



PHOT 222: Quantum Photonics

LECTURE 02

Michaël Barbier, Spring semester (2025-2026)

OVERVIEW OF THE COURSE

week	topic	Serway	Young
Week 1	Relativity	Ch. 39	Ch. 37
Week 2	Waves and Particles	Ch. 40	Ch. 38
Week 3	Wave packets and Uncertainty		
Week 4	The Schrödinger equation and Probability		
Week 5	Midterm exam 1		
Week 6	Quantum particles in a potential		
Week 7	Harmonic oscillator		
Week 8	Tunneling through a potential barrier		
Week 9	The hydrogen atom, absorption/emission spectra		
Week 10	Midterm exam 2		
Week 11	Many-electron atoms		
Week 12	Pauli-exclusion principle		
Week 13	Atomic bonds and molecules		
Week 14	Crystalline materials and energy band structure		

Blackbody Radiation & the Ultraviolet Catastrophe

BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

- Any **object at nonzero temperature** emits **thermal radiation** from its surface
- **Black body**: Perfect absorber/emitter
- Its Radiation: **blackbody radiation**

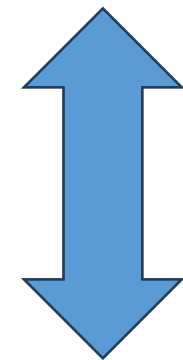


Adapted from <https://liberal-technology.com>



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- Color of daily objects: **Reflection of light**



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- Any **object at nonzero temperature** emits **thermal radiation** from its surface
- **Black body**: Perfect absorber/emitter
- Its Radiation: **blackbody radiation**
- **Stefan's law**: Total power of emitted radiation grows with temperature as T^4

$$P = \sigma A e T^4$$

Stefan-Boltzmann: $\sigma = 5.7 \times 10^{-8} \frac{W}{m^2} K^4$

Surface area: A , emissivity: e



BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

- **Stefan's law:** Total power of emitted radiation

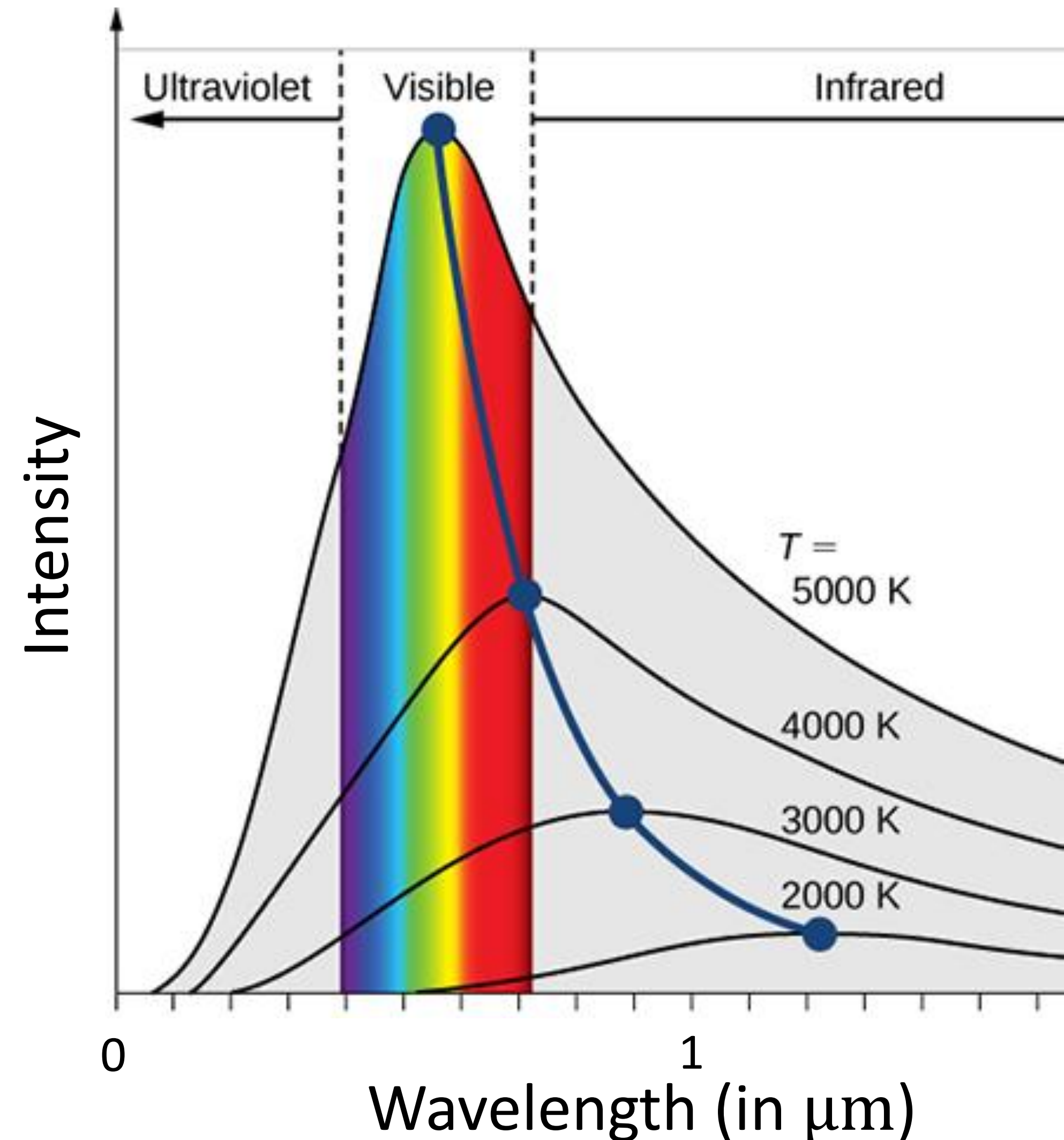
$$P = \sigma A e T^4$$

Stefan-Boltzmann: $\sigma = 5.7 \times 10^{-8} \frac{W}{m^2} K^4$

Surface area: A , emissivity: $0 \leq e \leq 1$

- **Wien's displacement law:**
Spectrum shifts to shorter wavelength at higher temperature

$$\lambda_{\max} T = 2.898 \times 10^{-3} m \cdot K$$

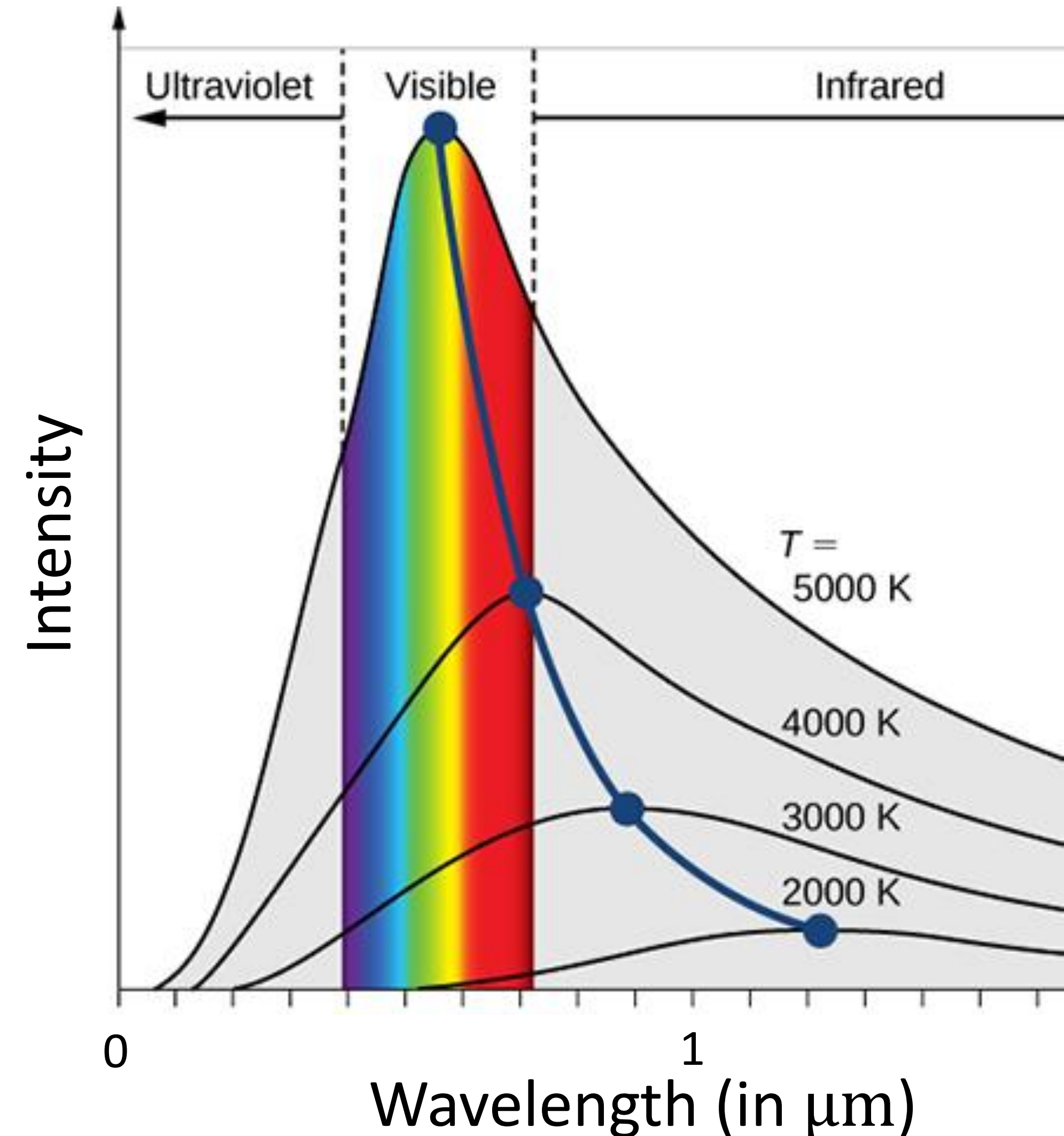


BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

- **Wien's displacement law:**
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- Temperature determines color, i.e. dominating wavelength



BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

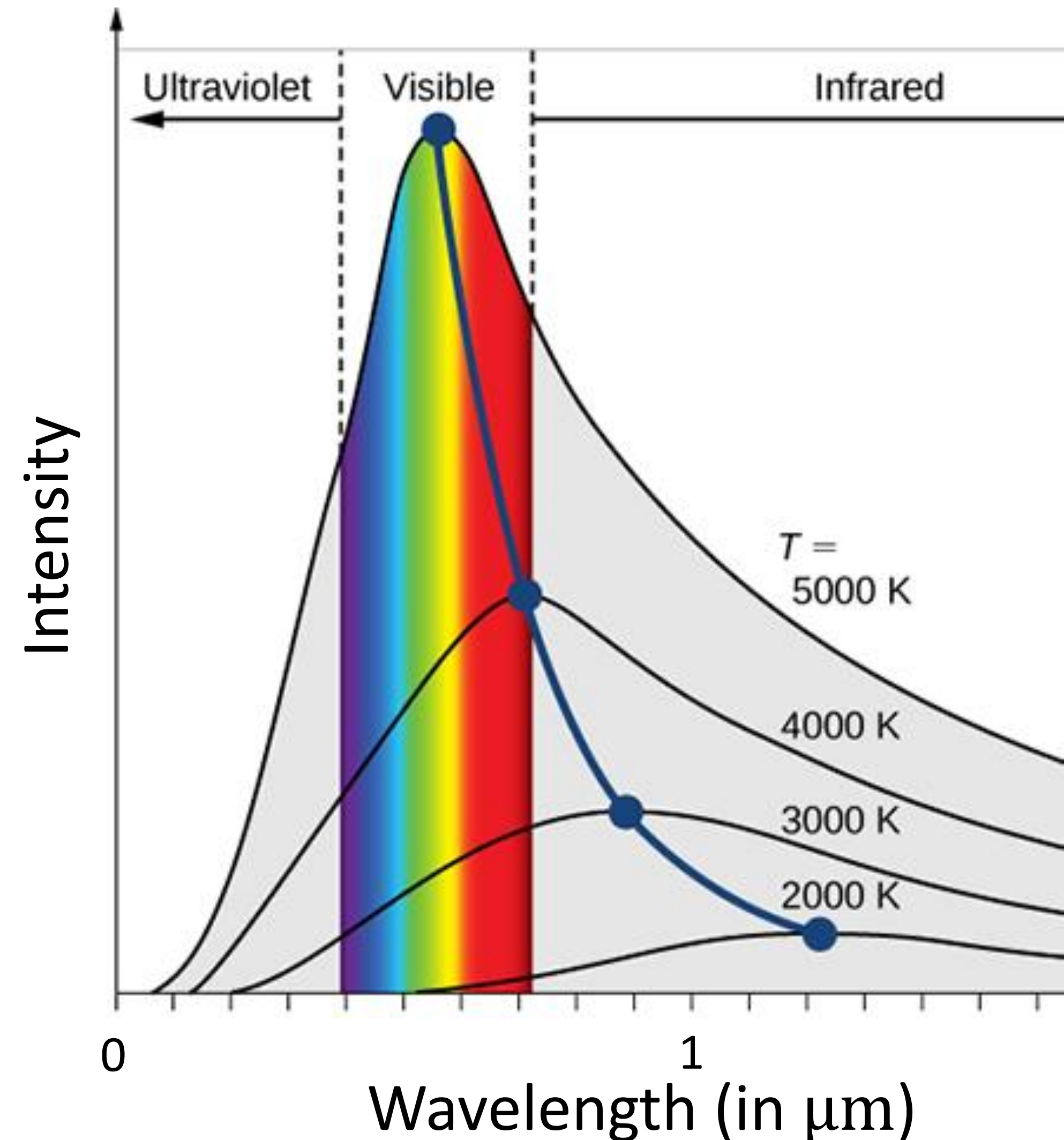
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- Theory: **Rayleigh-Jeans law**

$$I(\lambda, T) = \frac{8\pi c k_B T}{\lambda^4}$$

Intensity increases for shorter wavelengths, goes to infinity



BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

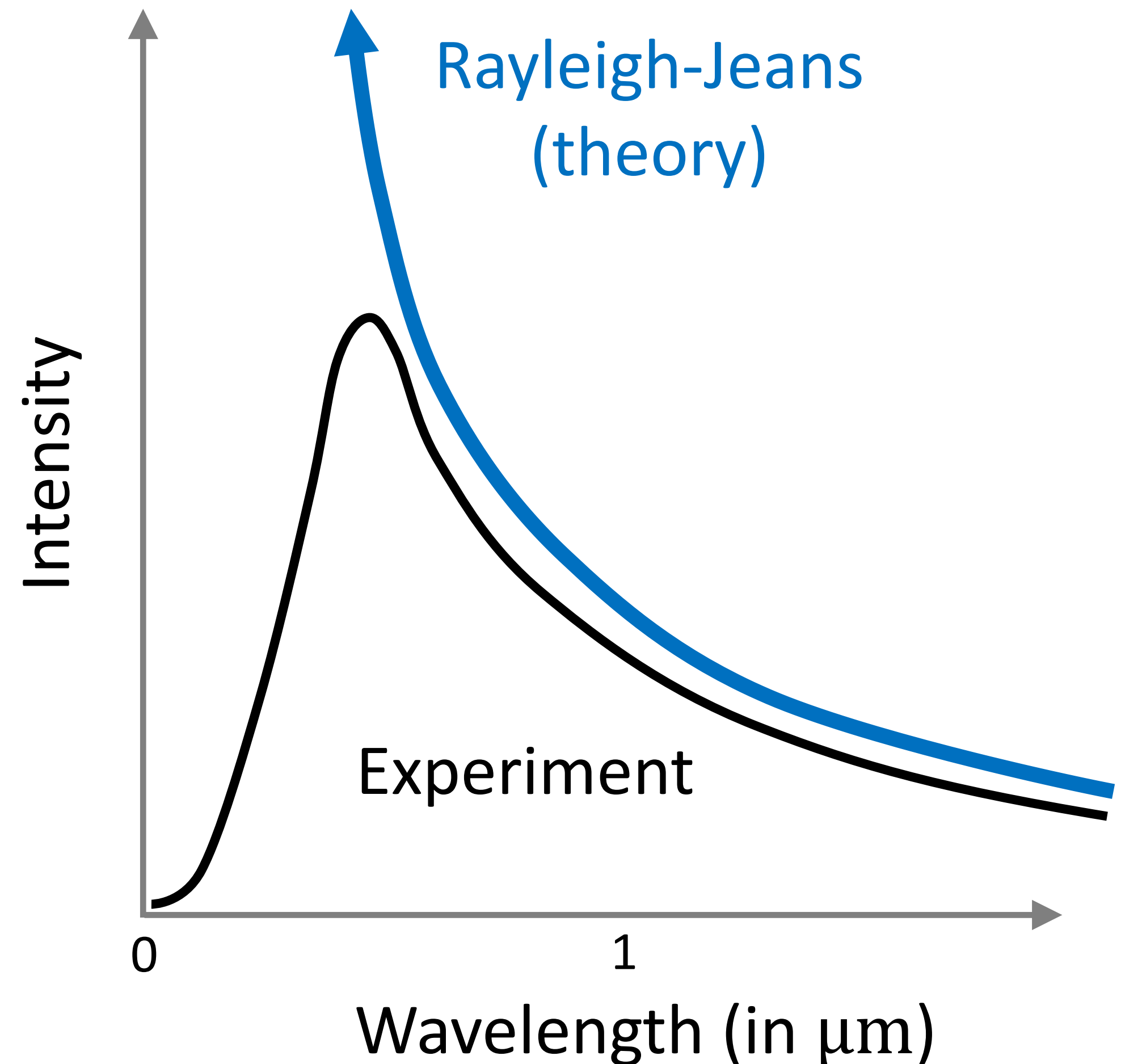
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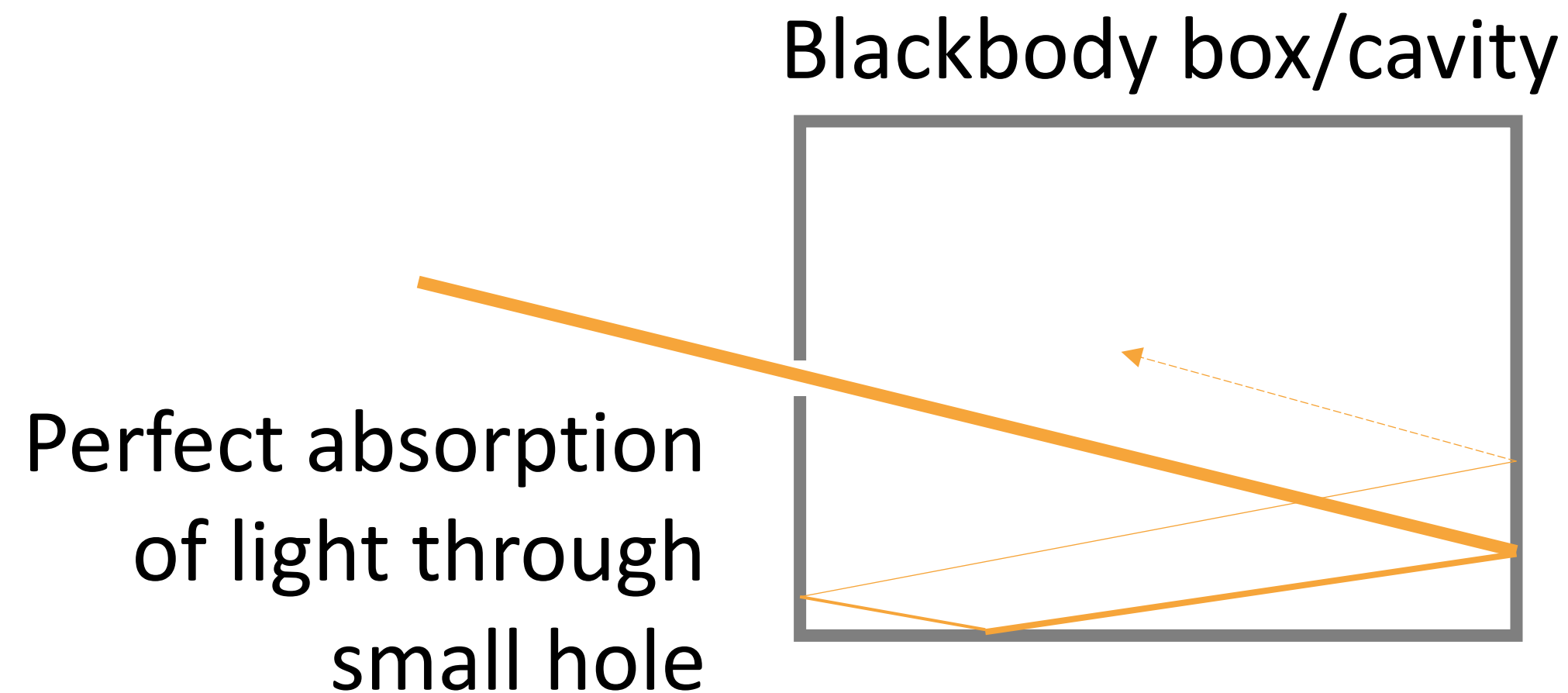


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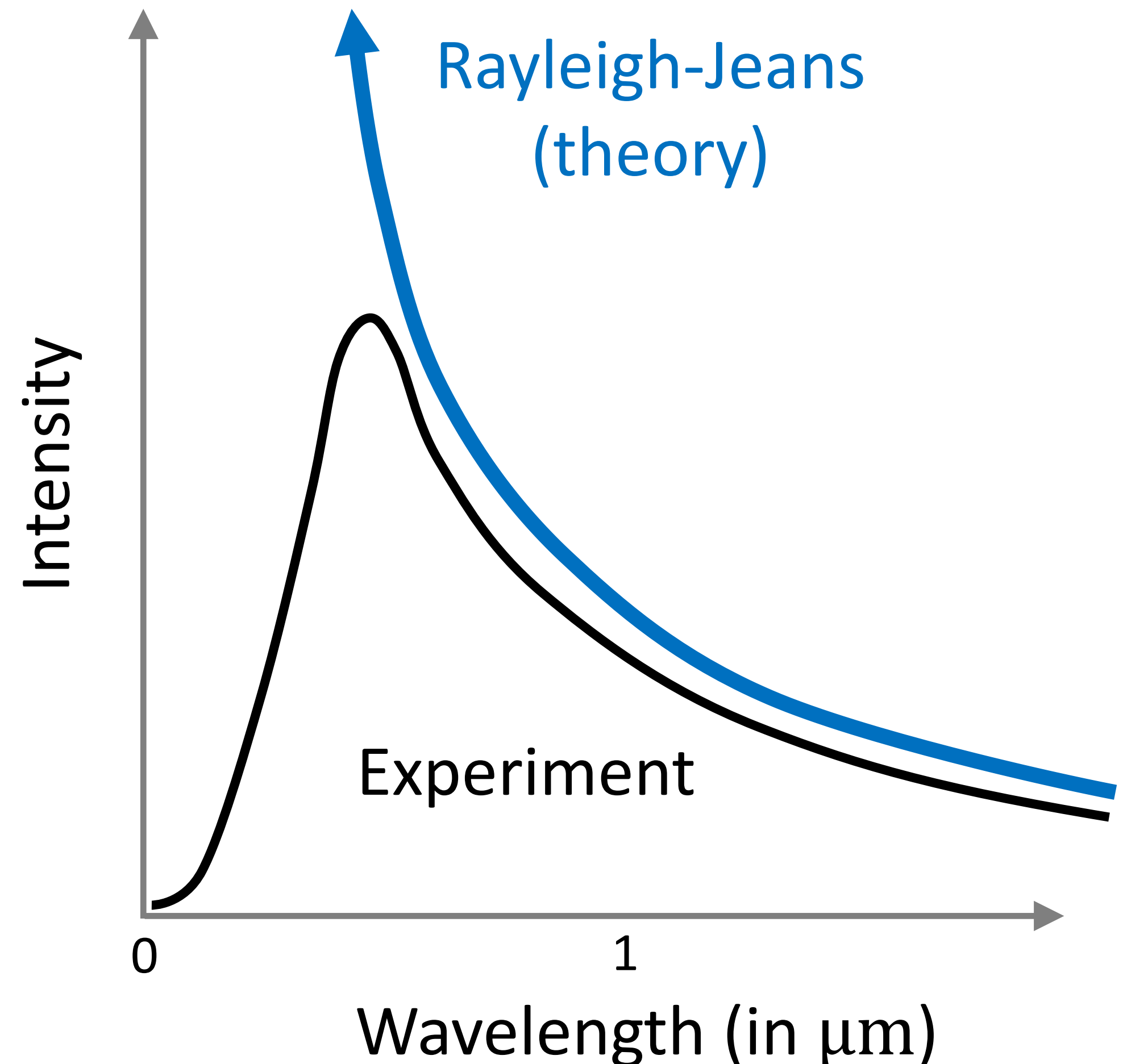
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- Theoretical model



Kirchoff's law: absorption = emission

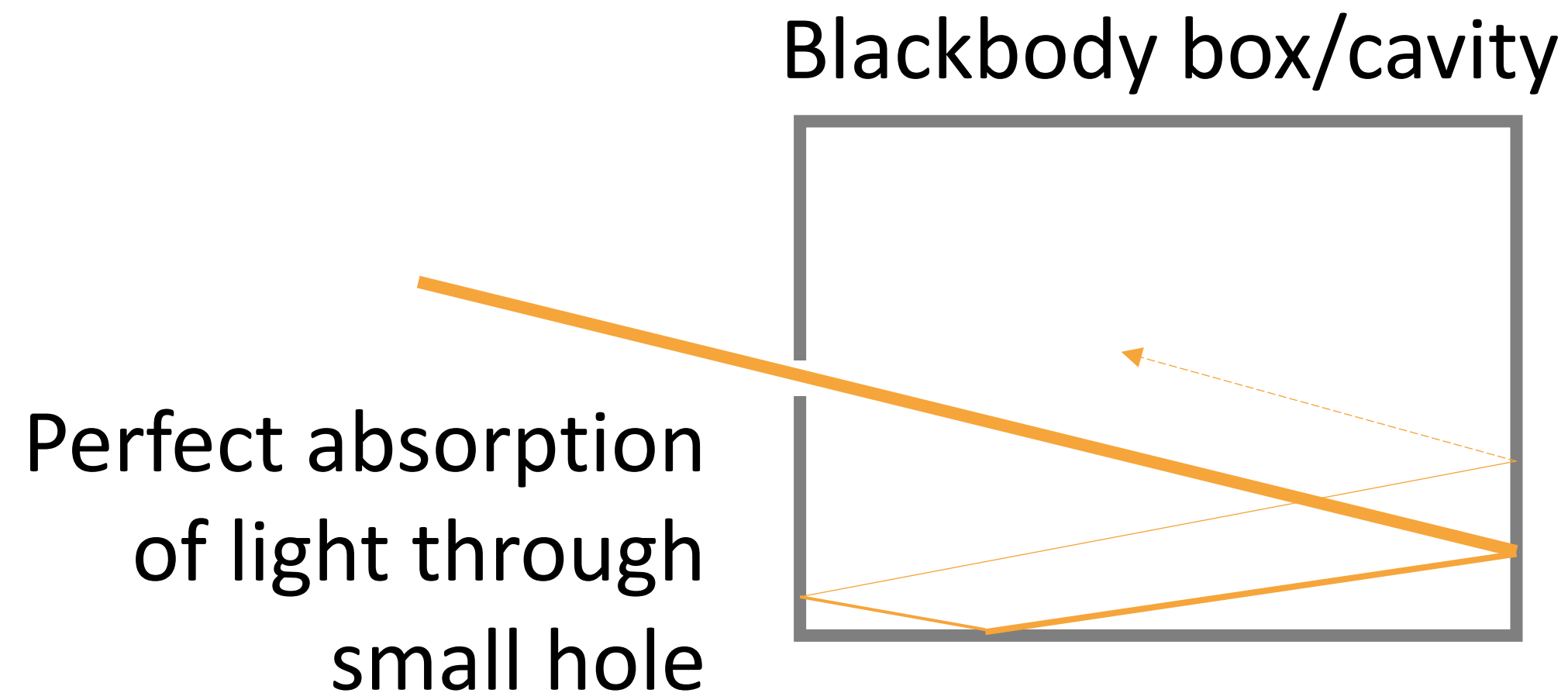


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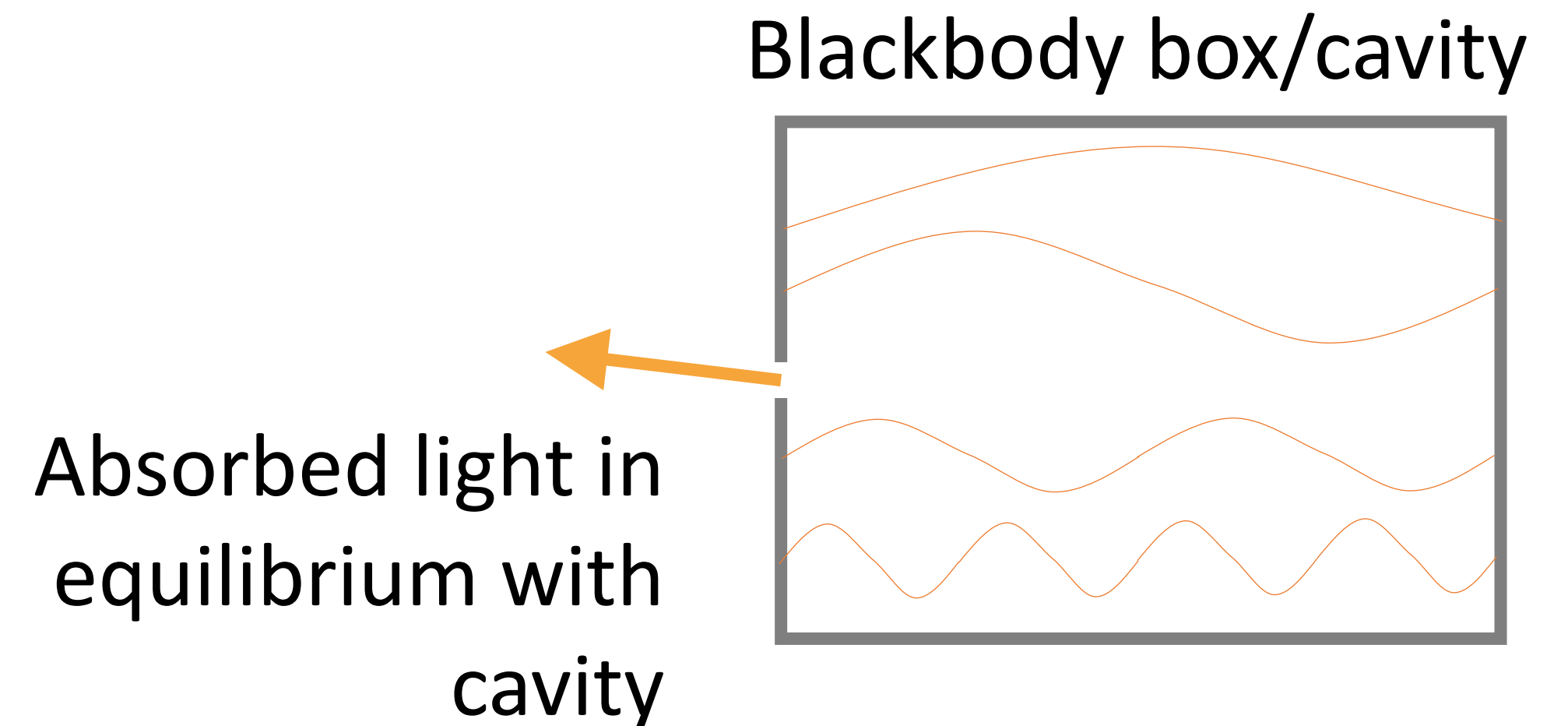
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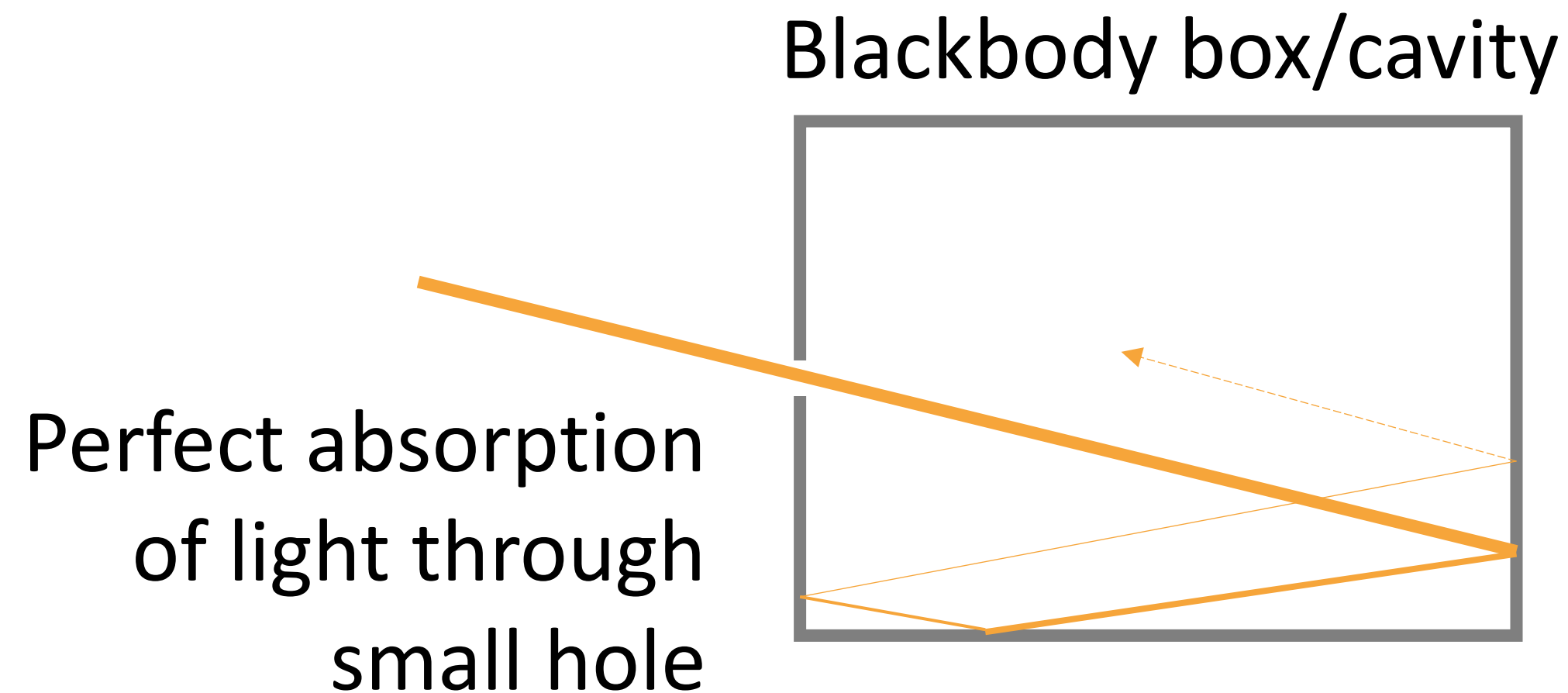


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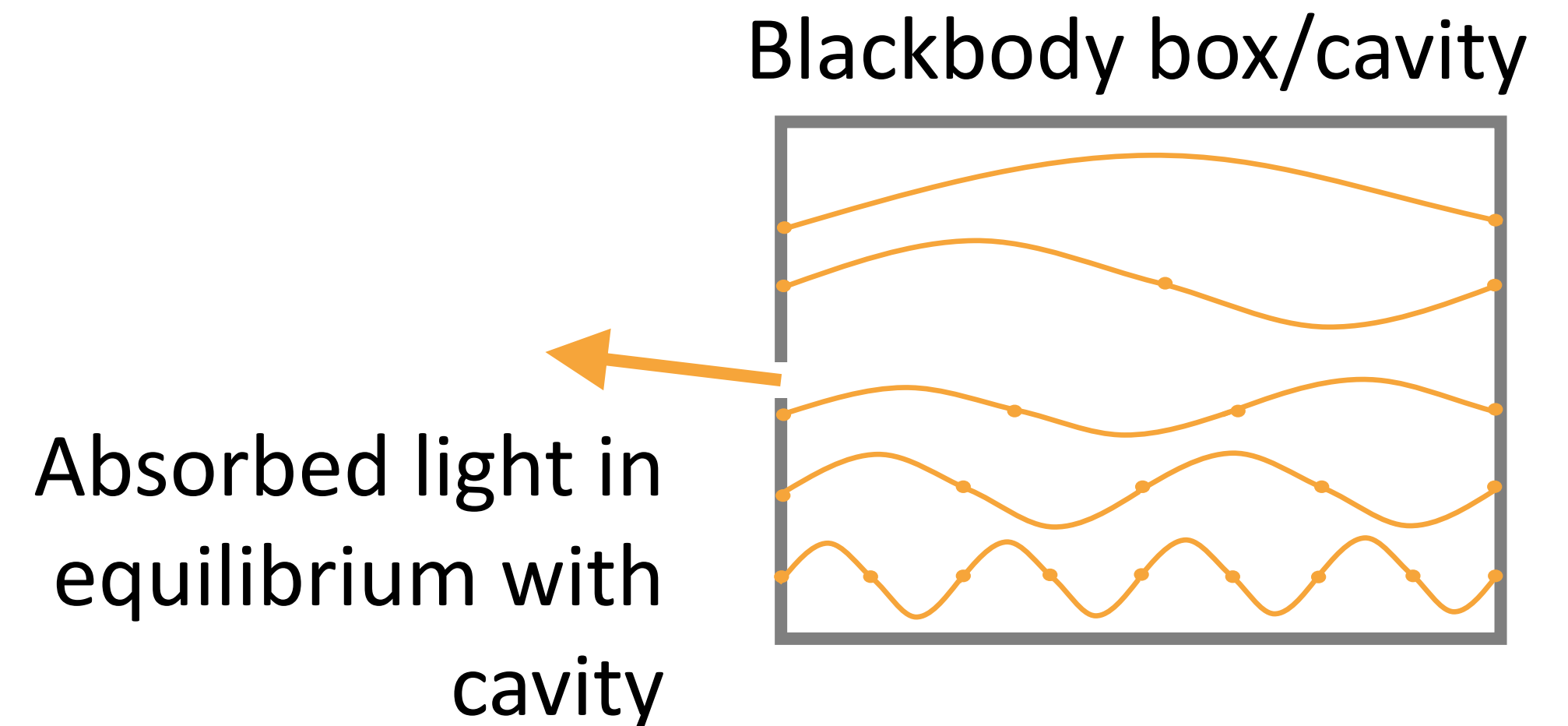
$$I(\lambda, T) = \frac{8\pi c k_B T}{\lambda^4}$$

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Kirchoff's law: absorption = emission

- Inside wall in equilibrium with cavity
- Number of standing waves
- Each wave $2 \times \frac{1}{2} k_B T$



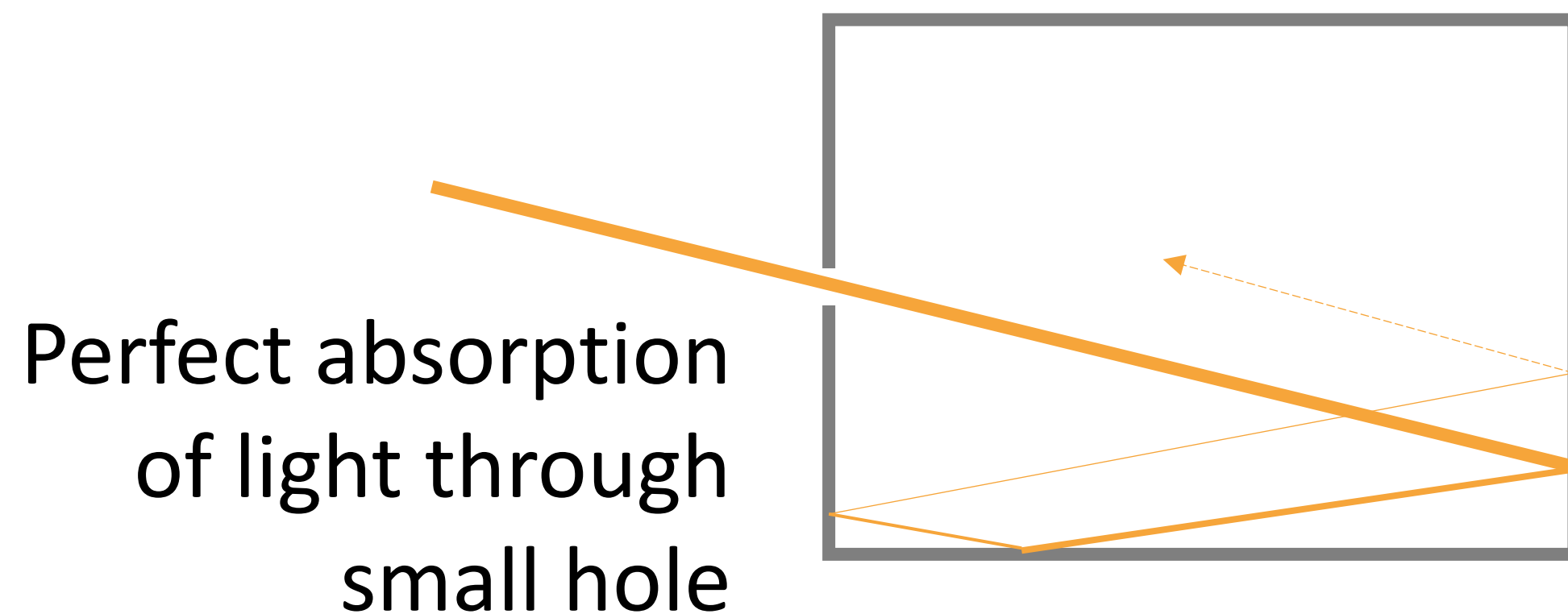
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Blackbody box/cavity



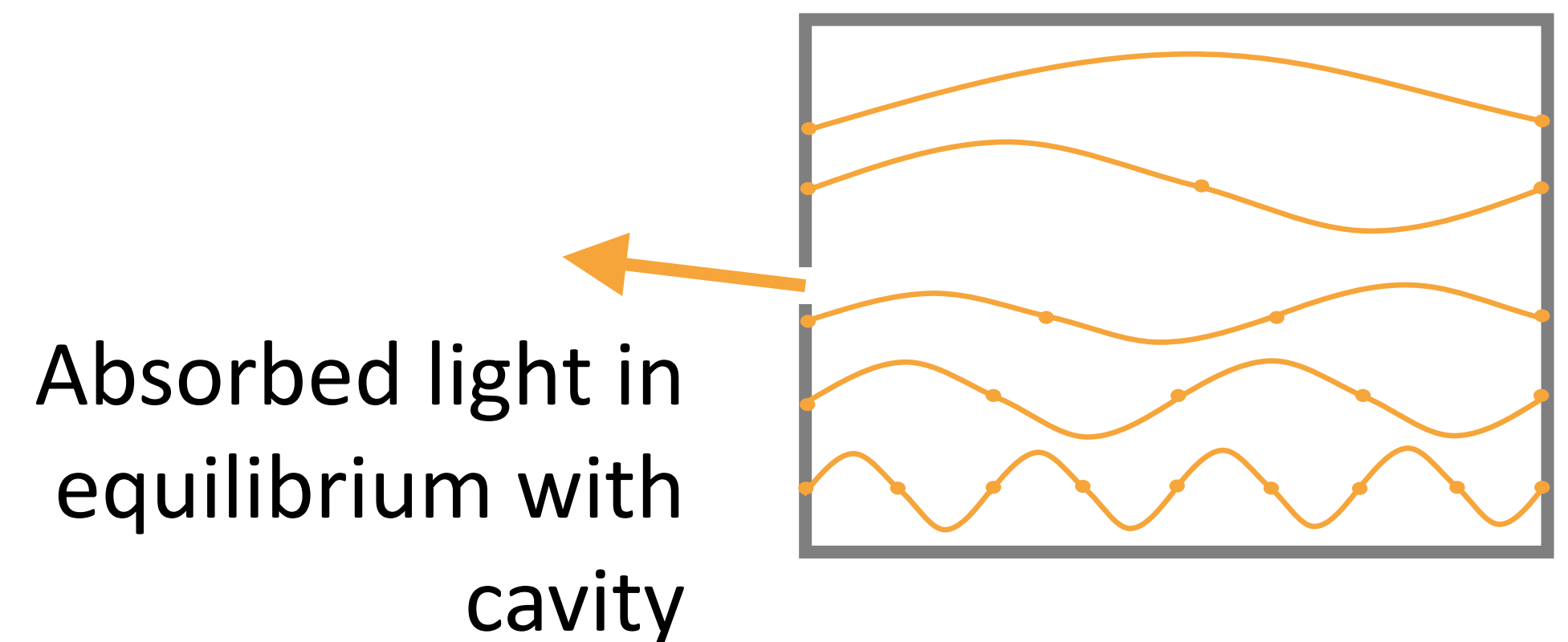
Kirchoff's law: absorption = emission

- Inside wall in equilibrium with cavity

Classical

- Number of standing waves
- Each wave $2 \times \frac{1}{2} k_B T$

Blackbody box/cavity

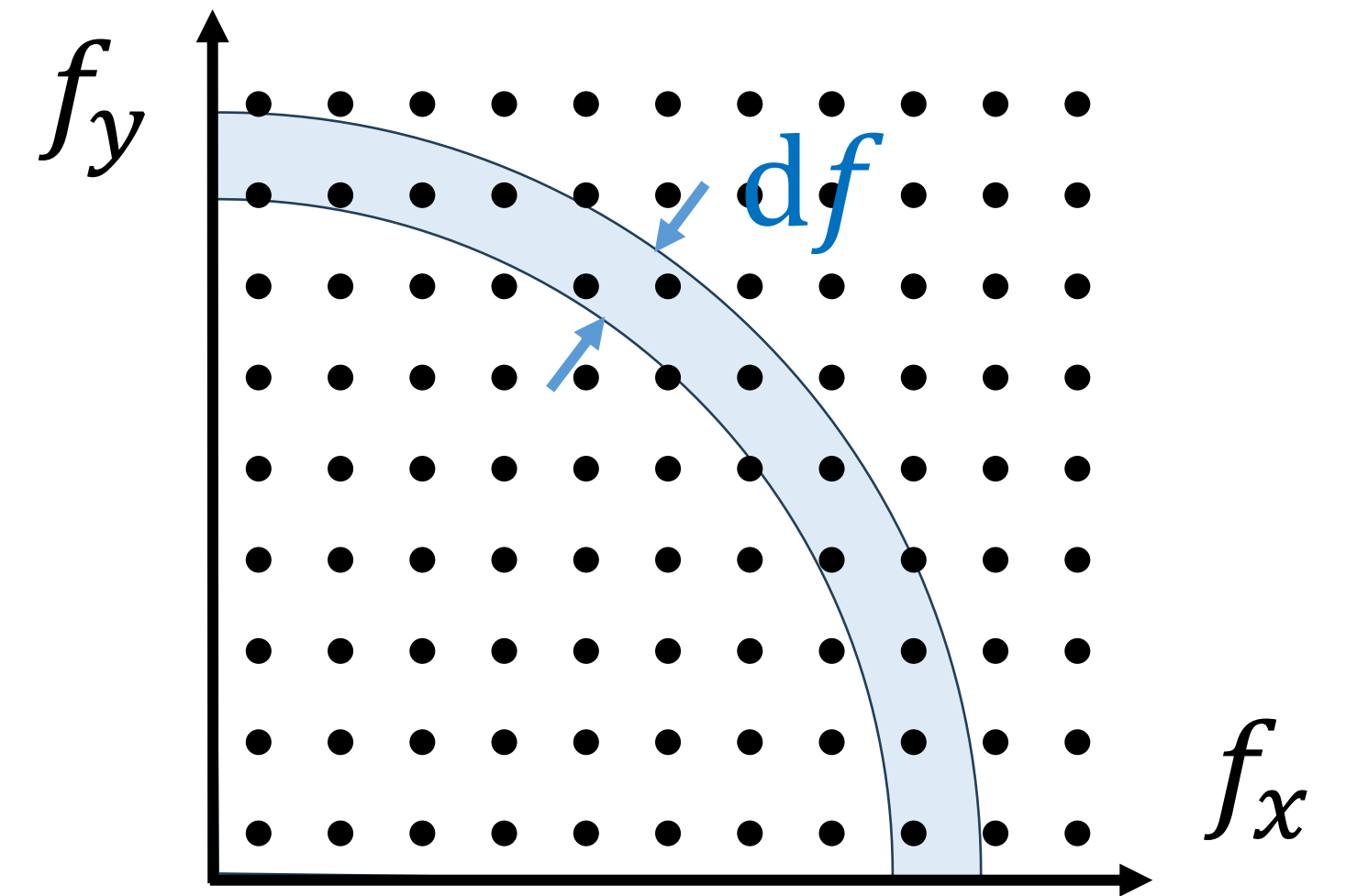


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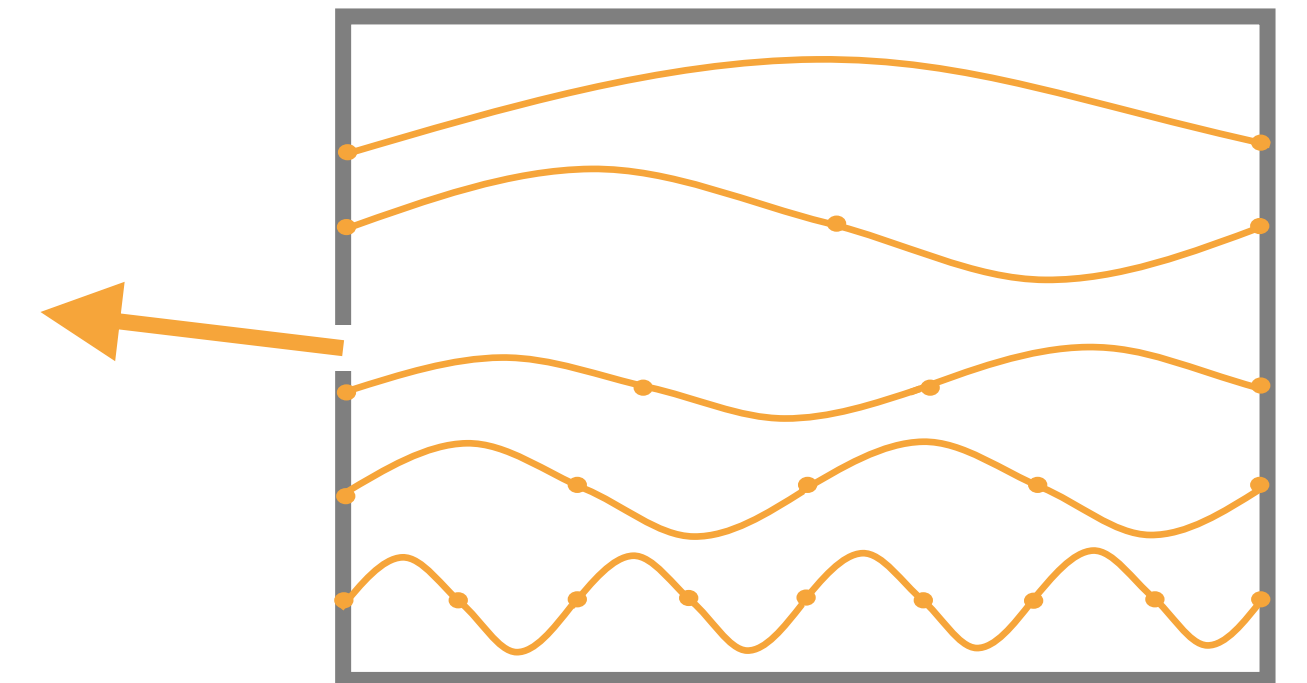
- Theory: **Rayleigh-Jeans law**

$$I(\lambda, T) = \frac{8\pi c k_B T}{\lambda^4}$$

- Number of standing waves
 - Frequencies: $f = (f_x, f_y, f_z)$
 - In shell with thickness $df \sim f^2$
→ More standing waves/modes at higher frequencies
- **Each wave/mode** $2 \times \frac{1}{2} k_B T$
- 2 degrees of freedom: K and V
- Contribution of shorter waves $\rightarrow \infty$



Blackbody box/cavity



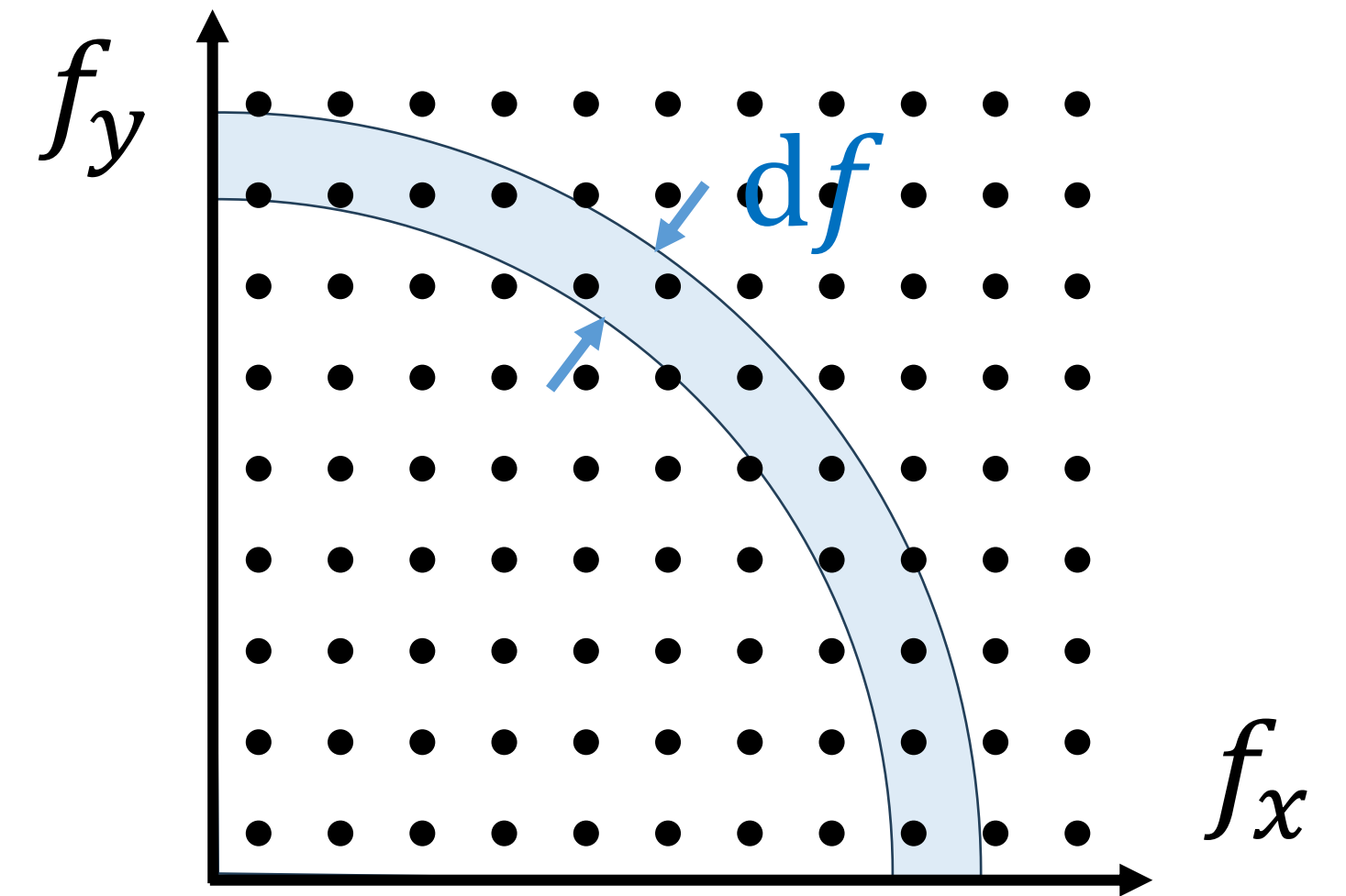
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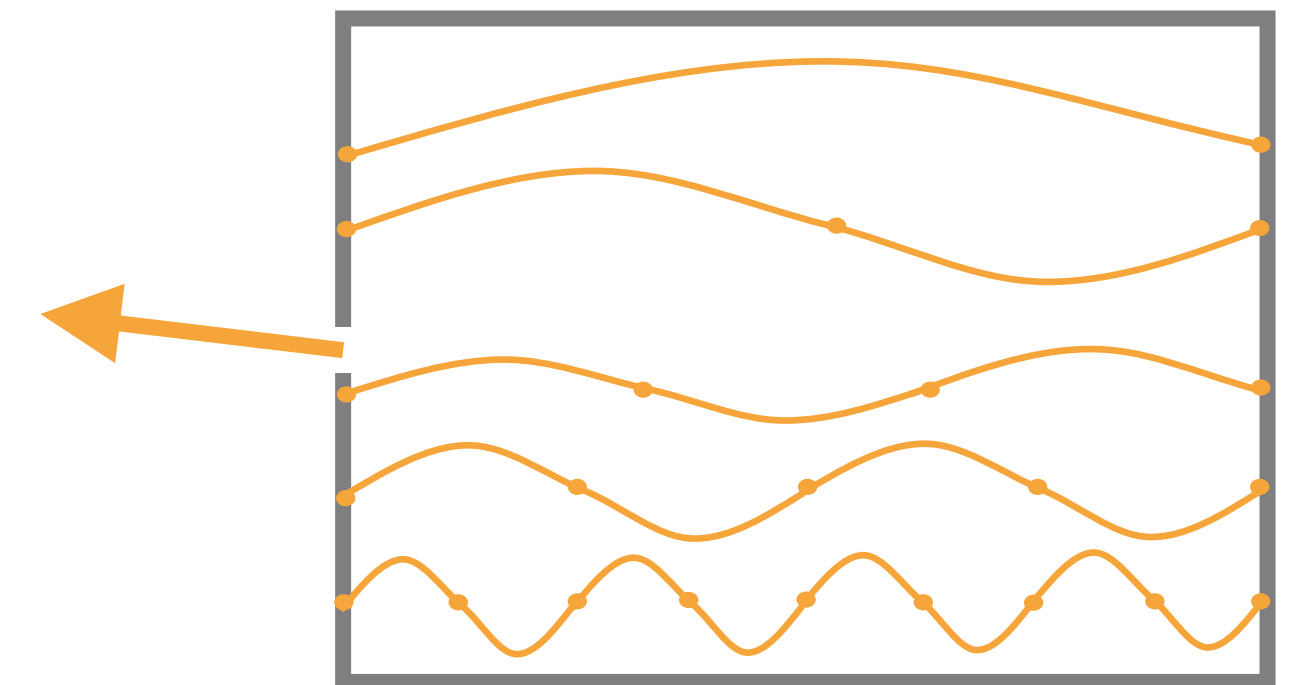
$$I(\lambda, T) = \frac{8\pi c k_B T}{\lambda^4}$$

- Number of standing waves
 - Frequencies: $f = (f_x, f_y, f_z)$
 - In shell with thickness $df \sim 4\pi f^2$
→ More standing waves/modes at higher frequencies
 - Energy per unit volume:

$$\frac{8\pi f^2 k_B T df}{c^3} \quad \& \quad df = d\left(\frac{c}{\lambda}\right) = -\frac{2c}{\lambda^2} d\lambda$$



Blackbody box/cavity



BLACKBODY RADIATION & THE ULTRAVIOLET CATASTROPHE

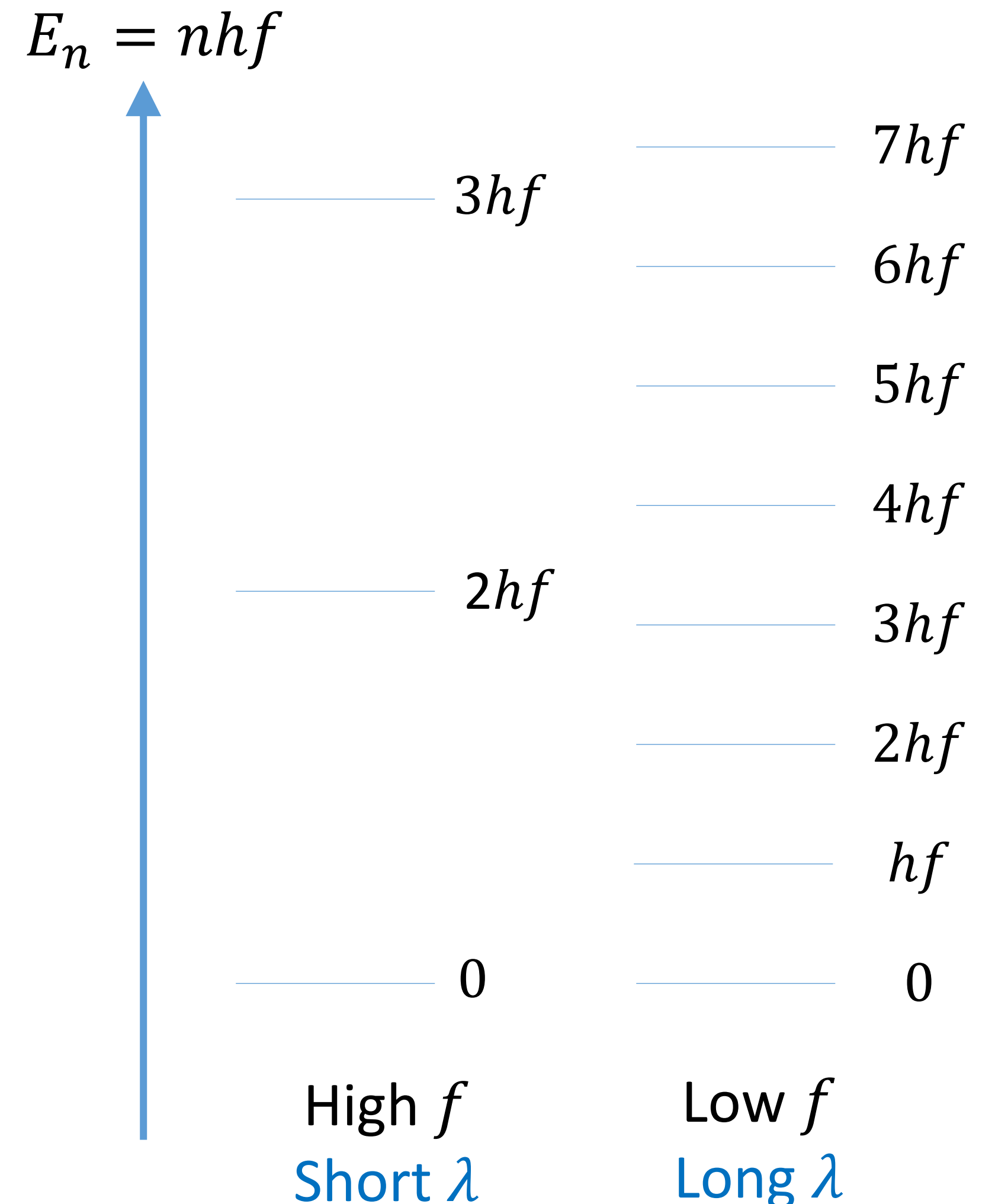
- Theory: **Planck's radiation law**

$$I(\lambda, T) = \frac{8\pi hc^2}{\lambda^5 (e^{hc/(\lambda k_B T)} - 1)}$$

- Similar to Rayleigh: $df \sim 4\pi f^2$

BUT

- Energy quantized: $E_n = nhf$
- Number of oscillators $\propto e^{-E_n/k_B T}$
- Avg. energy/oscillator $\langle E \rangle = \frac{hf}{e^{\frac{hf}{k_B T}} - 1}$



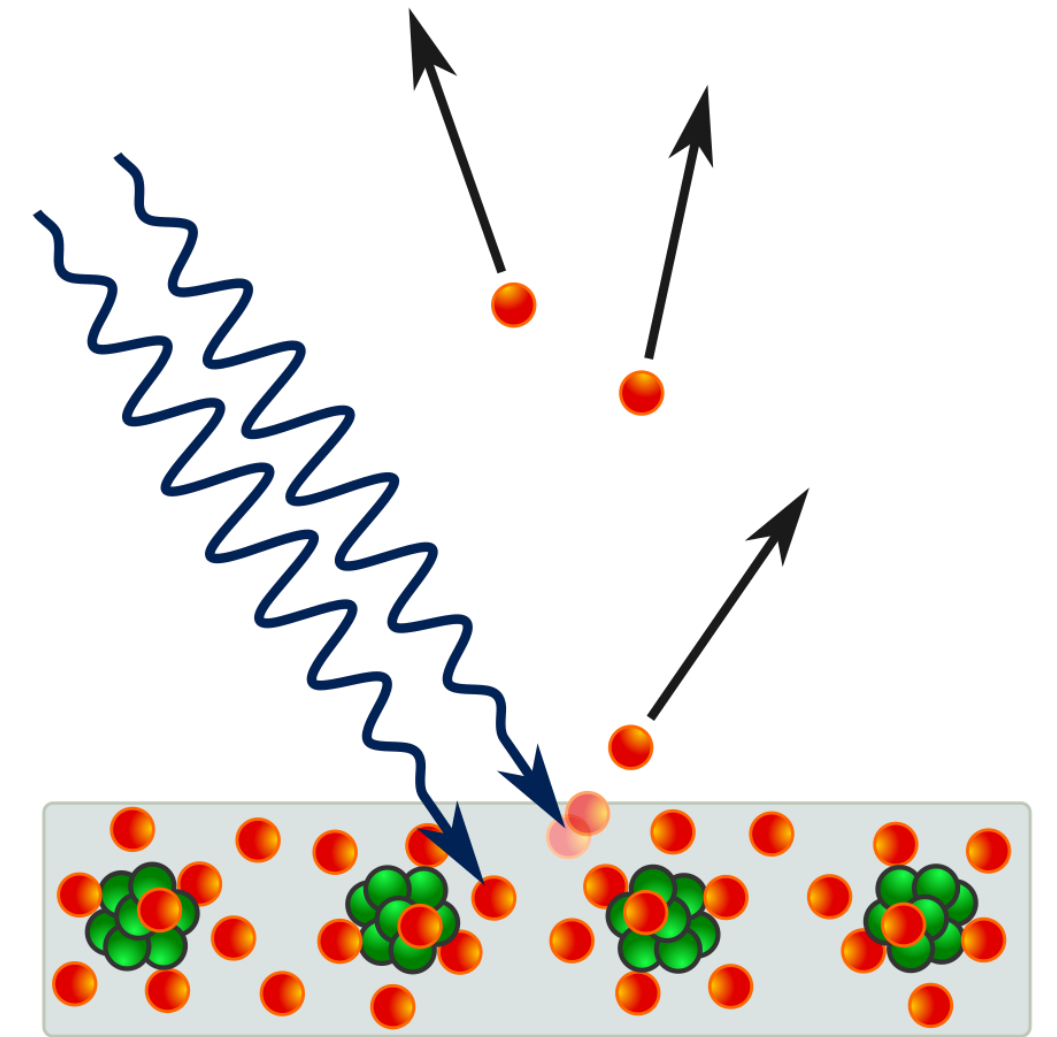
SUMMARY HISTORICAL ARGUMENTS

- Planck found energy packets of light emitted by matter
- Light quantized (particles?) or atoms quantized?

Photoelectric Effect

PHOTOELECTRIC EFFECT

- **Electrons ejected** from metallic surface **by light**
- Light gives electrons energy to escape the material
- The kinetic energy of the electrons is the energy beyond that ionizing energy



PHOTOELECTRIC EFFECT

- Light gives electrons energy to escape the material ΔU
- **Accelerate or stop** electrons: V_c
- The kinetic energy K of the electrons is the energy beyond ionizing energy

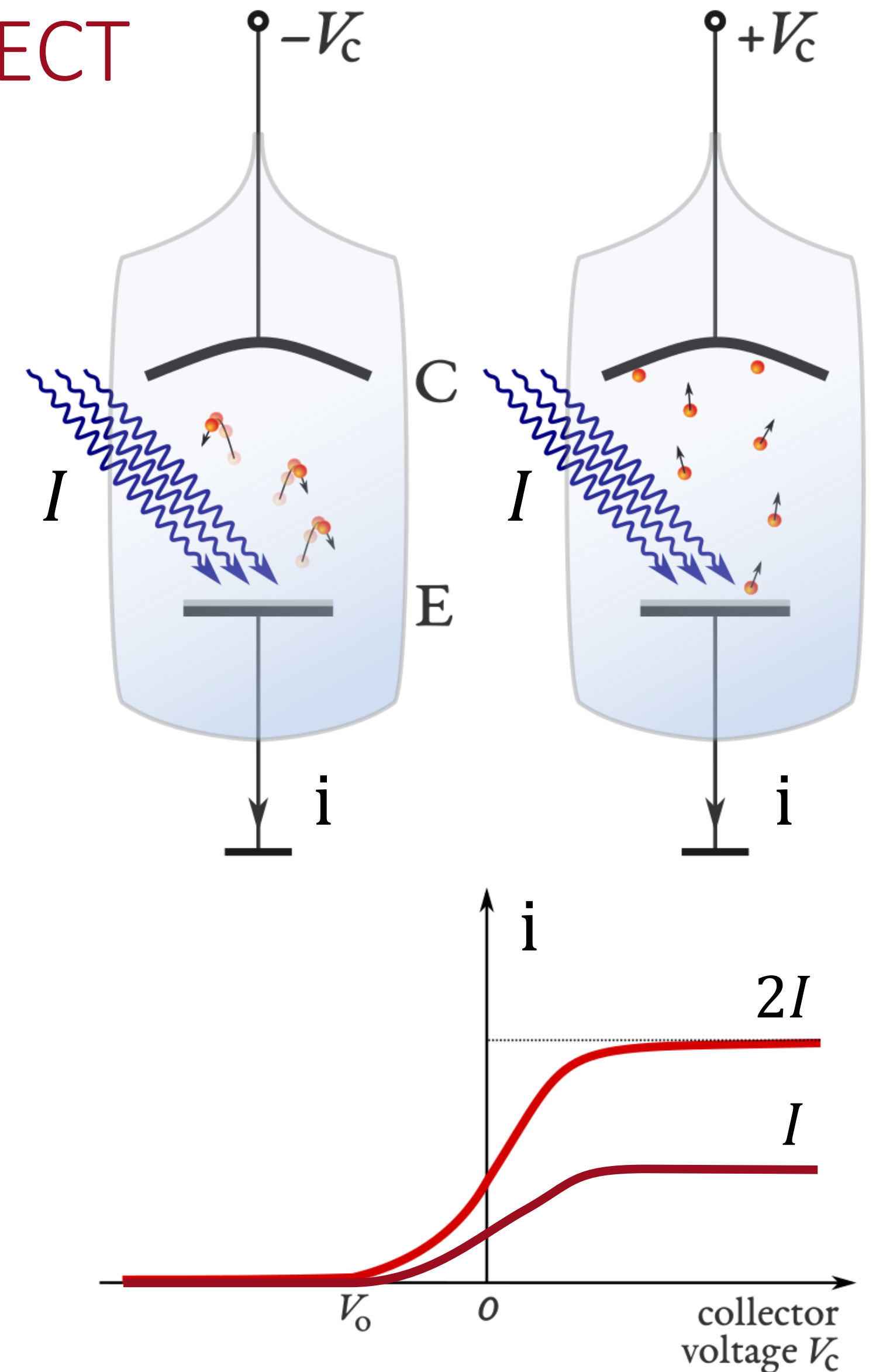
$$\Delta E = K + \Delta U$$

- **Stopping potential** for electrons

$$V_c = -V_0$$

- Kinetic energy K is:

$$K_{\max} = eV_0$$



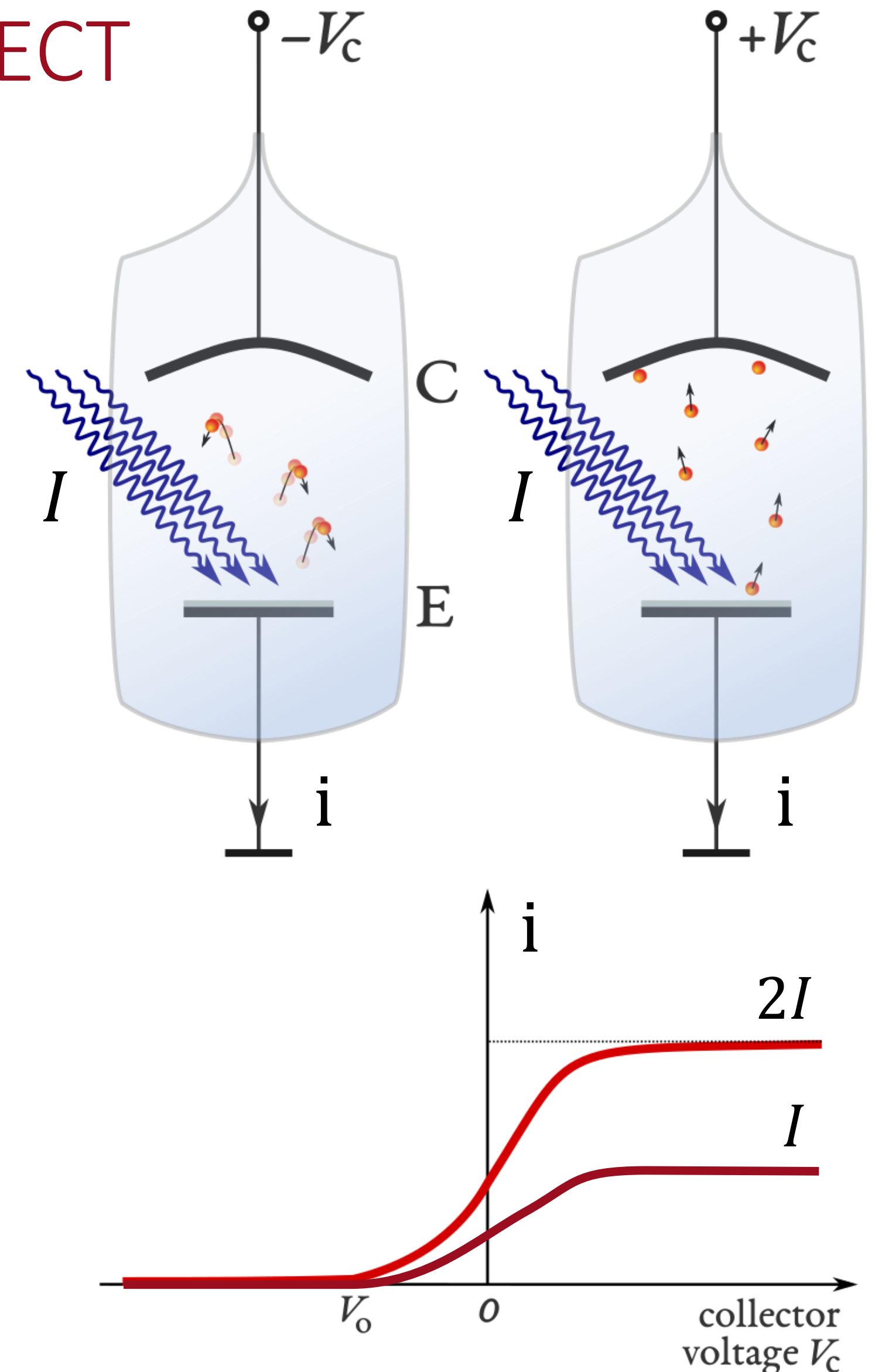
PHOTOELECTRIC EFFECT

Classical expectations:

1. Magnitude of photocurrent independent from frequency
2. Faint light $\rightarrow$ delayed electron ejection
3. Higher intensity, increasing K electrons

Experimentally we observe:

1. Minimum **threshold frequency** for freeing electrons
2. Photocurrent $\propto$ intensity: $i \propto I$
3. Stopping potential V_0 : required voltage to have no photocurrent same for all I



PHOTOELECTRIC EFFECT

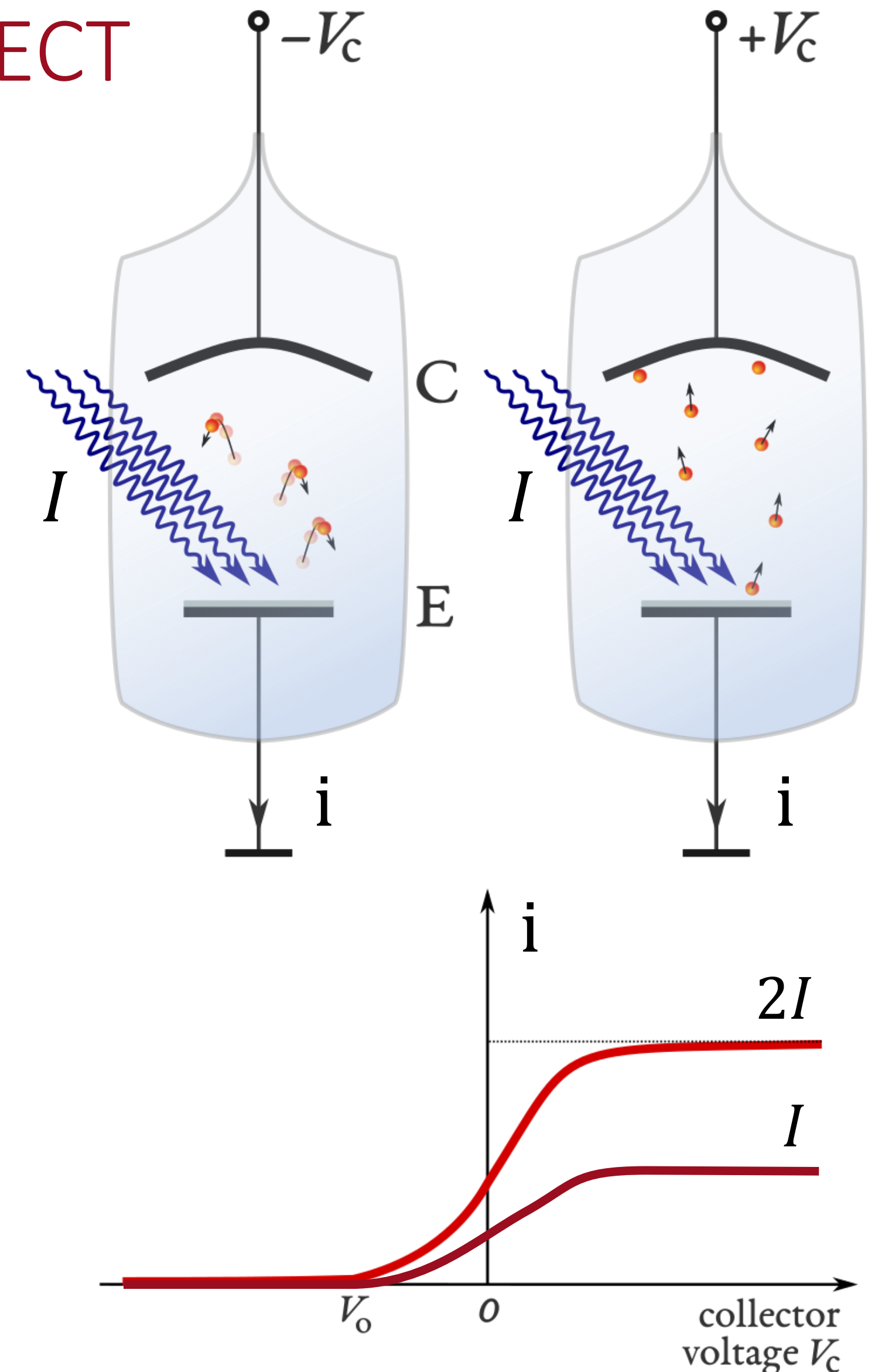
Einstein's idea:

- **Photon** gives all energy hf to one **single electron** in the metal
- Energy to release electron from metal:
Work function ϕ
- Kinetic energy of ejected electron K :

$$K_{\max} = hf - \phi$$

- Kinetic energy $K \leq K_{\max}$
- No delay of photocurrent

Consistent with experimental observations



PHOTOELECTRIC EFFECT

- **Kinetic energy** of ejected electron K :

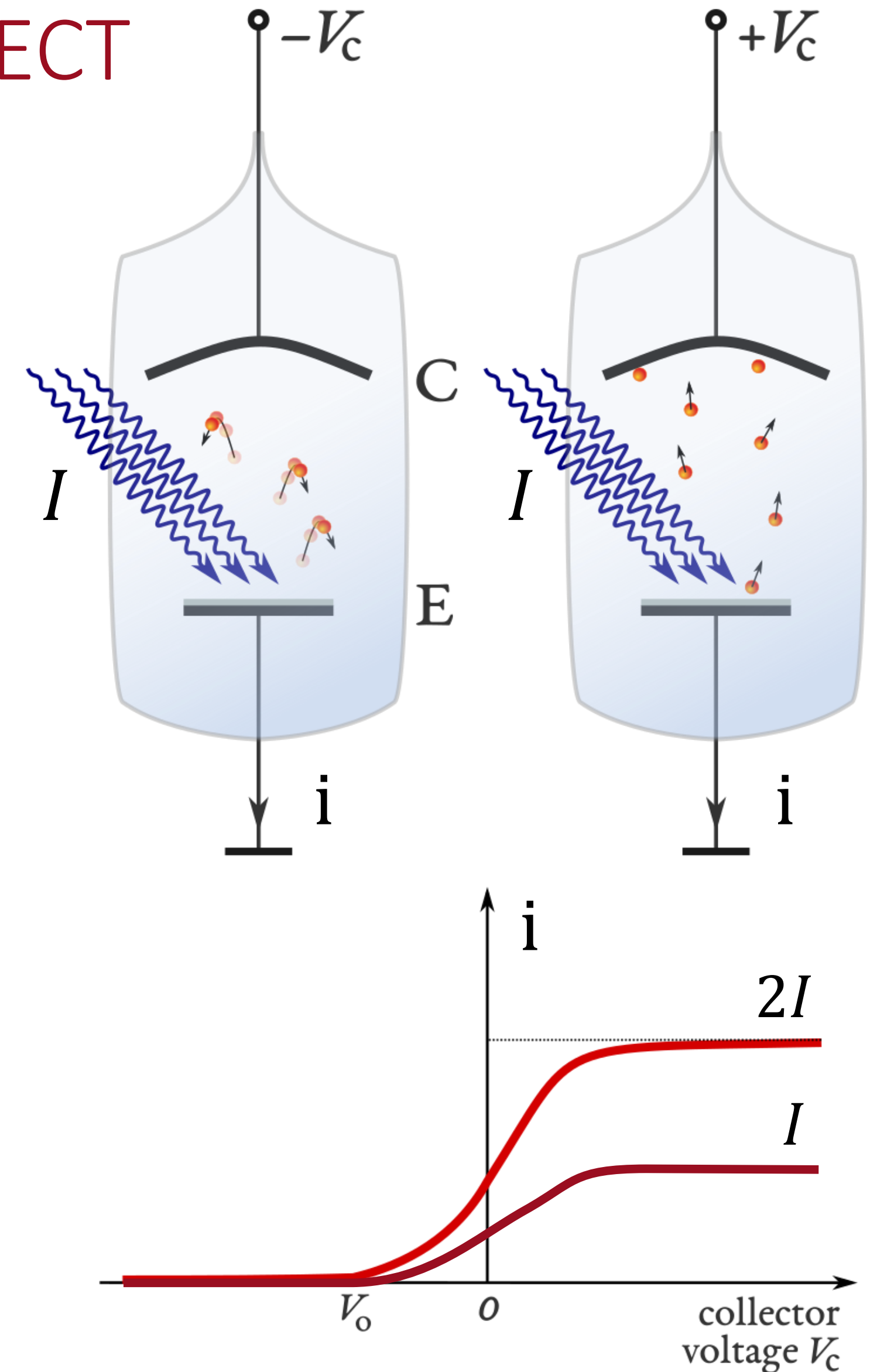
$$K_{\max} = hf - \phi$$

- Cut-off frequency and wavelength:

$$K_{\max} = 0 \Rightarrow f_c = \frac{\phi}{h}, \quad \lambda_c = \frac{hc}{\phi}$$

- Converting frequency to wavelength:

$$hc = 1240 \text{ eV} \cdot \text{nm}$$



PHOTOELECTRIC EFFECT

- **Kinetic energy** of ejected electron K :

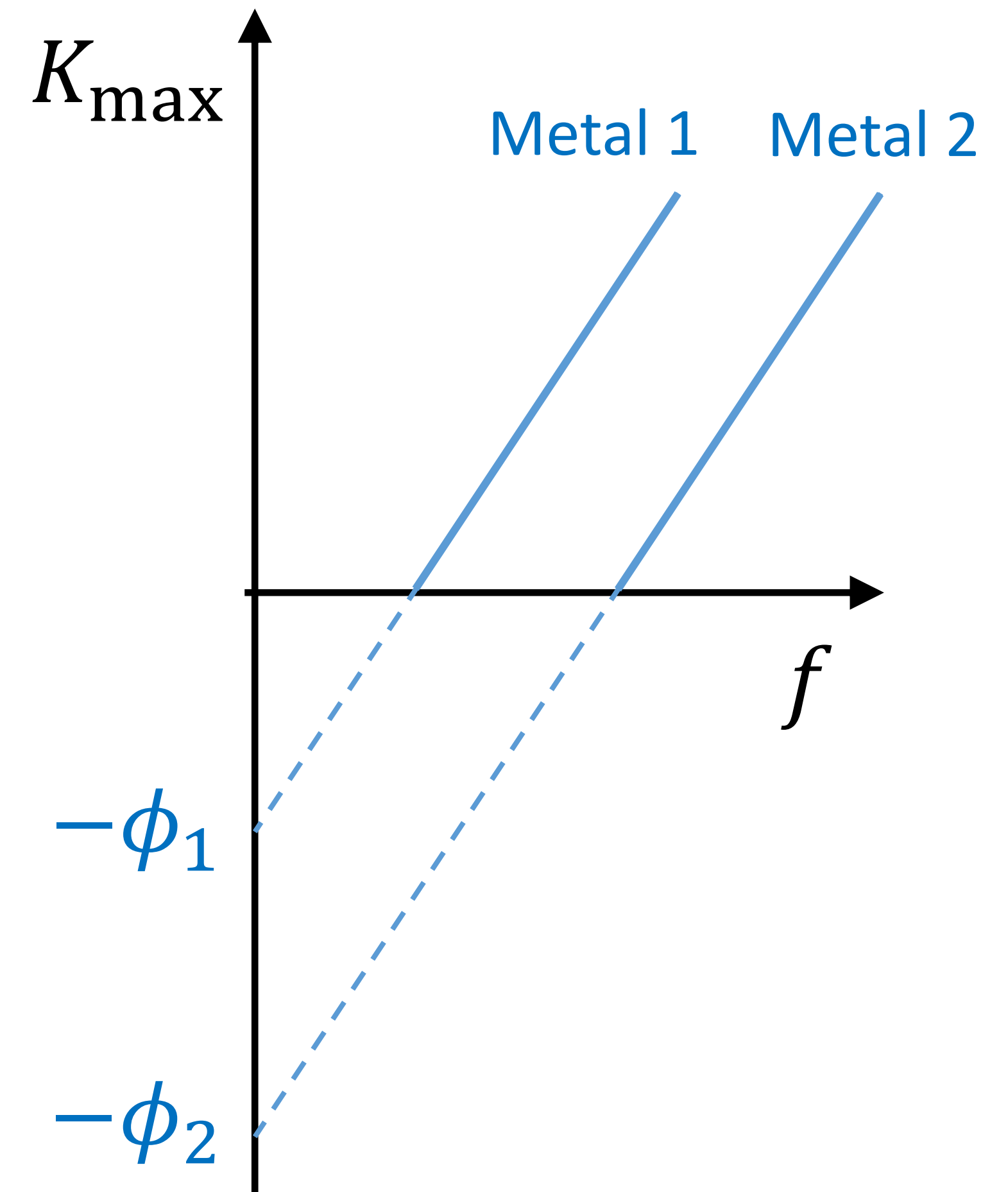
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PHOTOELECTRIC EFFECT: EXAMPLE

Problem: A Sodium surface ($\phi = 2.46$ eV) is illuminated with light with $\lambda = 300$ nm.

- What is the maximum kinetic energy?

$$K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV nm}}{300 \text{ nm}} - 2.46 \text{ eV} = 1.67 \text{ eV}$$

- What is cutoff wavelength?

$$\lambda_c = \frac{hc}{\phi} = \frac{1240 \text{ eV nm}}{2.46 \text{ eV}} \approx 500 \text{ nm}$$

Formulas:

$$K_{\max} = hf - \phi, \quad f_c = \frac{\phi}{h}, \quad \lambda_c = \frac{hc}{\phi}, \quad hc = 1240 \text{ eV} \cdot \text{nm}$$

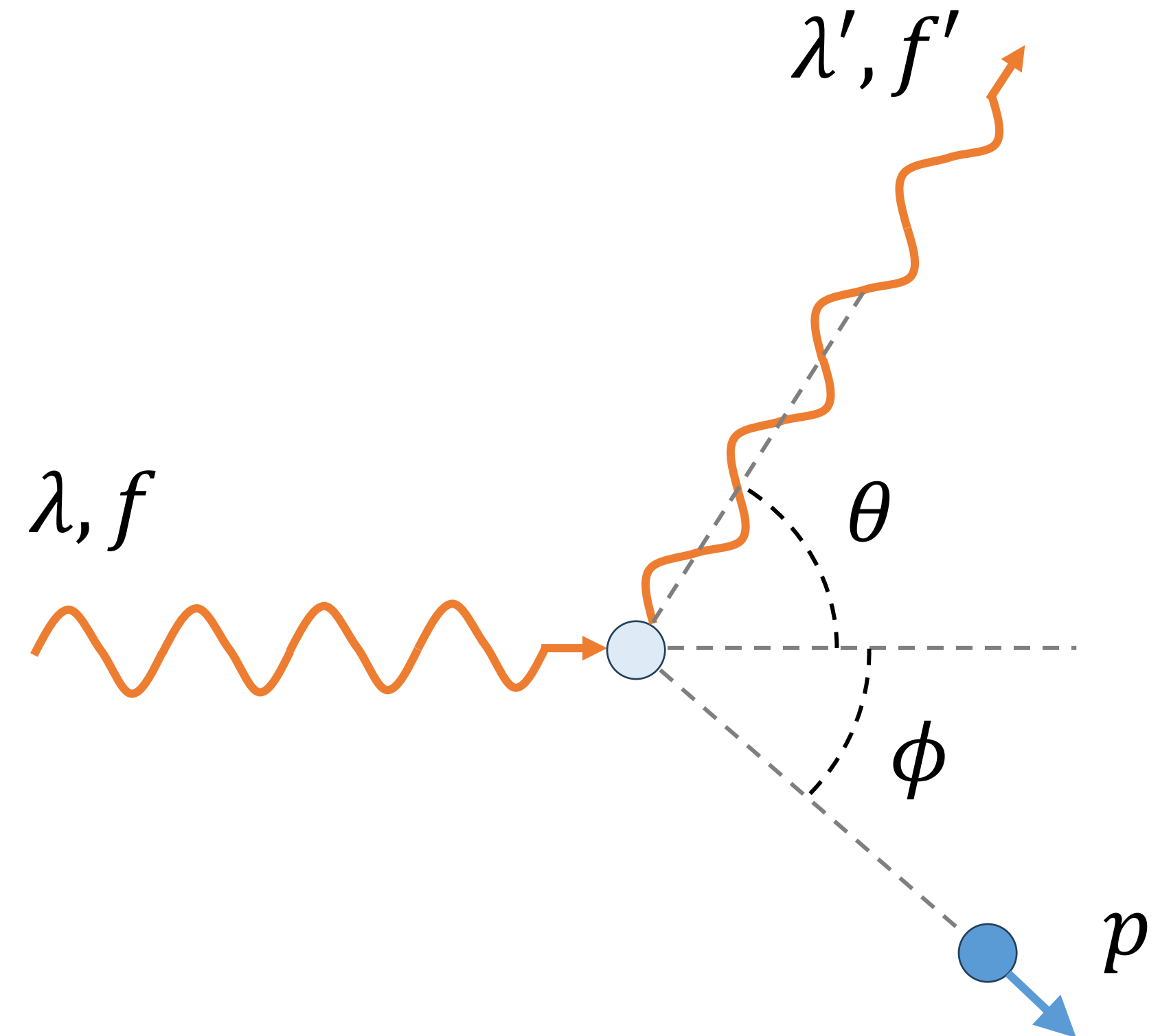
SUMMARY PHOTOELECTRIC EFFECT

- Einstein argued that light exists of photons (quantized)
- Photons react with electrons in material: transfer of all energy
- The work function of a material can be found by measuring the stopping potential

Compton Effect

COMPTON EFFECT

- Einstein in 1919: a photon has
 - Energy $E = hf = \frac{hc}{\lambda}$
 - Momentum $p = \frac{E}{c} = \frac{hf}{c}$
- Compton showed photon-electron collision is similar to a billiard and thus particle-like.
 - Conservation of momentum
 - Conservation of energy



COMPTON EFFECT

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- Energy $E = hf = \frac{hc}{\lambda}$

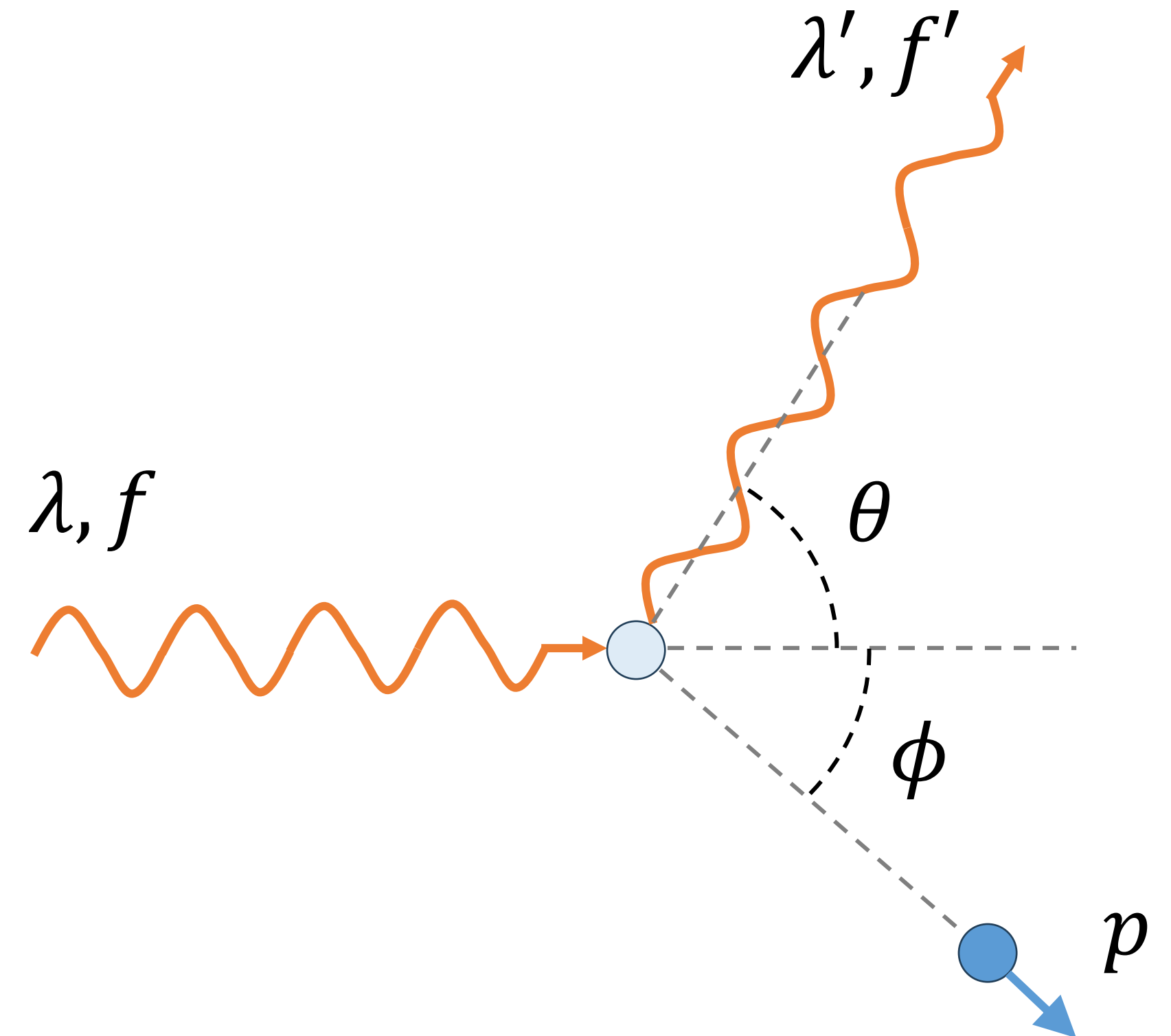
- Momentum $p = \frac{E}{c} = \frac{hf}{c}$

- **Compton shift equation:**

$$\lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta)$$

- **Compton wavelength of electrons:**

$$\lambda_c = \frac{h}{m_e c} = 0.00243 \text{ nm}$$



COMPTON EFFECT

- Derivation **Compton shift**:

$$\Delta K_{\text{photon}} + \Delta K_e = 0$$

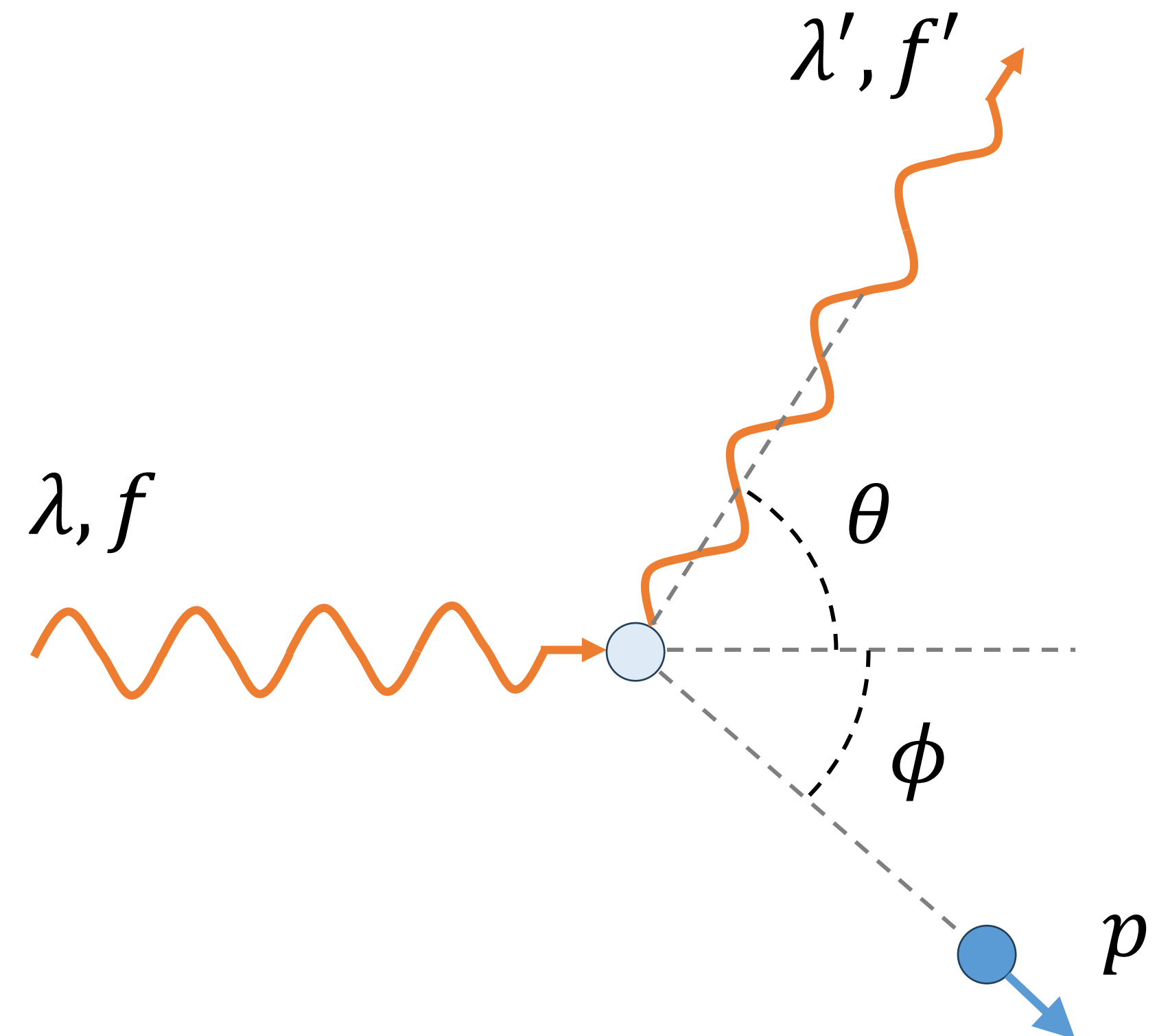
$$\Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda'} + K_e$$

$$\Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda'} + (\gamma - 1)m_e c^2$$

Formulas:

$$E = hf = \frac{hc}{\lambda}, \quad p = \frac{E}{c} = \frac{hf}{c}$$

$$\lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta)$$



COMPTON EFFECT

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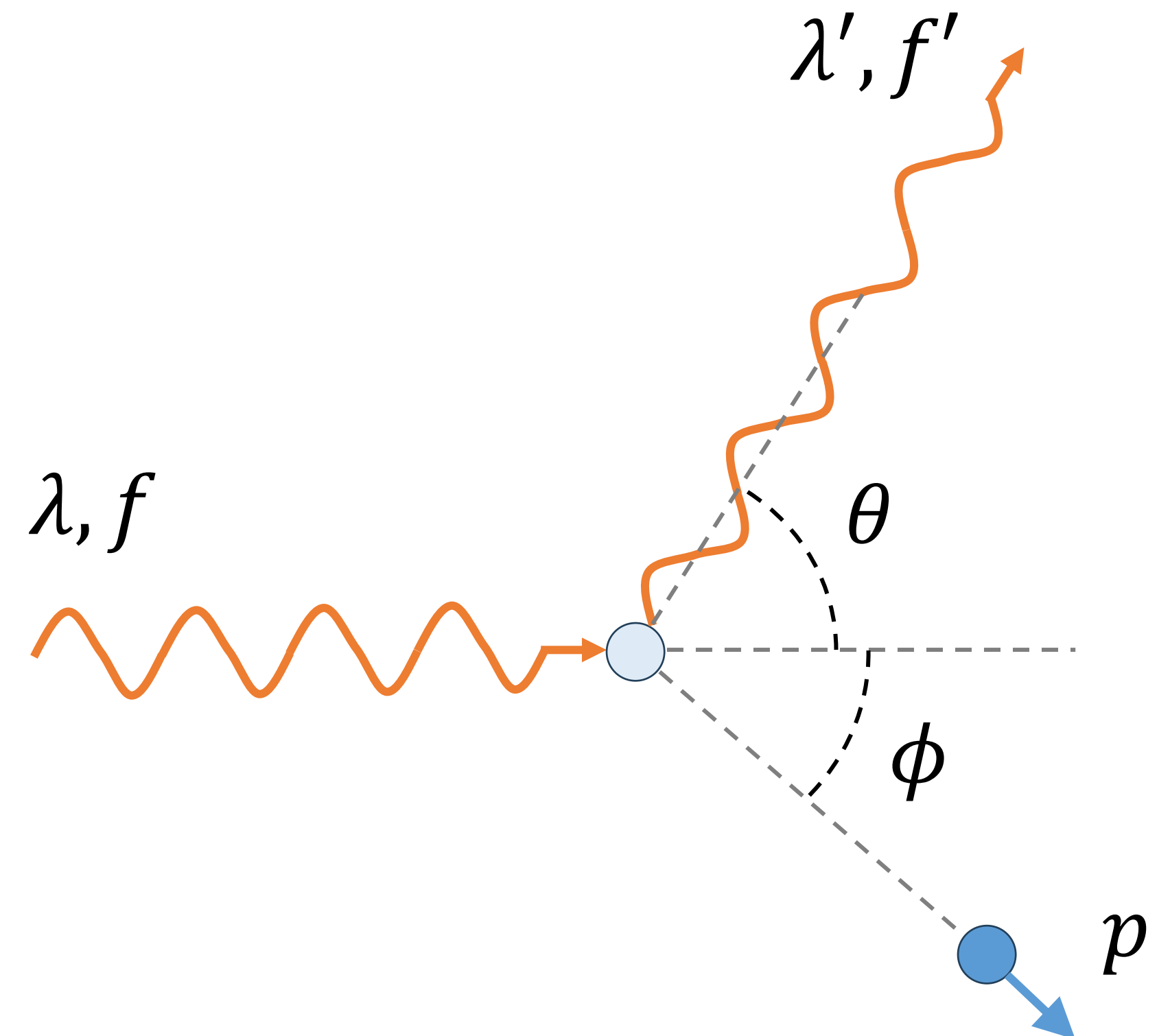
$$\Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda'} + (\gamma - 1)m_e c^2$$

With $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$

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COMPTON EFFECT

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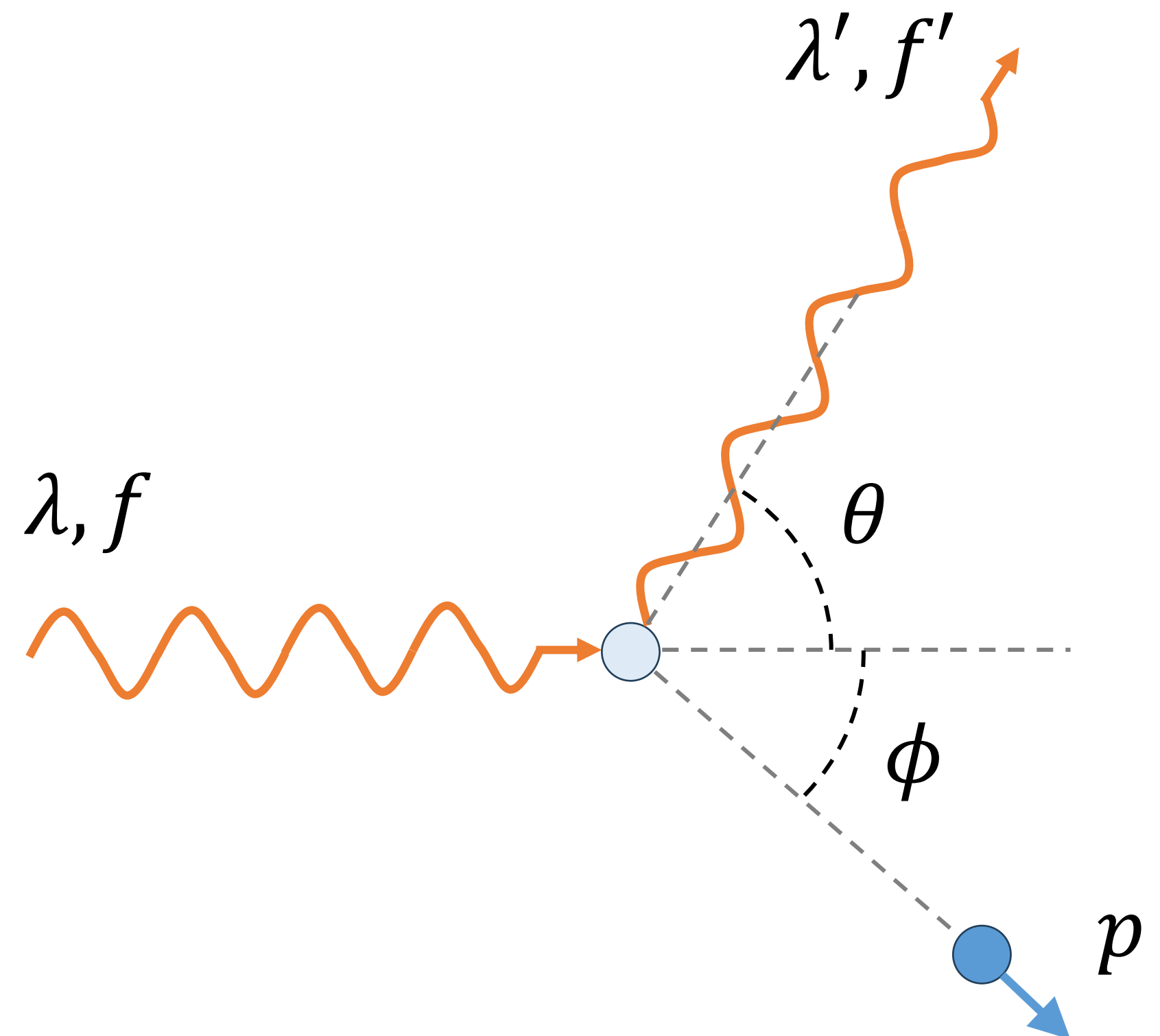
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Formulas:

$$E = hf = \frac{hc}{\lambda}, \quad p = \frac{E}{c} = \frac{hf}{c}$$

Momentum conservation in x and y:

$$\begin{cases} x: & \frac{h}{\lambda} = \frac{h}{\lambda'} \cos \theta + \gamma m_e u \cos \phi \\ y: & 0 = \frac{h}{\lambda'} \sin \theta - \gamma m_e u \sin \phi \end{cases}$$



SUMMARY COMPTON EFFECT

- Compton effect shows that photon-electron collisions are similar to billiard-balls
- Momentum/energy conserved during collision
- Relativistic treatment

ELECTROMAGNETIC RADIATION AS PARTICLES

- Blackbody: Quantized light ?
- Photoelectric effect: Photon gives full energy to atom
- Compton effect:
 - Photon collides with electron, partial energy transaction
 - Relativistic momentum/energy conserved during collision
- Further particle-like effects:
 - Electron-positron pair production and annihilation
 - Energy increase of photon by falling in a gravitational field