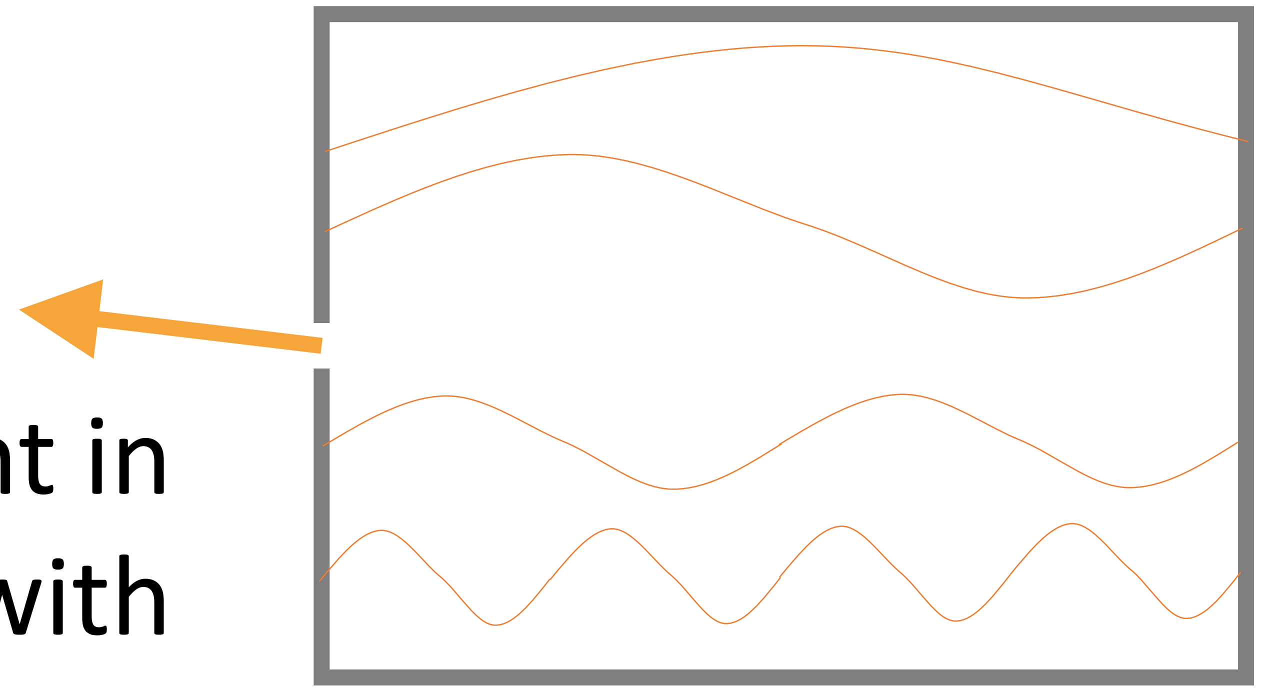
Blackbody box/cavity



Perfect absorption of light through small hole

Blackbody box/cavity



Absorbed light in equilibrium with cavity

Kirchoff’s law: absorption = emission

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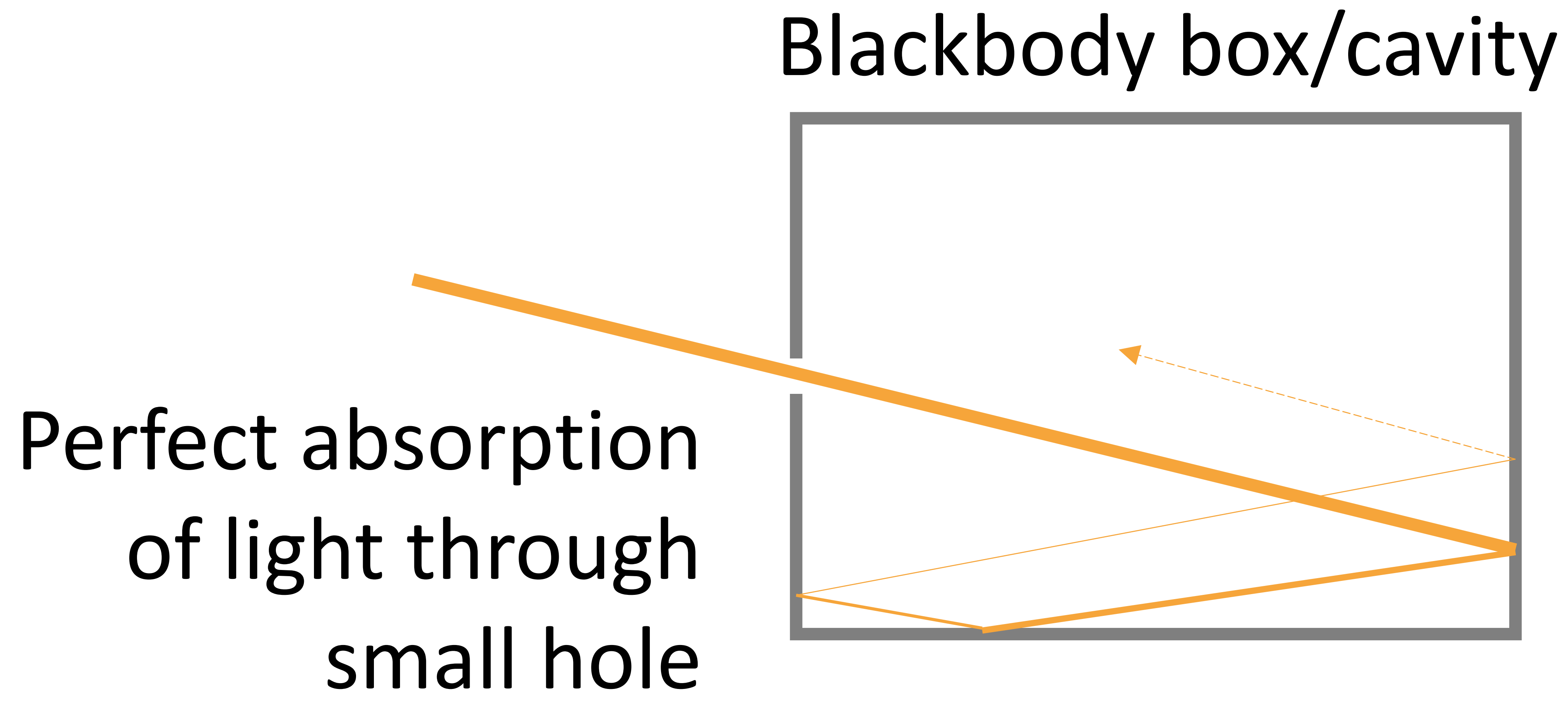
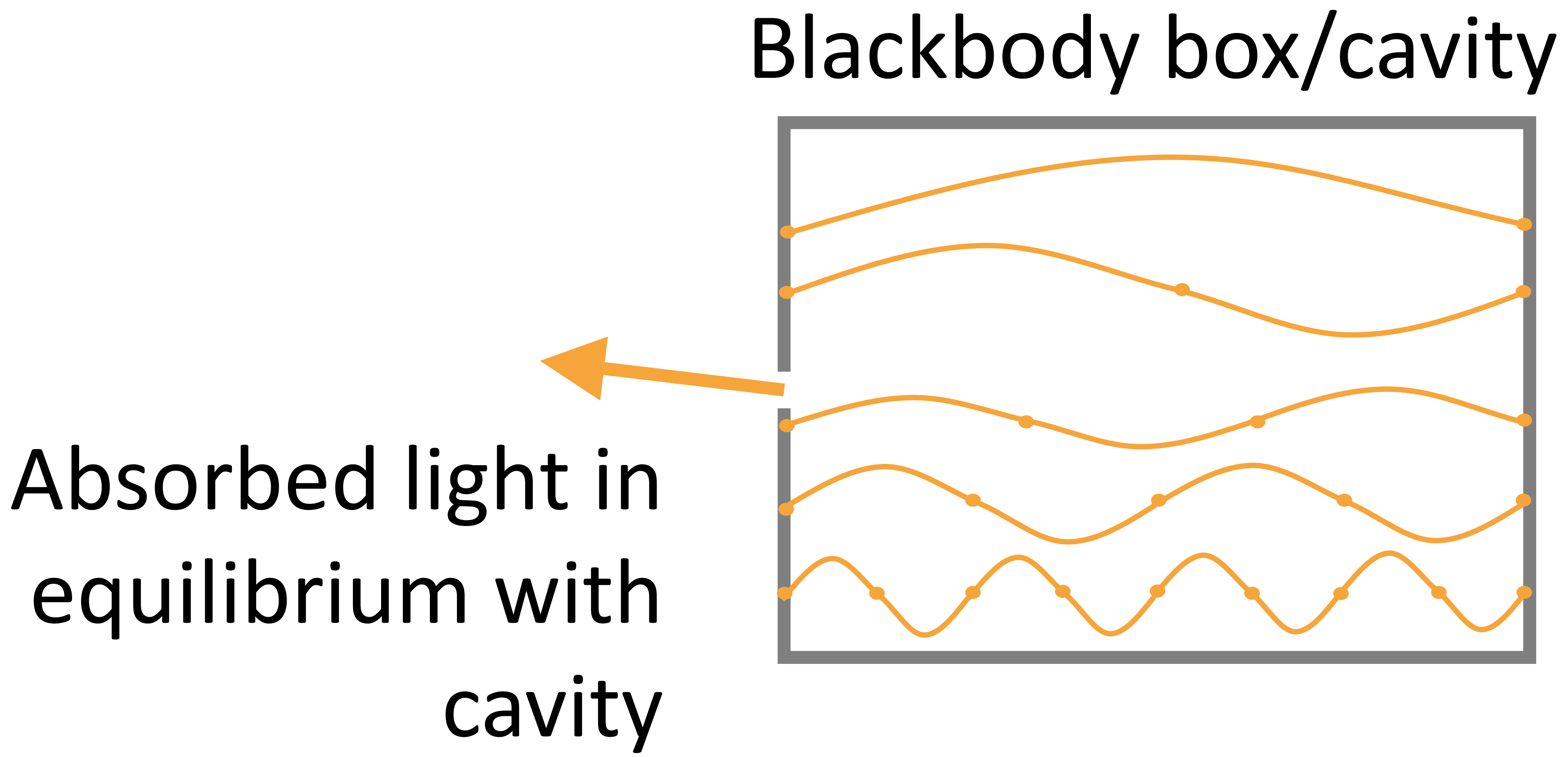


Kirchoff's law: absorption = emission

- Inside wall in equilibrium with cavity

- Number of standing waves
- Each wave $2 \times \frac{1}{2} k_B T$



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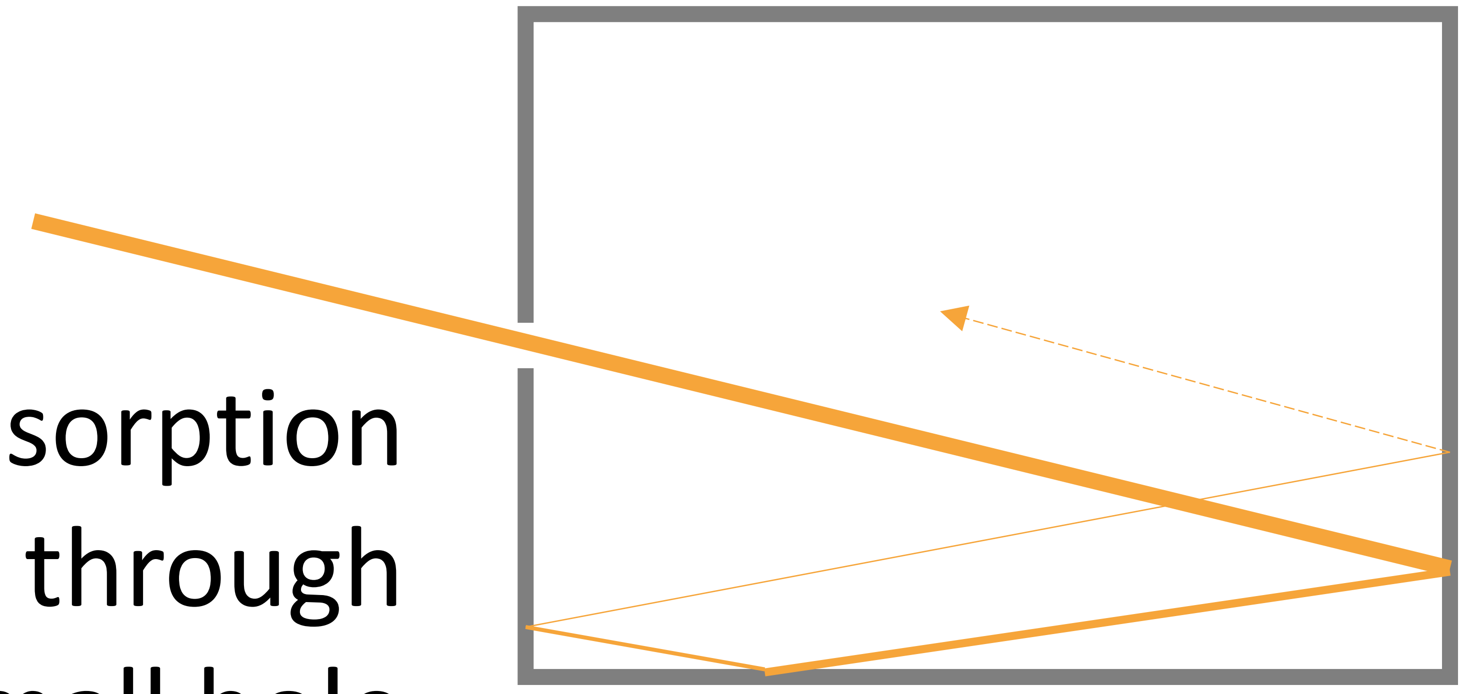


Perfect absorption of light through small hole

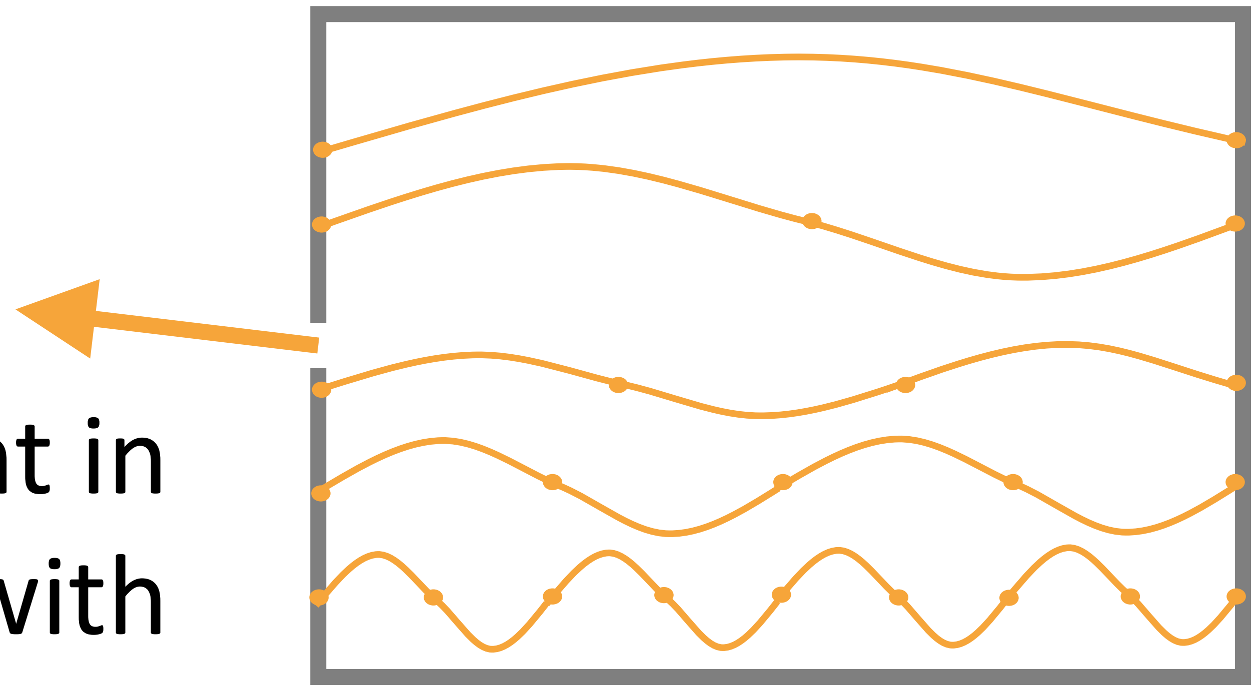
Kirchoff's law: absorption = emission

- Inside wall in equilibrium with cavity

Classical

- Number of standing waves
- Each wave $2 \times \frac{1}{2} k_B T$

Blackbody box/cavity



Absorbed light in equilibrium with cavity

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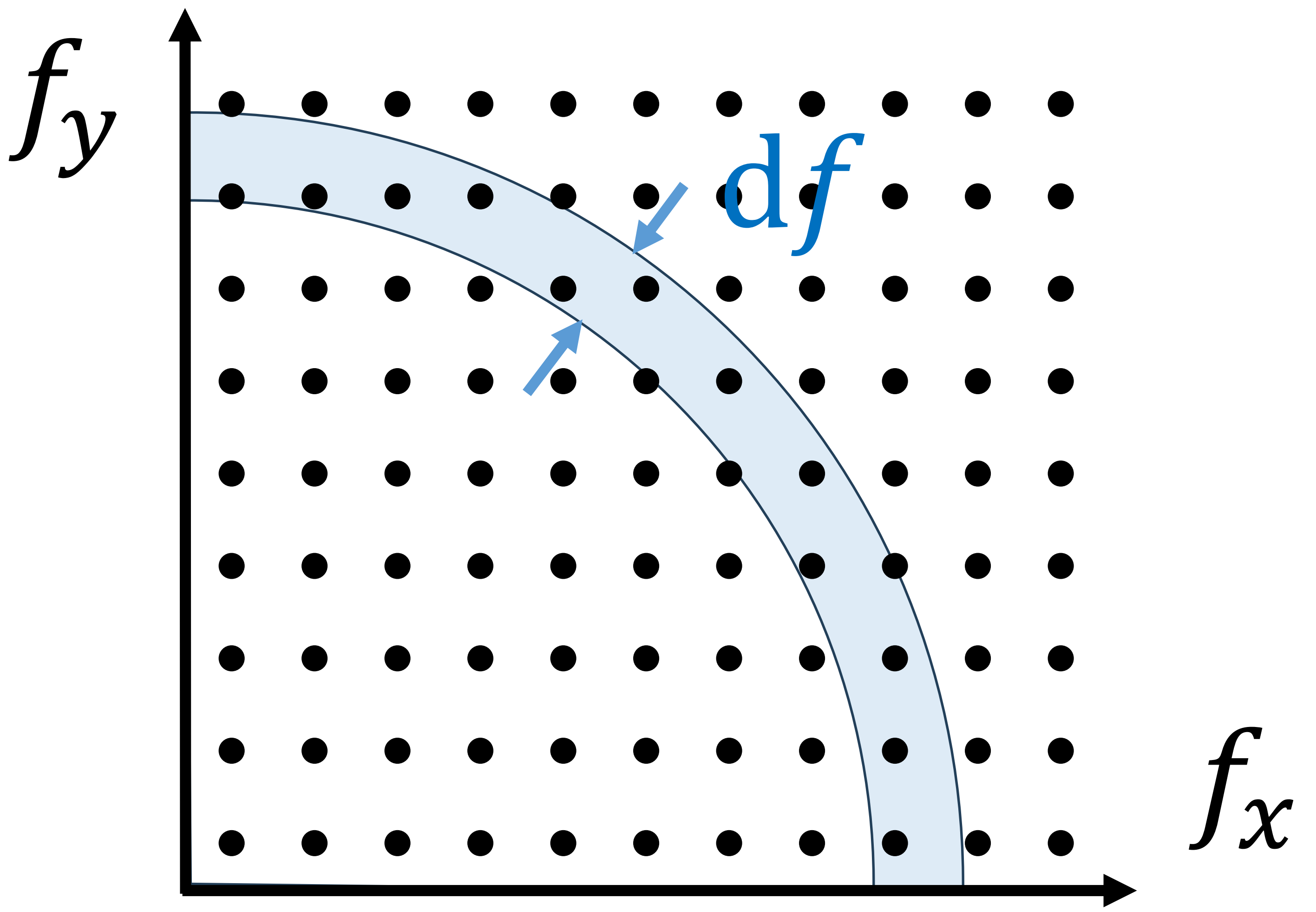
- Theory: **Rayleigh-Jeans law**

$$I(\lambda, T) = \frac{8\pi c k_B T}{\lambda^4}$$

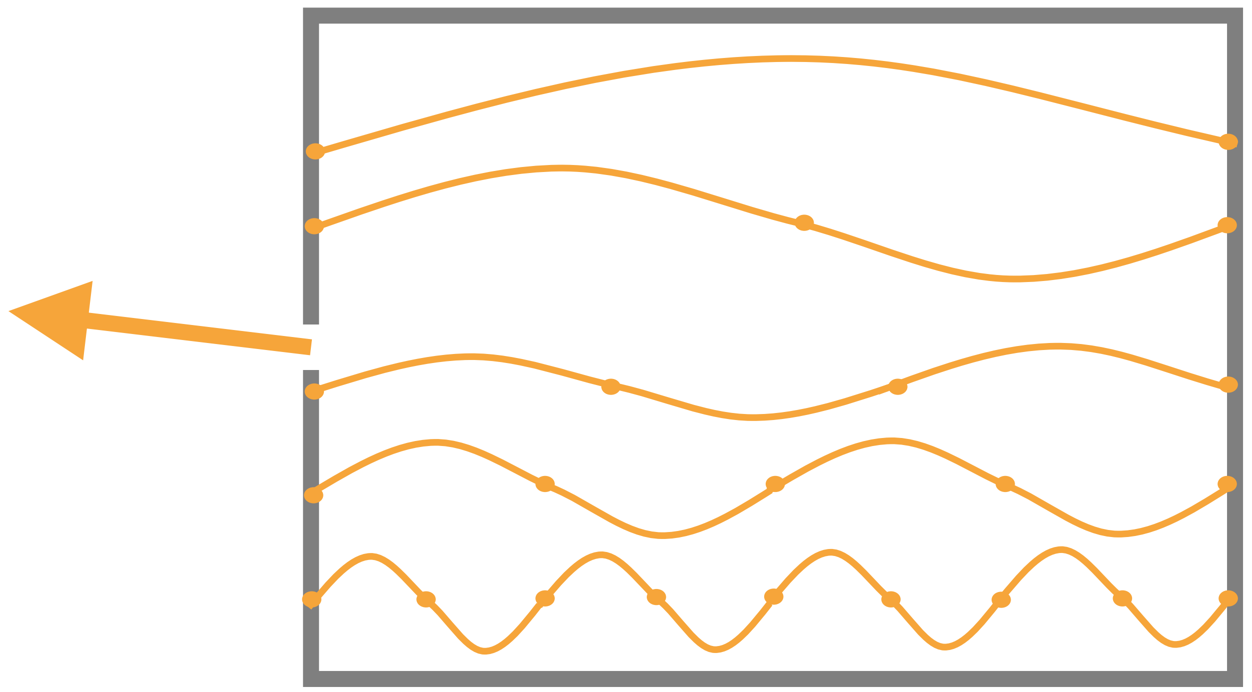
- Number of standing waves
  - Frequencies: $f = (f_x, f_y, f_z)$
  - In shell with thickness $\mathrm{d}f \sim f^2$

  → More standing waves/modes at higher frequencies

- **Each wave/mode** $2 \times \frac{1}{2} k_B T$
- 2 degrees of freedom: K and V
- Contribution of shorter waves → ∞



Blackbody box/cavity



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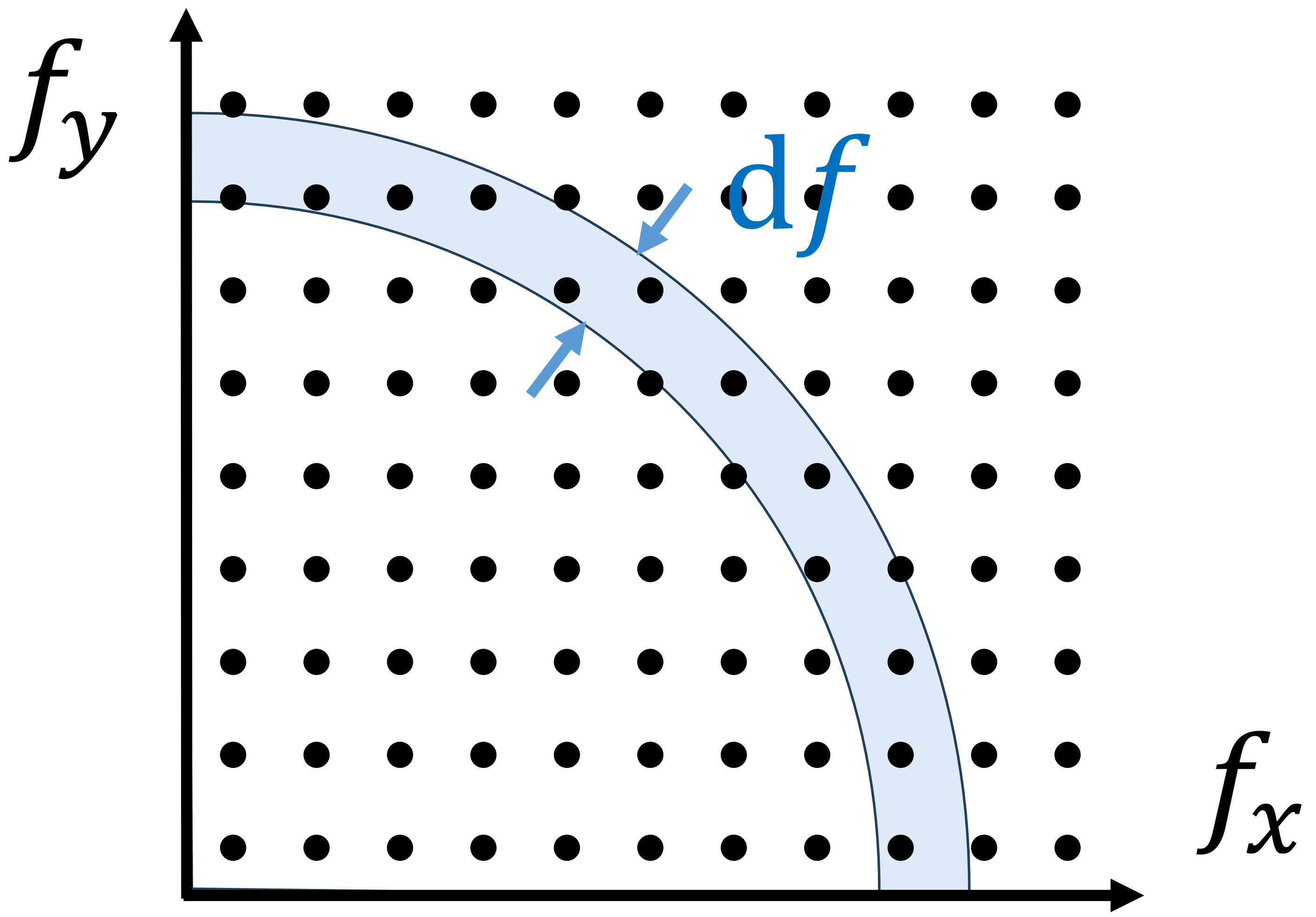
- Number of standing waves
  - Frequencies: $f = (f_x, f_y, f_z)$
  - In shell with thickness $\mathrm{d}f \sim 4\pi f^2$

  → More standing waves/modes at higher frequencies

  - Energy per unit volume:

$$\frac{8\pi\, f^2\, k_B T\, \mathrm{d}f}{c^3} \quad \& \quad \mathrm{d}f = \mathrm{d}\left(\frac{c}{\lambda}\right) = -\frac{2c}{\lambda^2}\mathrm{d}\lambda$$



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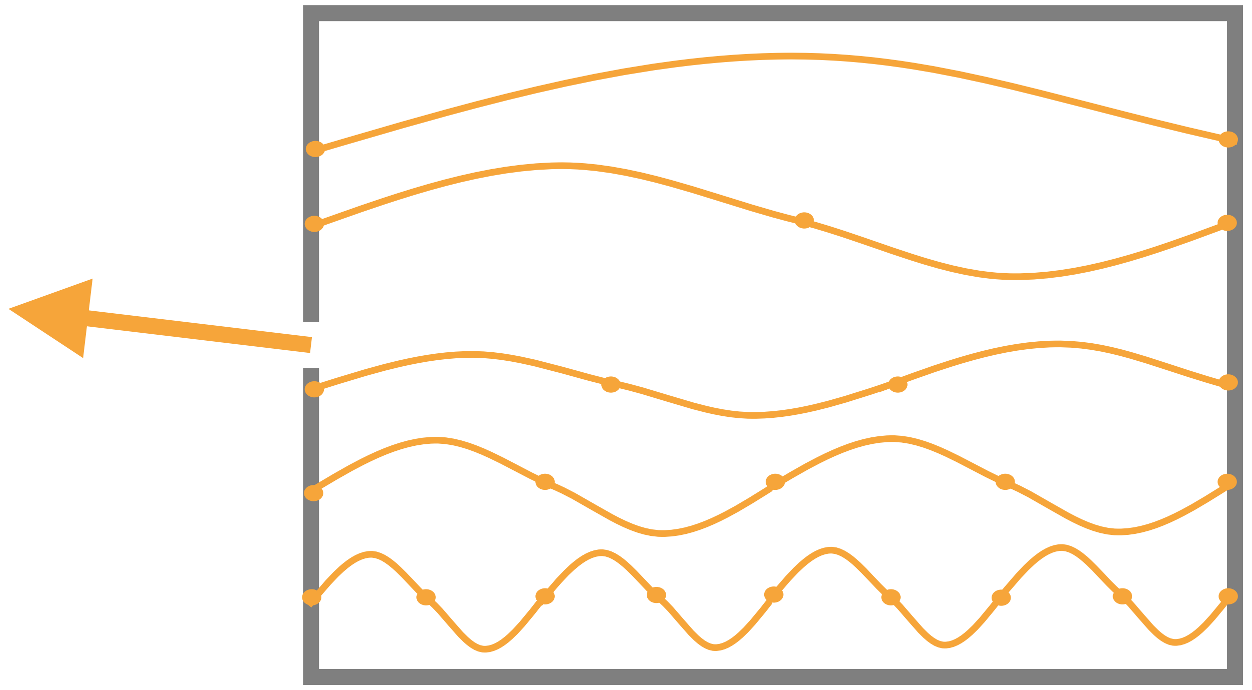


BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

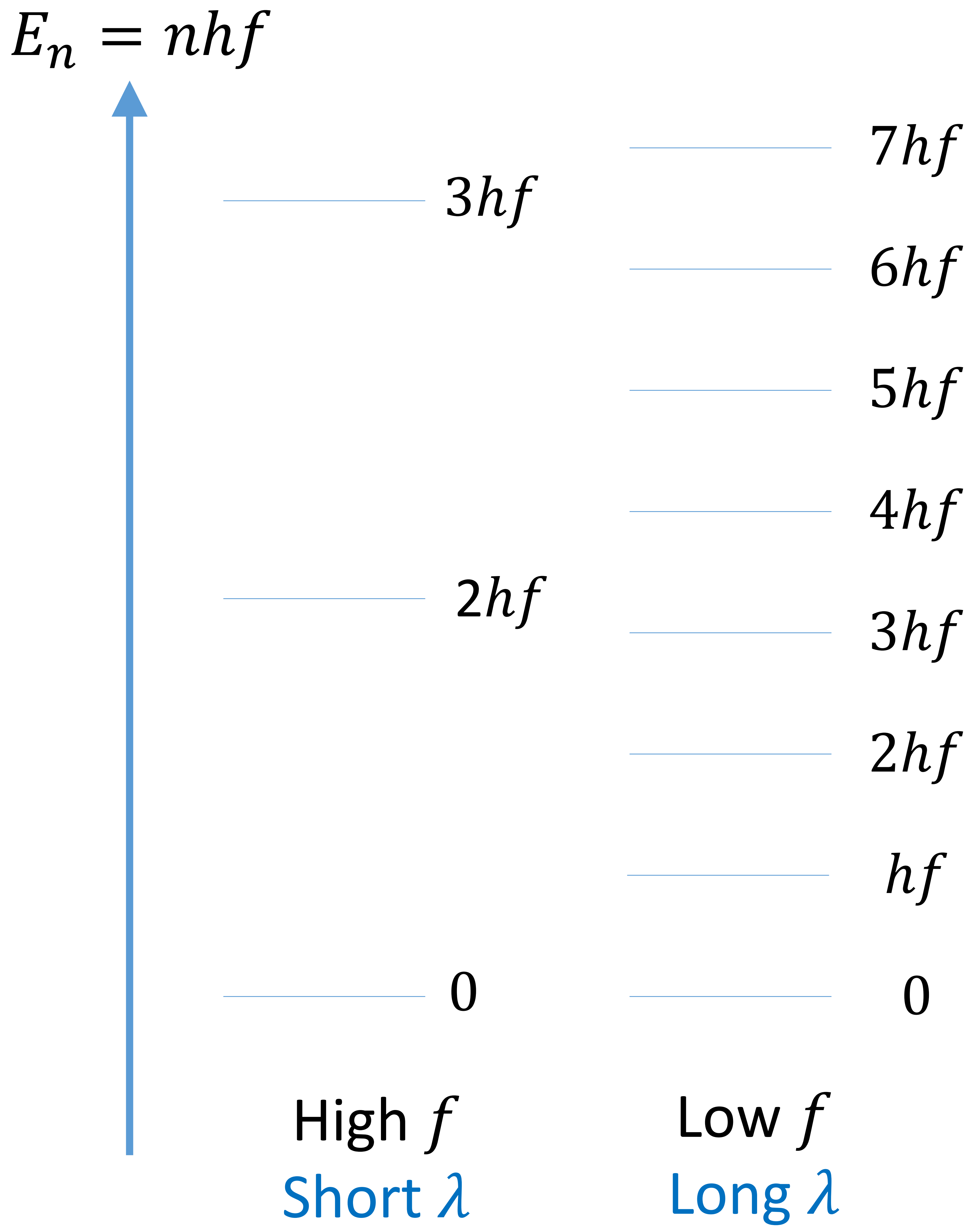
- Theory: **Planck's radiation law**

$$I(\lambda, T) = \frac{8\pi hc^2}{\lambda^5\left(e^{hc/(\lambda k_B T)} - 1\right)}$$

- Similar to Rayleigh: $\mathrm{d}f \sim 4\pi f^2$

BUT

- Energy quantized: $E_n = nhf$
- Number of oscillators $\propto \; e^{-E_n/k_B T}$
- Avg. energy/oscillator $\langle E \rangle = \frac{hf}{e^{\frac{hf}{k_B T}} - 1}$



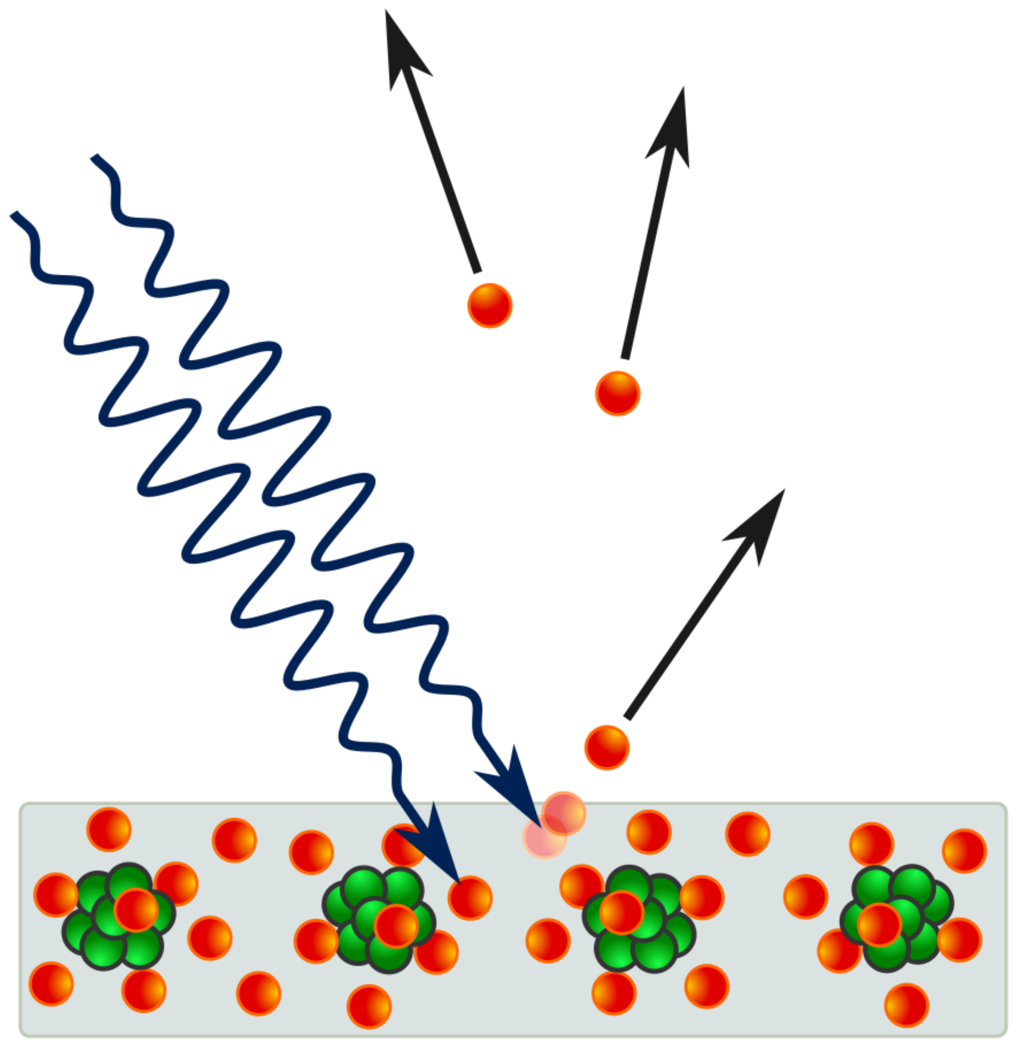
SUMMARY HISTORICAL ARGUMENTS

- Planck found energy packets of light emitted by matter
- Light quantized (particles?) or atoms quantized?

Photoelectric Effect

PHOTOELECTRIC EFFECT

- **Electrons ejected** from metallic surface **by light**
- Light gives electrons energy to escape the material
- The kinetic energy of the electrons is the energy beyond that ionizing energy



- Light gives electrons energy to escape the material ΔU
- **Accelerate** or **stop** electrons: V_c
- The kinetic energy K of the electrons is the energy beyond ionizing energy

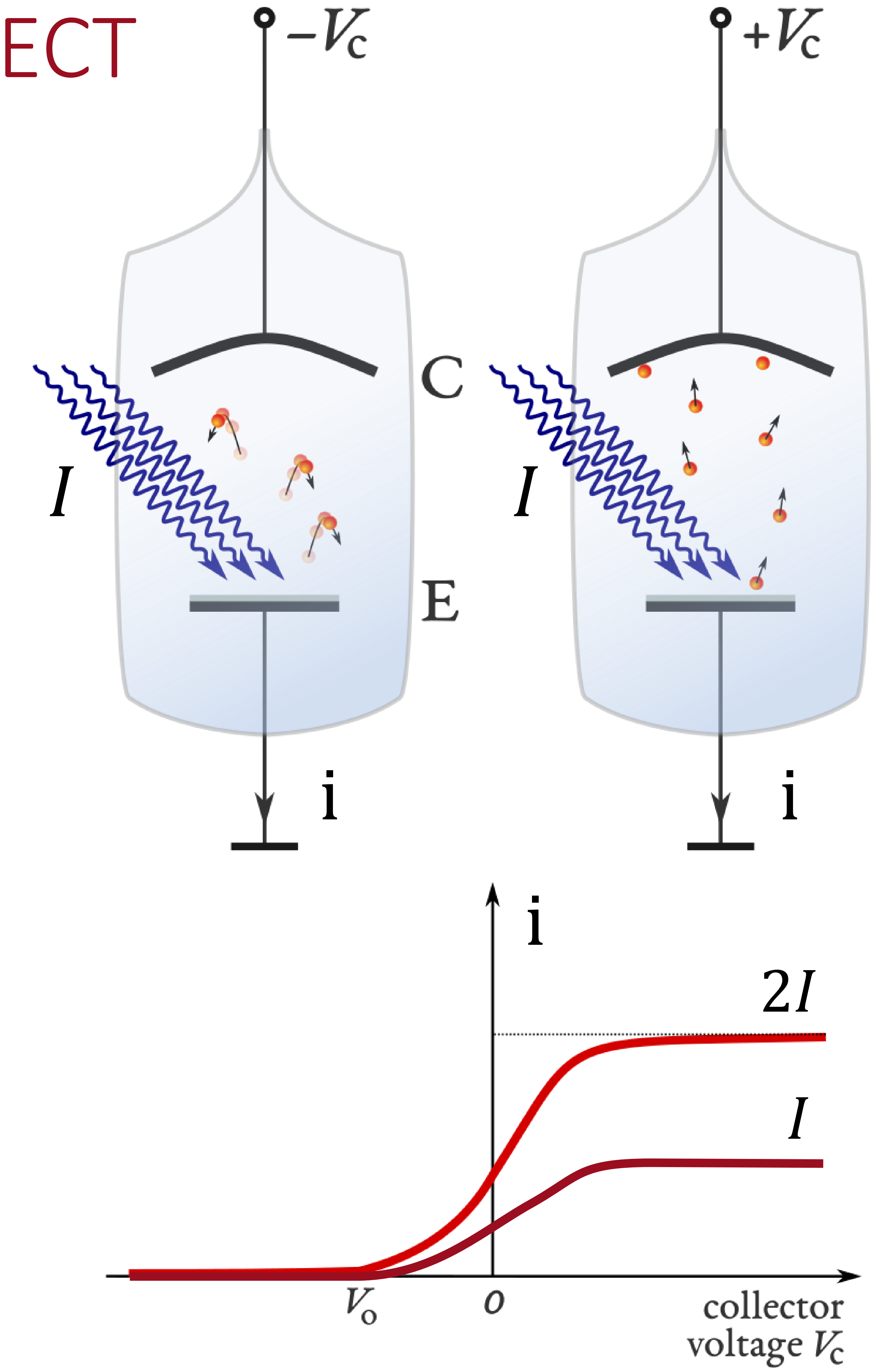
$$\Delta E = K + \Delta U$$

- **Stopping potential** for electrons

$$V_c = -V_0$$

- Kinetic energy K is:

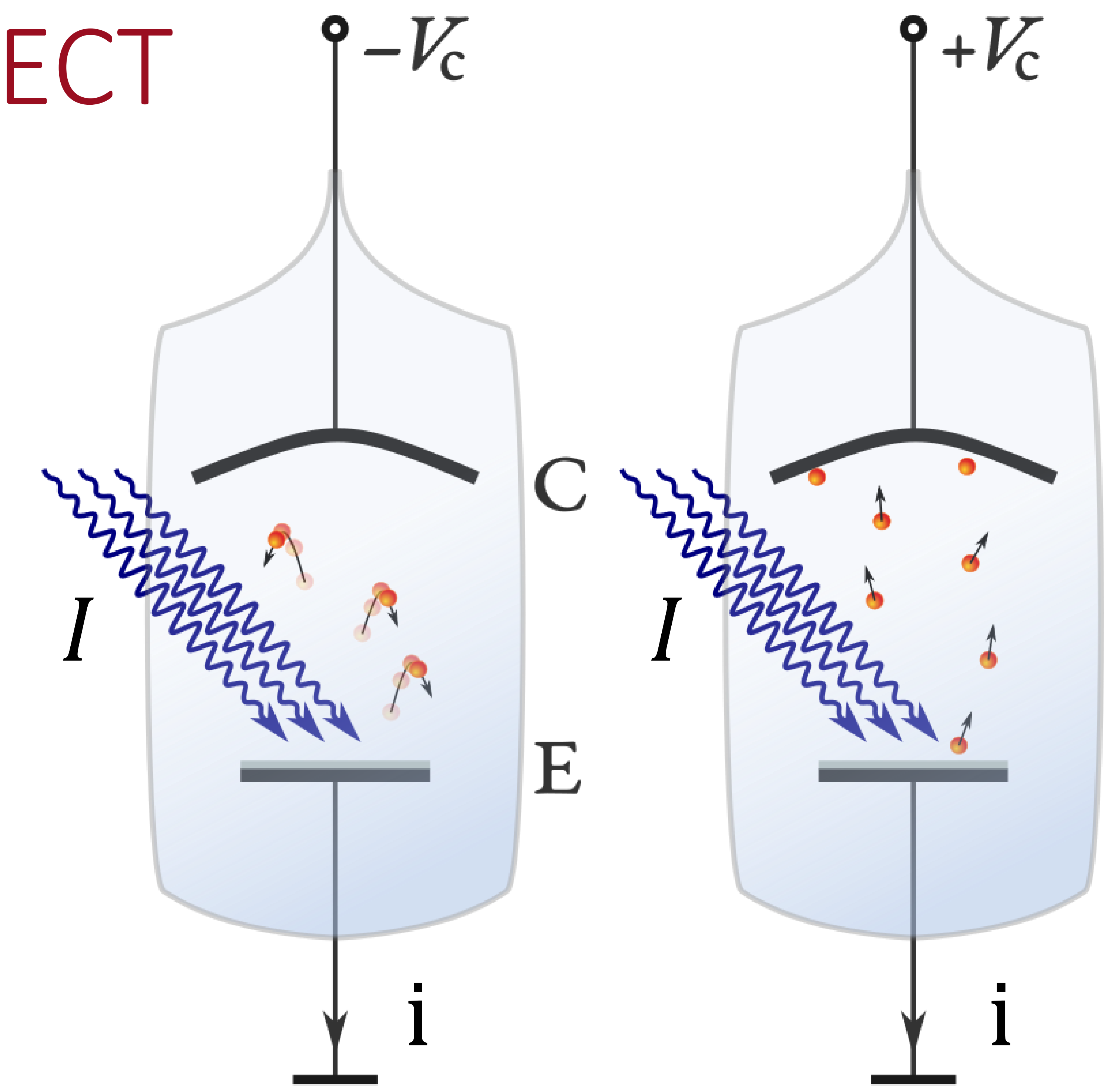
$$K_{\max} = eV_0$$



PHOTOELECTRIC EFFECT

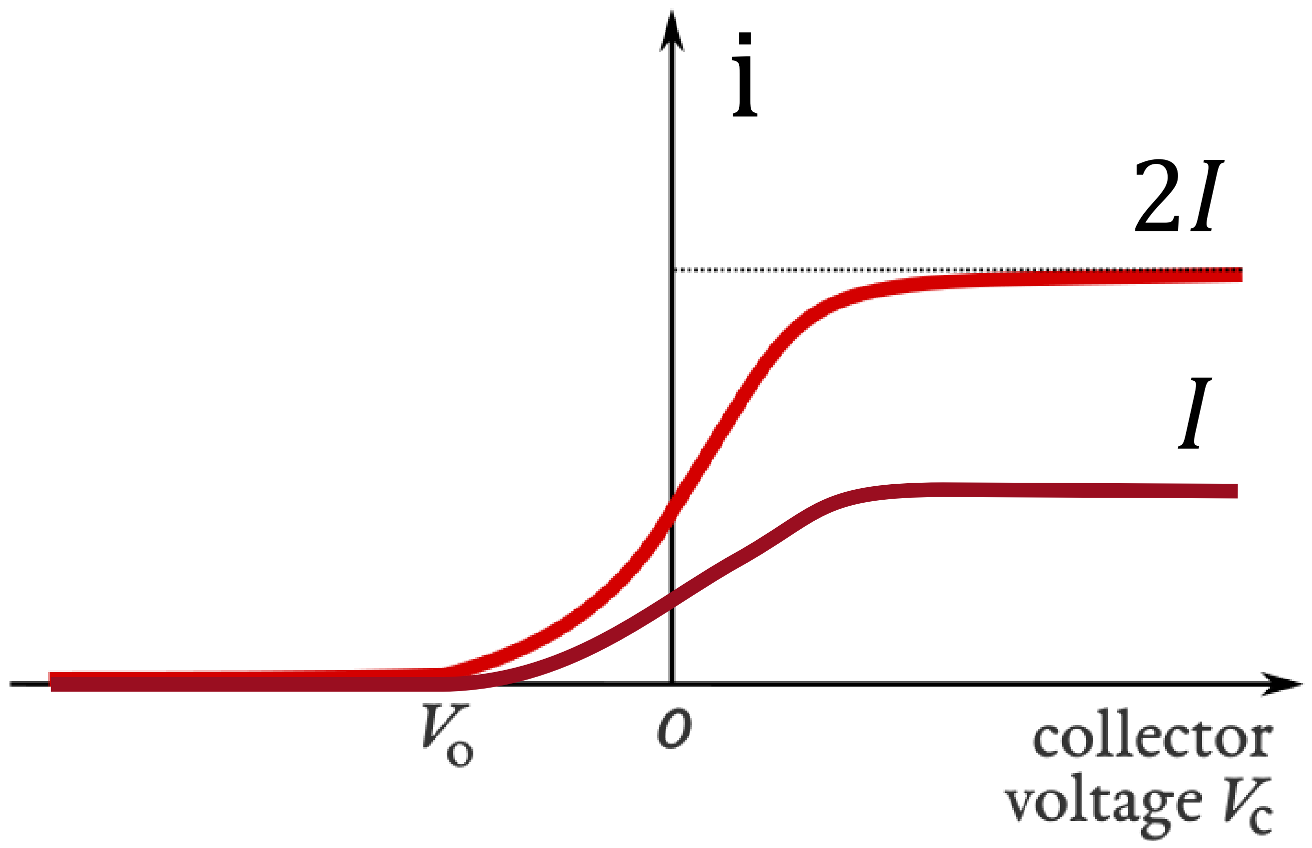
Classical expectations:

1. Magnitude of photocurrent independent from frequency
2. Faint light → delayed electron ejection
3. Higher intensity, increasing K electrons



Experimentally we observe:

1. Minimum **threshold frequency** for freeing electrons
2. Photocurrent ∝ intensity: $\mathrm{i} \propto I$
3. Stopping potential V_0: required voltage to have no photocurrent same for all I



PHOTOELECTRIC EFFECT

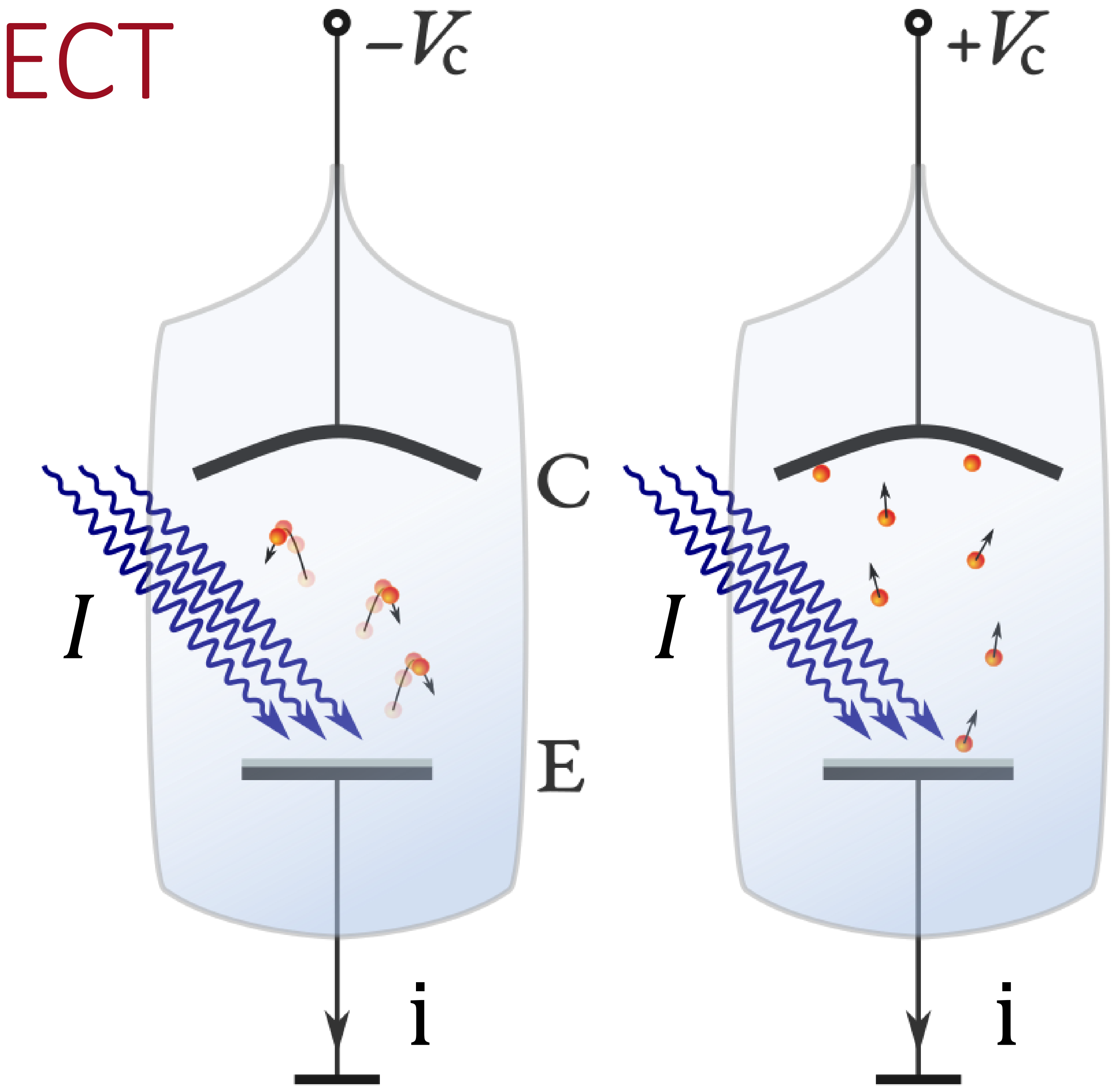
Einstein's idea:

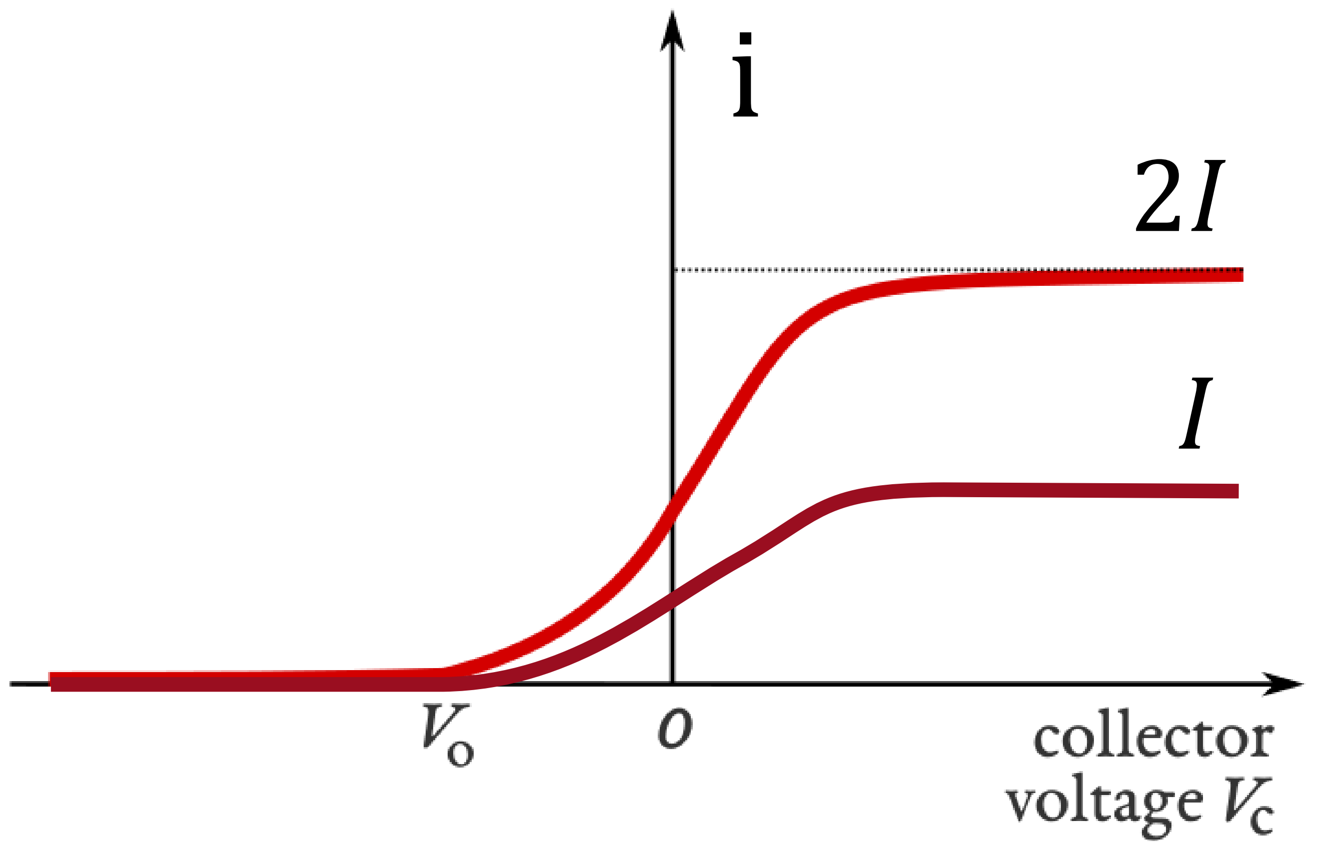
- **Photon** gives all energy hf to one **single electron** in the metal
- Energy to release electron from metal: Work function ϕ
- Kinetic energy of ejected electron K:

$$K_{\max} = hf - \phi$$

- Kinetic energy $K \leq K_{\max}$
- No delay of photocurrent

Consistent with experimental observations





PHOTOELECTRIC EFFECT

- **Kinetic energy** of ejected electron K:

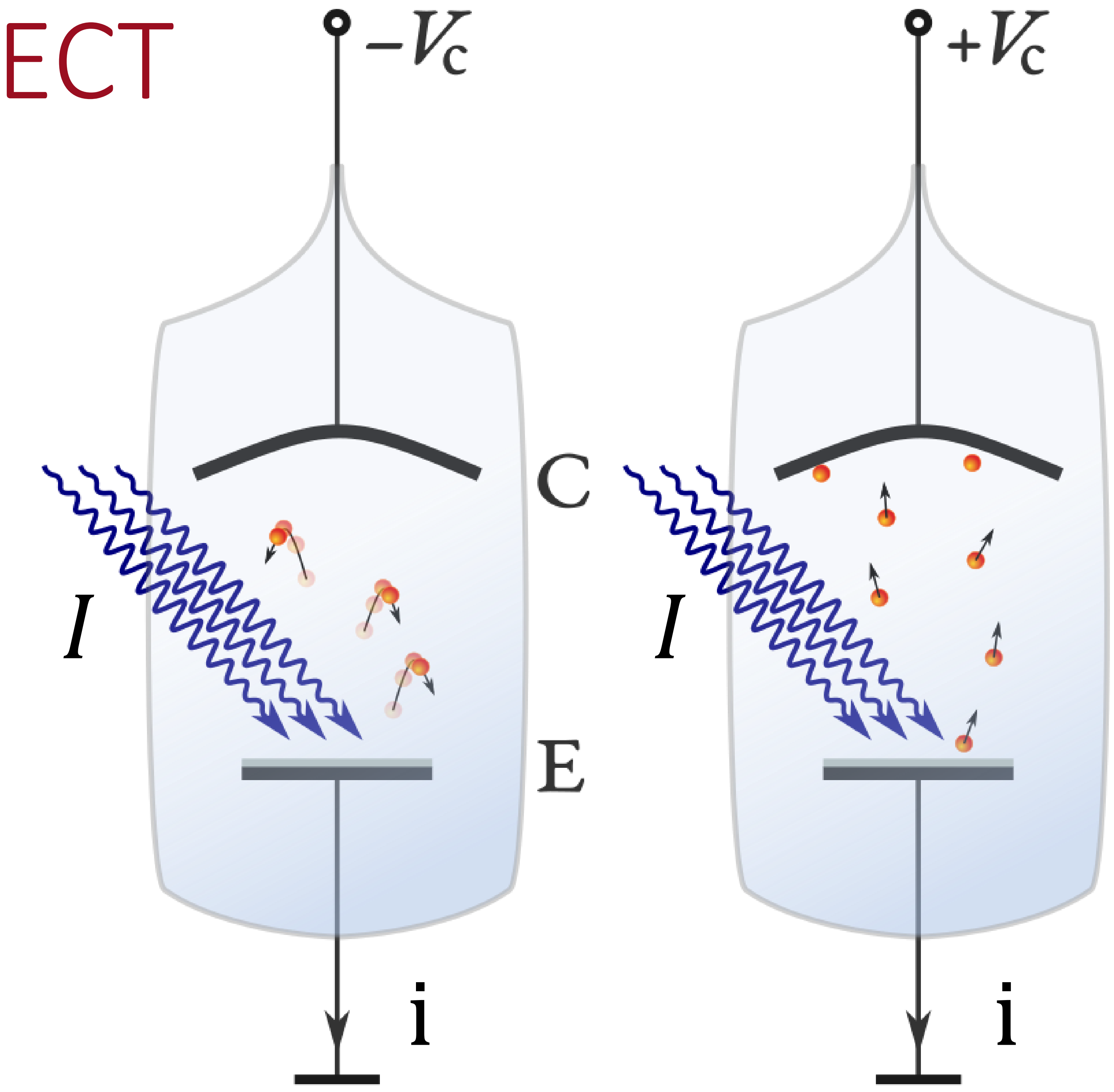
$$K_{\text{max}} = hf - \phi$$

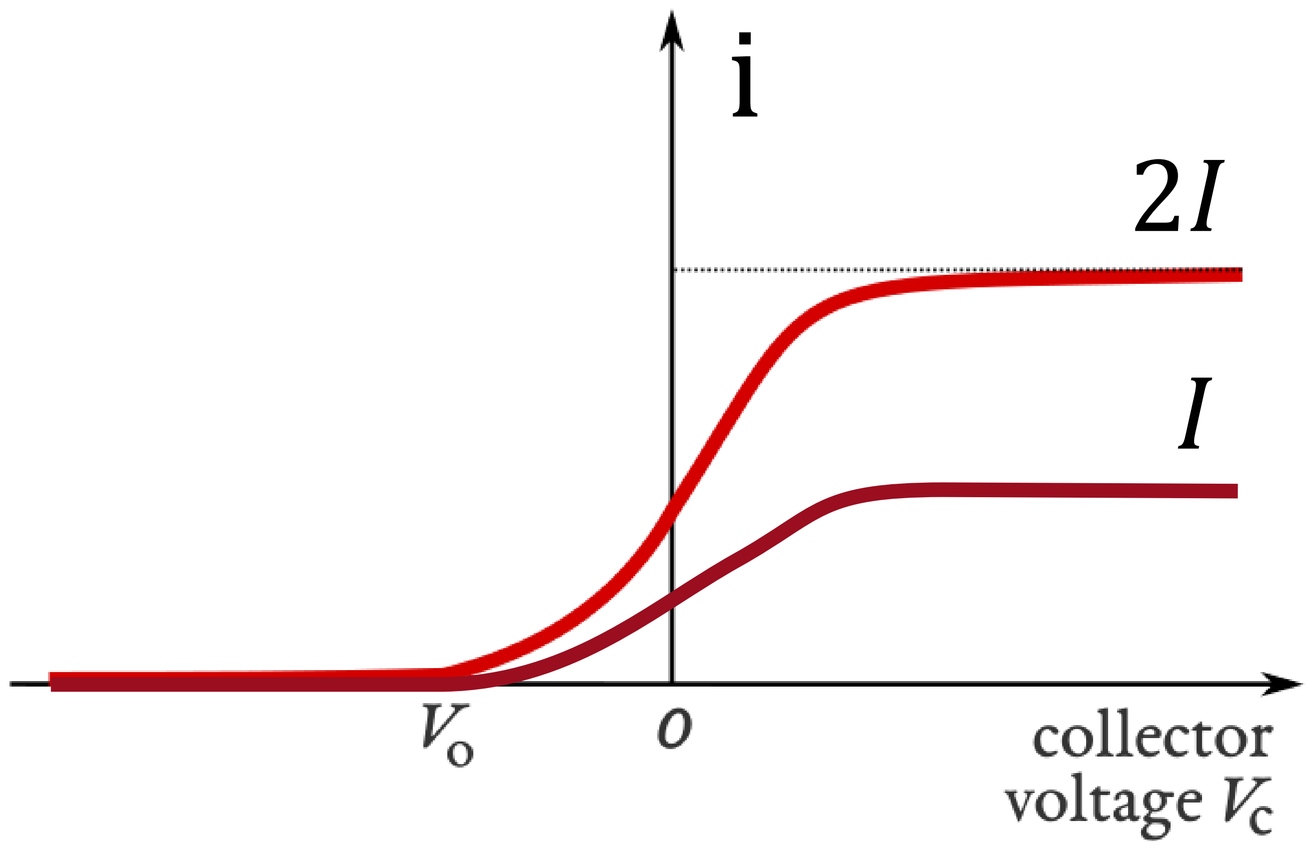
- Cut-off frequency and wavelength:

$$K_{\text{max}} = 0 \;\Rightarrow\; f_c = \frac{\phi}{h}, \quad \lambda_c = \frac{hc}{\phi}$$

- Converting frequency to wavelength:

$$hc = 1240\ \text{eV} \cdot \text{nm}$$





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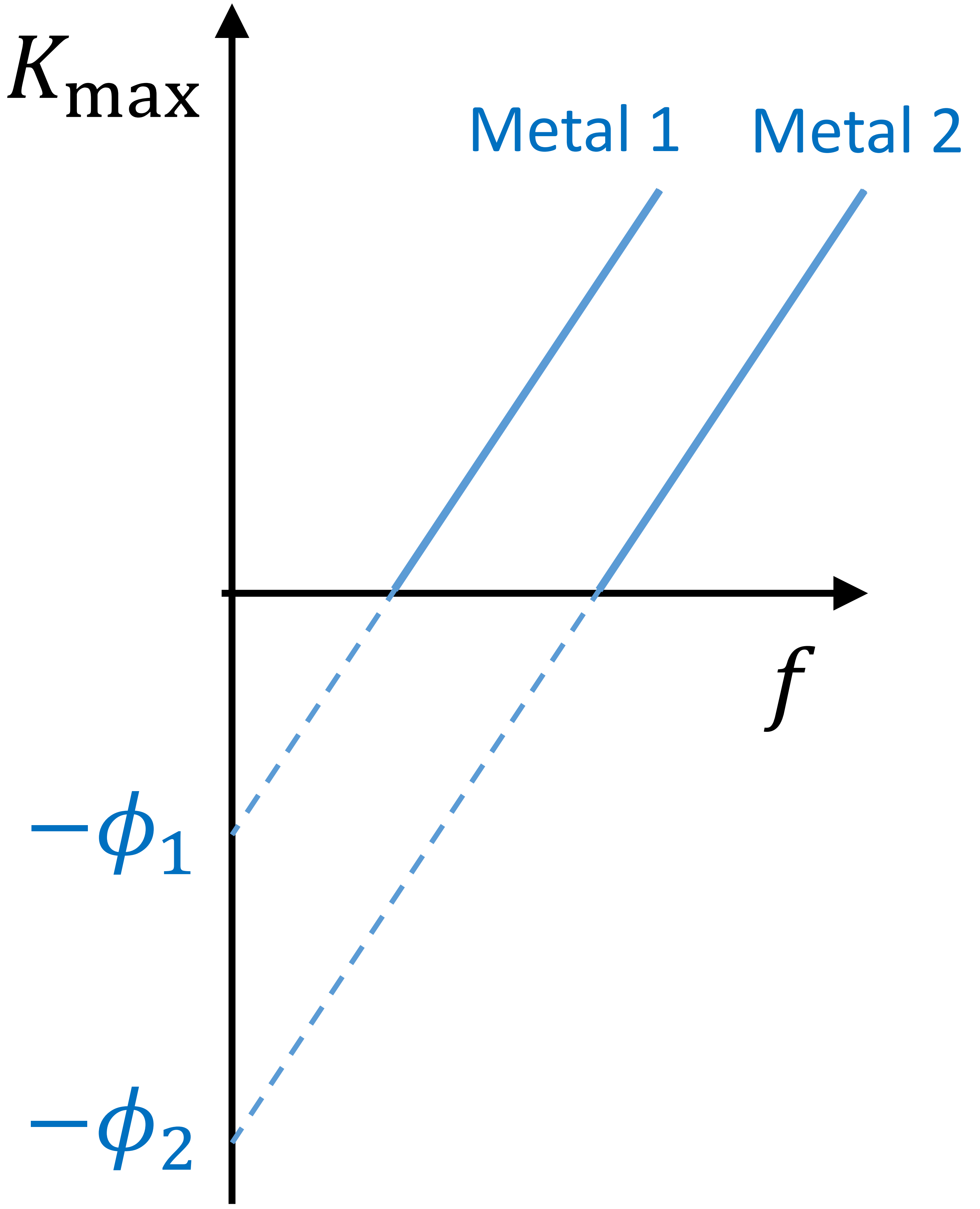


PHOTOELECTRIC EFFECT: EXAMPLE

Problem: A Sodium surface ($\phi = 2.46$ eV) is illuminated with light with $f = 300$ nm.

- What is the maximum kinetic energy?

$$K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV nm}}{300 \text{ nm}} - 2.46 \text{ eV} = 1.67 \text{ eV}$$

- What is cutoff wavelength?

$$\lambda_c = \frac{hc}{\phi} = \frac{1240 \text{ eV nm}}{2.46 \text{ eV}} \approx 500 \text{ nm}$$

Formulas:

$$K_{\max} = hf - \phi, \quad f_c = \frac{\phi}{h}, \quad \lambda_c = \frac{hc}{\phi}, \quad hc = 1240 \text{ eV} \cdot \text{nm}$$

SUMMARY PHOTOELECTRIC EFFECT

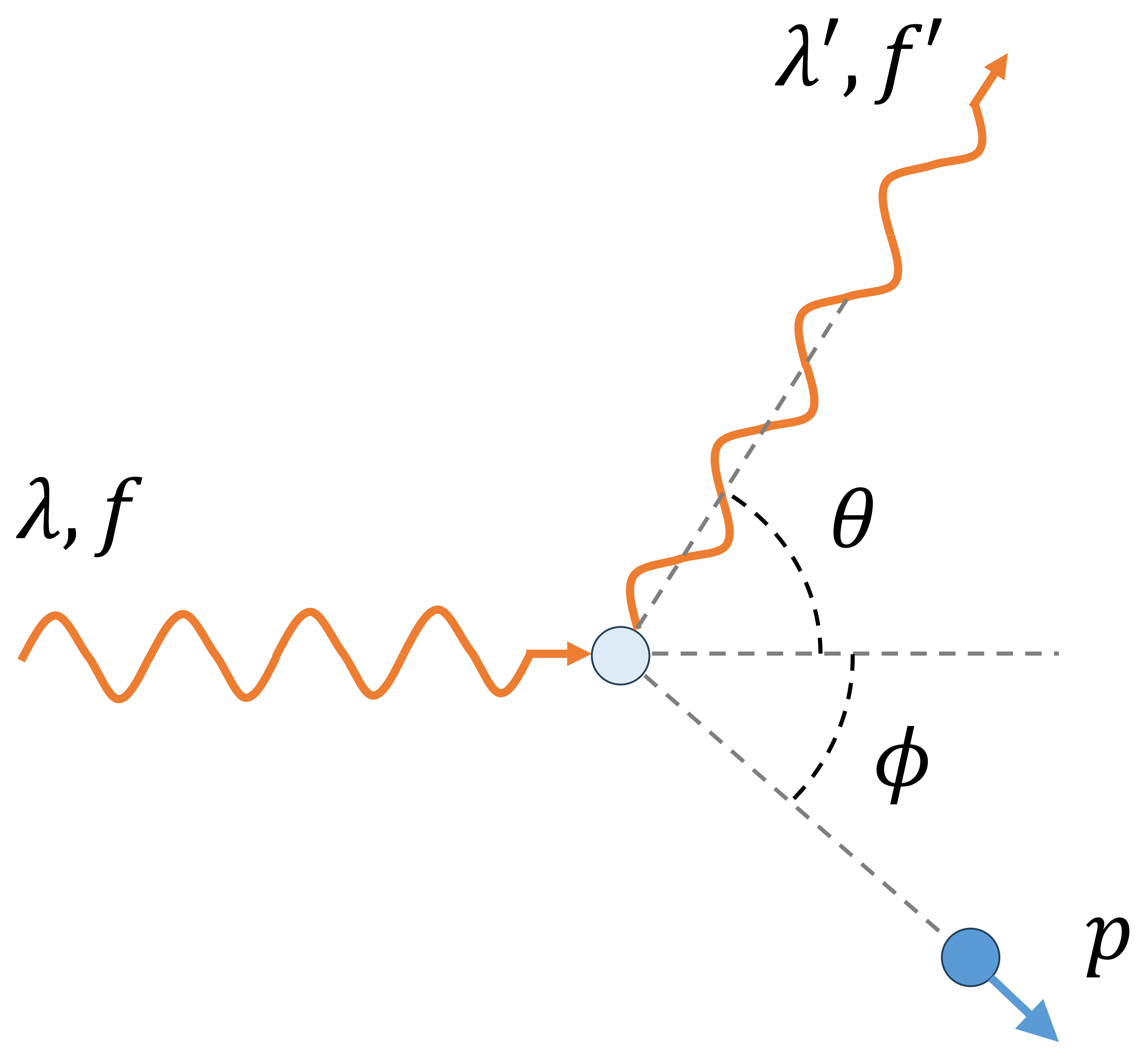
- Einstein argued that light exists of photons (quantized)
- Photons react with electrons in material: transfer of all energy
- The work function of a material can be found by measuring the stopping potential

Compton Effect

COMPTON EFFECT

- Einstein in 1919: a photon has
  - Energy $E = hf = \frac{hc}{\lambda}$
  - Momentum $p = \frac{E}{c} = \frac{hf}{c}$

- Compton showed photon-electron collision is similar to a billiard and thus particle-like.
  - Conservation of momentum
  - Conservation of energy



COMPTON EFFECT

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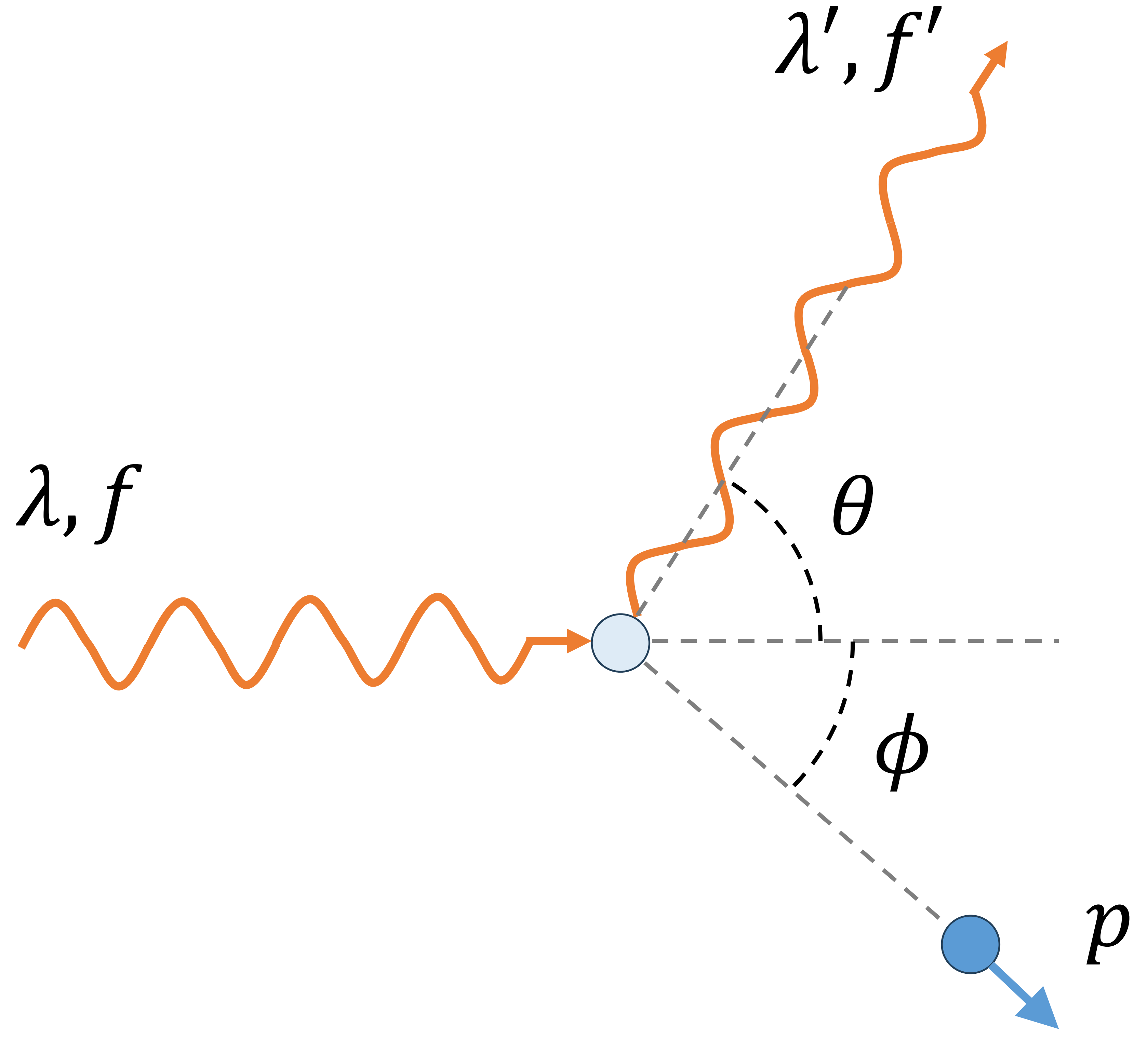
- **Compton shift** equation:

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

- **Compton wavelength** of electrons:

$$\lambda_C = \frac{h}{m_e c} = 0.00243\ \text{nm}$$



COMPTON EFFECT

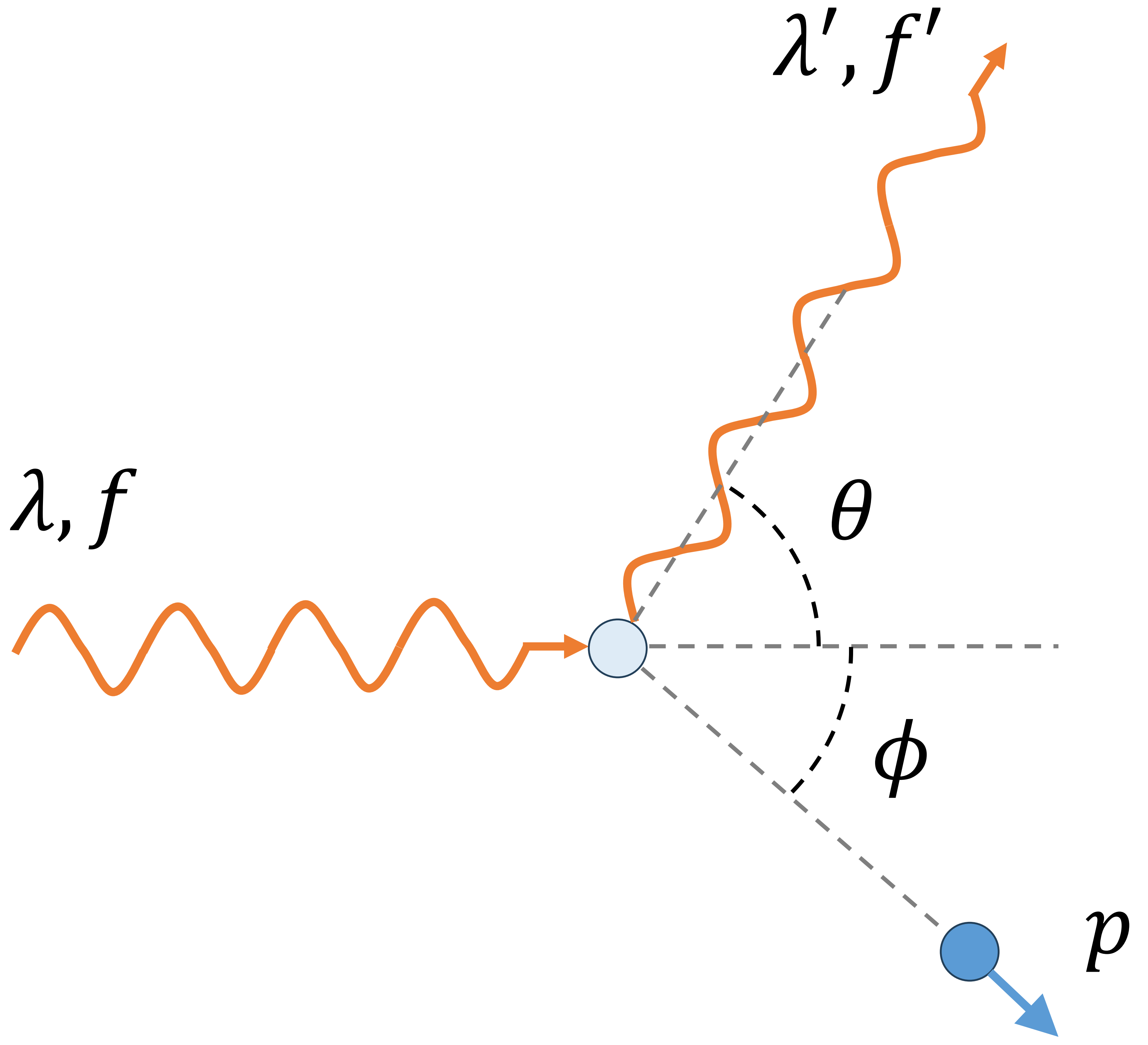
- Derivation **Compton shift**:

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\Delta K_{photon} + \Delta K_e = 0$$

$$\Rightarrow \quad \frac{hc}{\lambda} = \frac{hc}{\lambda'} + K_e$$

$$\Rightarrow \quad \frac{hc}{\lambda} = \frac{hc}{\lambda'} + (\gamma - 1)m_e c^2$$



Formulas:

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COMPTON EFFECT

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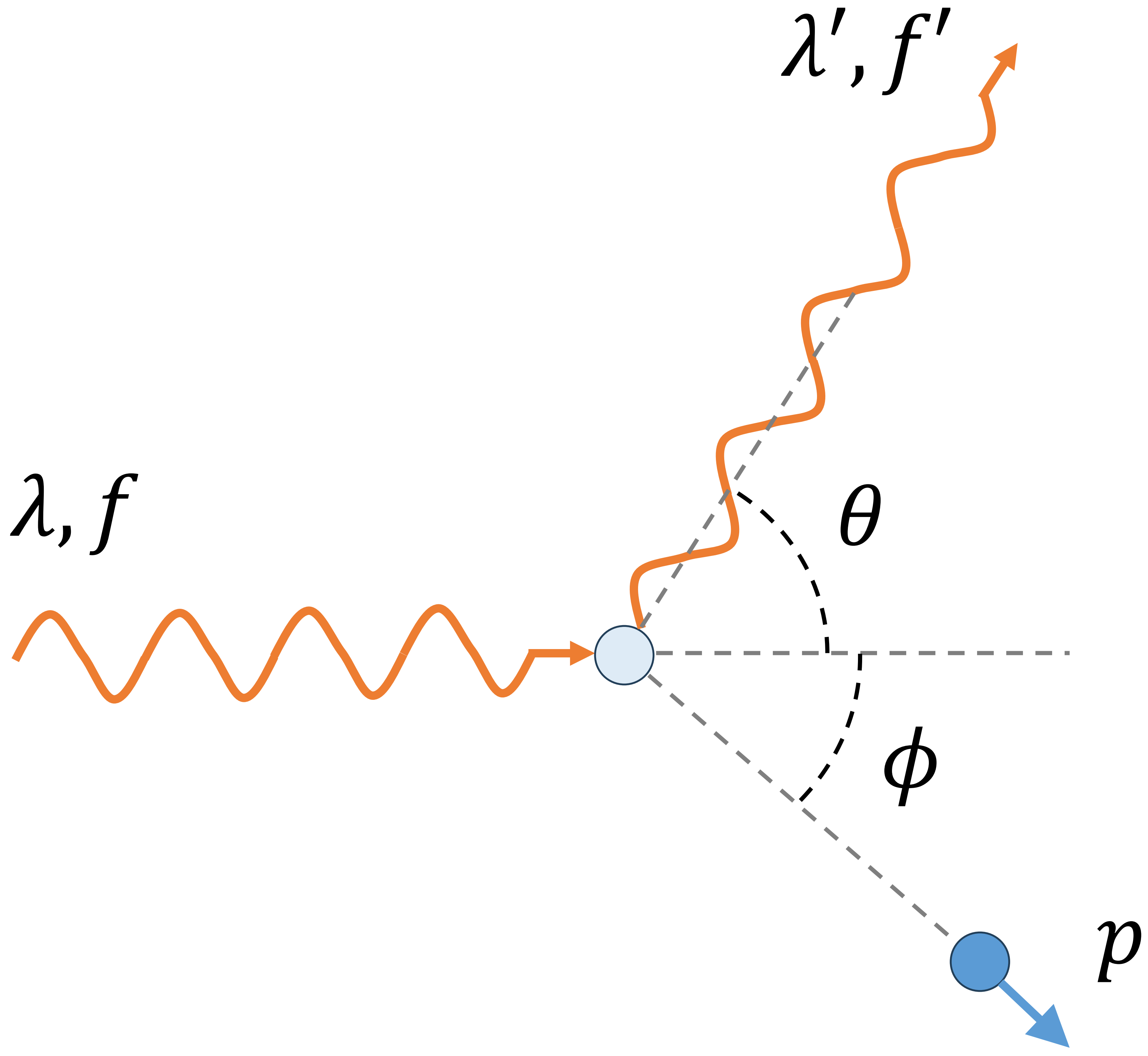

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COMPTON EFFECT

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + (\gamma - 1)m_e c^2$$

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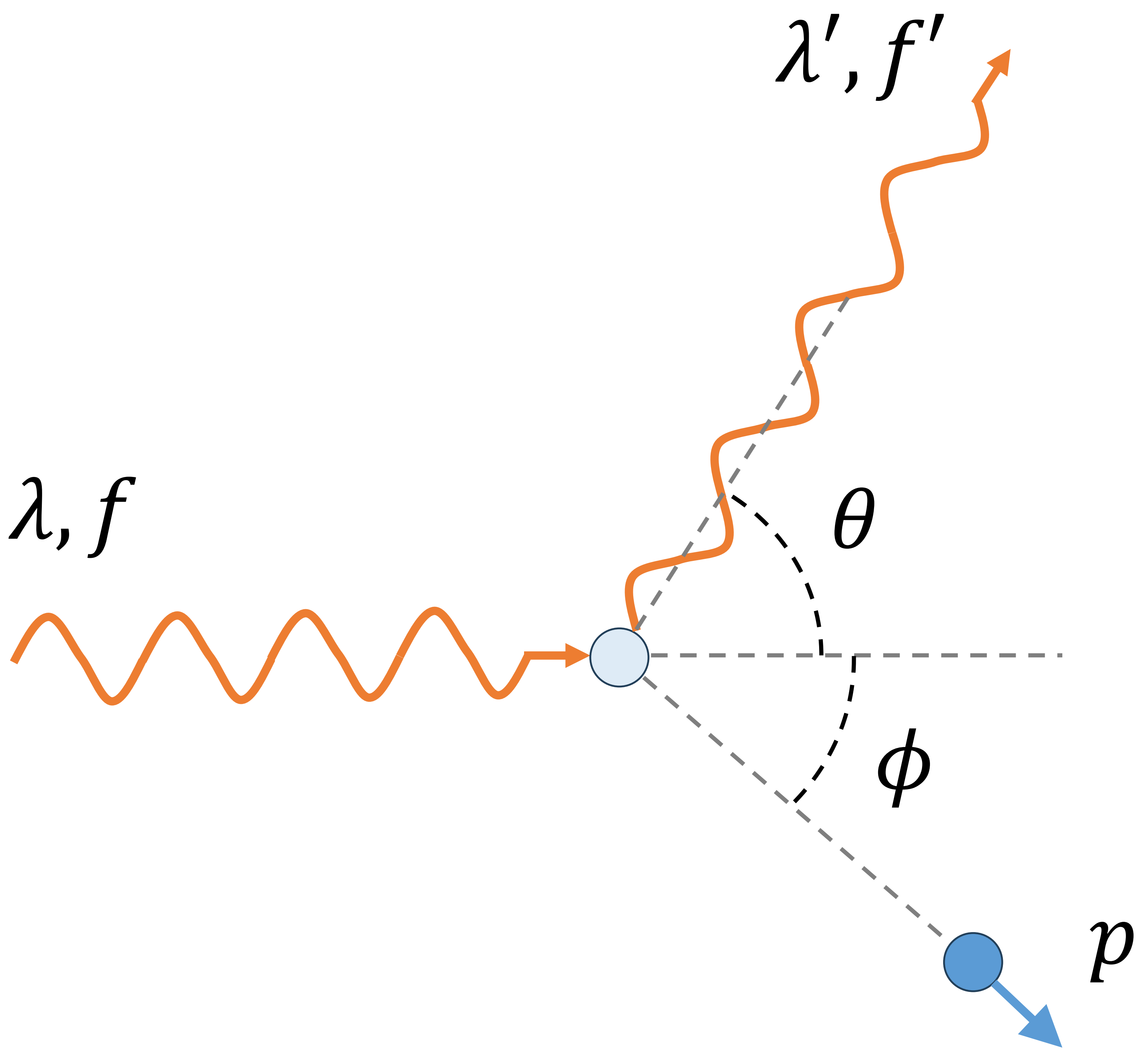

Momentum conservation in x and y:

$$\begin{cases} x: & \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \gamma m_e u \cos\phi \\ y: & 0 = \frac{h}{\lambda'}\sin\theta - \gamma m_e u \sin\phi \end{cases}$$

SUMMARY COMPTON EFFECT

- Compton effect shows that photon-electron collisions are similar to billiard-balls
- Momentum/energy conserved during collision
- Relativistic treatment