PHOT 222: Quantum Photonics

Midterm exam 1: example questions

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General information on the exam

Grading: This midterm exam will count for 20% of your total grade.

Exam type: The midterm exam consists of 4 open questions/problems. The exam is a written exam and all questions can be answered using only pen and paper. Calculators, mobile phones, laptops are not needed, and are not allowed to be used during the exam.

The duration of the midterm exam is 2 hours.

Exam questions

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam.

Please tell if any question is unclear or ambiguous.

Question 1: Relativistic energie

(1/3) What is the rest energy of a photon?
(2/3) What is the total energy of an electron traveling at $v = 0.8c$. Assume that the electron has a mass m_e.
(3/3) Consider an electron with mass m_e travels at velocity v and a proton that is at rest. What is the velocity v of the electron when the total energy of both particles is equal?

Question 2: Waves and particles

According to de Broglie particles having momentum p have a corresponding wavelength $\lambda = h/p$.

(1/2) What is the wavelength λ for an electron traveling at a velocity $v = 1000$ m/s?
(2/2) How fast should a proton approximately go to obtain the same corresponding wavelength λ of the electron traveling at a velocity $v = 1000$ m/s?

Question 3: Photoelectric effect

When we shine UV light onto a silver (Ag) plate photoelectrons are emitted from the surface. Assume that Silver has a workfunction $\phi = 4.3$ eV.

(1/2) If we use UV light with a wavelength of 200 nm, what is the maximum kinetic energy K_{max} of the electrons?
(2/2) What is the cutoff wavelength to have photoemission for a silver plate?

Question 4: Wave functions and probability

Consider the following wave function defined with $x \in \mathbb{R}$ and $a > 0$:

$$\psi(x) = A \frac{1}{\sqrt{x^2 + a^2}},$$

with A a normalization constant.

(1/3) Calculate the normalization constant A of the wave function.
(2/3) Determine the probability for the particle to be found within interval $[0, a]$.
(3/3) Afterwards calculate the expectation value for the function $f(x) = x$, that is, calculate $\langle x \rangle$.

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg, 1 eV $= 1.602 \times 10^{-19}$ J
Mass of a proton: $m_p \approx 1836\ m_e$
Values: $h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s, $c = 3 \times 10^8$ m/s, $hc = 1240$ eV nm

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b f(x)\, |\psi(x)|^2\, dx.$$

You can also make use of following indefinite integrals (anti-derivatives):

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \frac{1}{2}\left[\log\left(1+\frac{x}{\sqrt{x^2+a^2}}\right)+\log\left(1-\frac{x}{\sqrt{x^2+a^2}}\right)\right]$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\,\arctan\left(\frac{x}{a}\right)$$

$$\int \frac{x}{\sqrt{x^2+a^2}}\,dx = \sqrt{x^2+a^2}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\,\log\left(x^2+a^2\right)$$