PHOT 222: Quantum Photonics

Midterm exam 2: questions

Michaël Barbier, Spring semester (2024-2025)

General information on the exam

Grading: This midterm exam will count for 20% of your total grade.

Exam type: The midterm exam consists of 4 open questions/problems. The exam is a written exam and all questions can be answered using only pen and paper. Calculators, mobile phones, laptops are not needed, and are not allowed to be used during the exam.

The duration of the midterm exam is 2 hours.

Exam questions

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam.

Please tell if any question is unclear or ambiguous.

Question 1: Particle in a Box

Assume a particle that is confined within a 1D box of width L. According to de Broglie a quantum particle has a corresponding wave with wavelength $\lambda = h/p$ and energy $E = hf$.

(a) If the particle is a proton and the box has $L = 1$ nm, what is its minimum velocity according to de Broglie?
(b) If the "particle" is a football (weight: $m = 0.4$ kg) and the box has $L = 1$ m, what is its minimum velocity?

Question 2: Particle in a Box

An electron in a 1D infinitely deep square well of width L has wave function solutions $\psi_n(x)$ and E_n given by

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}), \text{ with } x \in [0, L], \qquad E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2}$$

Assume that $L = 1$ nm and the electron is in the ground state $\psi_1(x)$, and gets excited to the $\psi_4(x)$ state by absorbing a photon.

(a) What is the energy required to excite the electron?
(b) What the corresponding wavelength of the photon to excite the electron?

Question 3: Quantum Harmonic Oscillator

Consider a 1D quantum harmonic oscillator in the first excited state, which has wave function $\psi_1(x) = A_1 2\beta x e^{-\beta^2 x^2/2}$ where $\beta = \sqrt{\frac{m\omega}{\hbar}}$ and A_1 a normalization constant.

(a) Determine the value of the normalization constant A_1.
(b) Calculate the expectation value $\langle x^2 \rangle$. Simplify the resulting formula.

Question 4: Tunneling probability

The tunneling probability T through a potential barrier for $T \ll 1$ can be approximated by following formula:

$$T \approx 16\,\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2\kappa L}, \qquad \text{with} \quad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

Assume a quantum particle with energy $E = 1$ eV and that is incident on a 1D potential barrier with height $V_0 = 2$ eV, and width $L = 2$ nm.

(a) What is the tunneling probability T_a?
(b) If you change the width of the potential barrier to $L = 1$ nm, how much larger becomes the tunneling probability T_b? Give the formula for the ratio T_b/T_a and simplify as much as possible.

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg
Mass of a proton: $m_p \approx 1836\ m_e$
1 eV $= 1.602 \times 10^{-19}$ J
Joule in SI units: [J = kg m^2/s^2]
$h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s
$c = 3 \times 10^8$ m/s
$hc = 1240$ eV nm
$m_e c^2 = 0.511$ MeV

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b \psi(x)^* \, f(x) \, \psi(x) dx.$$

Definite integrals:

$$\int_{-\infty}^{+\infty} x e^{-ax^2} \, dx = \frac{1}{a}$$
$$\int_{-\infty}^{+\infty} x^2 e^{-ax^2} \, dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$
$$\int_{-\infty}^{+\infty} x^3 e^{-ax^2} \, dx = \frac{1}{a^2}$$
$$\int_{-\infty}^{+\infty} x^4 e^{-ax^2} \, dx = \frac{3}{4a^2}\sqrt{\frac{\pi}{a}}$$

Particle in a box: A quantum particle in a 1D infinitely deep square well of width L has wave function solutions $\psi_n(x)$ and E_n given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}), \text{ with } x \in [0, L], \qquad E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$