

PHOT 222: Quantum Photonics

Midterm exam 1: questions

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General information on the exam

Grading: This midterm exam will count for 20% of your total grade.

Exam type: The midterm exam consists of 4 open questions/problems. The exam is a written closed-book exam and all questions can be answered using only pen and paper. Calculators, mobile phones, laptops are not needed, and are not allowed to be used during the exam.

The duration of the midterm exam is 2 hours.

Exam questions

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam.

Please tell if any question is unclear or ambiguous.

Question 1: Relativistic energie

An electron is traveling at a velocity $v = 0.6 c$.

(a) What is the (relativistic) kinetic energy K of the electron?

(b) Then calculate the kinetic energy nonrelativistic ($K_{\text{classical}}$). What is the ratio between them:

$$\text{ratio} = \frac{K}{K_{\text{classical}}}$$

Question 2: Photoelectric effect

When we shine UV light onto a carbon (C) plate photoelectrons are emitted from the surface. Assume that carbon has a workfunction $\phi = 5.0$ eV.

- (a) What is the minimum frequency of the light to have photoemission for this carbon surface?
- (b) If we use UV light with a wavelength of 100 nm, what is the maximum velocity v_{\max} of the electrons?

Question 3: de Broglie

A proton travels at $v_p = 0.01c$ with c the speed of light.

- (a) What is the de Broglie wavelength $\lambda = h/p$ for this proton?
- (b) An electron with the same de Broglie wavelength λ would go faster, what is its approximate velocity $v_e = ?$ (Use a relativistic description for the electron).
- (c) If you would calculate v_e in a nonrelativistic manner, what would be the result? Is that faster or slower than the speed of light?

Question 4: Wave functions and probability

Consider the following wave function defined with $x \in [0, \infty[$.

$$\psi(x) = A x e^{-x/2},$$

with A a normalization constant.

- (a) Calculate the normalization constant A of the wave function.
- (b) Determine the probability for the particle to be found within interval $[0, 1]$. The Euler constant $e \approx 2.718$, but you can write the end-result as a formula.
- (c) Afterwards calculate the expectation value for the position x , that is, calculate $\langle x \rangle$.

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg

Mass of a proton: $m_p \approx 1836 m_e$

1 eV = 1.602×10^{-19} J

A Joule has units of: [J = kg m²/s²]

$h = 6.63 \times 10^{-34}$ J s = 4.14×10^{-15} eV s

$c = 3 \times 10^8$ m/s

$hc = 1240$ eV nm

$m_e c^2 = 0.511$ MeV

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b f(x) |\psi(x)|^2 dx.$$

You can also make use of following indefinite integrals (anti-derivatives):

$$\int x e^{-x} dx = -(x + 1) e^{-x}$$

$$\int x^2 e^{-x} dx = -(x^2 + 2x + 2) e^{-x}$$

$$\int x^3 e^{-x} dx = -(x^3 + 3x^2 + 6x + 6) e^{-x}$$