PHOT 222: Quantum Photonics

Midterm exam 2: example questions

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General information on the exam

Grading: This midterm exam will count for 20% of your total grade.

Exam type: The midterm exam consists of 4 open questions/problems. The exam is a written exam and all questions can be answered using only pen and paper. Calculators, mobile phones, laptops are not needed, and are not allowed to be used during the exam.

The duration of the midterm exam is 2 hours.

Exam questions

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam.

Please tell if any question is unclear or ambiguous.

Question 1: Particle in a Box

An electron in a 1D infinitely deep square well of width L has wave function solutions $\psi_n(x)$ and E_n given by

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}),\ \text{with } x \in [0, L], \qquad E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2}$$

(1/3) Assume that the electron has the wave function $\psi_3(x)$, what is the probability for the electron to be found in interval $[0, L/3]$?
(2/3) Sketch graphically the probability density for an electron with wave function $\psi_2(x)$.
(3/3) Is the probability of an electron with wave function $\psi_2(x)$ to be in interval $[0, L/3]$, larger than 1/3, equal to 1/3, or smaller than 1/3?

Question 2: Simple Harmonic Oscillator

A single electron is oscillating in a 1D quantum "simple" harmonic oscillator with $\omega = \sqrt{k/m_e}$.

(1/2) What is the energy of the system if the electron is in the third excited state $n = 3$?
(2/2) What is the frequency of the light emitted if the electron comes from the second excited state $n2$ and transitions to the ground state.

Question 3: The Bohr Model

A hydrogen atom is in the first state ($n = 2$).

(1/2) What is the radius of the orbit of the electron according to Bohr's theory?
(2/3) Calculate the energy E of the electron.
(3/3) What is the kinetic energy of the electron after we ionize the electron by hitting it with a photon with an energy of 6.4 eV?

Question 4: Hydrogen Model and Quantum Numbers

Consider the quantum model of the hydrogen atom.

(1/3) What are the possible values for the orbital magnetic quantum number m_l when $l = 3$?
(2/3) Determine the magnitude of the angular momentum $\vec{L}$ and L_z component for each possible value of l.
(3/3) Give the relation between L_z and the angle that $\vec{L}$ makes with the z-axis. How is this angle related to the orbital quantum number l?

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg, 1 eV $= 1.602 \times 10^{-19}$ J
Mass of a proton: $m_p \approx 1836\ m_e$
Values: $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ J s, $\quad c = 3 \times 10^8$ m/s, $\quad hc = 1240$ eV nm

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b \psi(x)^* \, f(x) \, \psi(x) dx.$$

The "orbital quantum number" l is also sometimes called the "azimuthal quantum number".