

PHOT 222: Quantum Photonics

Midterm exam 1B: questions

Michaël Barbier, Spring semester (2024-2025)

General information on the exam

Grading: This midterm exam will count for 20% of your total grade.

Exam type: The midterm exam consists of 4 open questions/problems. The exam is a written closed-book exam and all questions can be answered using only pen and paper. Calculators, mobile phones, laptops are not needed, and are not allowed to be used during the exam.

The duration of the midterm exam is 2 hours.

Exam questions

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam.

Please tell if any question is unclear or ambiguous.

Question 1: Relativistic energie

A proton has a total energy E of two times its kinetic energy K .

- (a) What is the (relativistic) kinetic energy K of the proton?
- (b) What is the corresponding velocity of the proton?

Question 2: The Compton shift

A photon with wavelength $\lambda = 0.1\text{nm}$ hits a standing still free electron during a Compton measurement. The photon scatters under an angle $\theta = 60^\circ$. The Compton shift is given by the following formula:

$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos(\theta))$$

(a) What is wavelength λ' of the photon after the collision?

Question 3: de Broglie

(a) A photon has energy $E = 5$ eV. What is its wavelength λ ?
(b) What is the de Broglie wavelength of an electron that has the same kinetic energy.
(c) What is the velocity of the electron?

Question 4: Wave functions and probability

Consider the following wave function defined with $x \in [0, 1]$:

$$\begin{aligned}\psi(x) &= Ax(1-x), & \text{If } x \in [0, 1] \\ \psi(x) &= 0, & \text{If } x > 1 \text{ or } x < 0\end{aligned}$$

with A a normalization constant.

(a) Calculate the normalization constant A of the wave function.
(b) Afterwards calculate the expectation value for $f(x) = x^2$, that is, calculate $\langle x^2 \rangle$.

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg

Mass of a proton: $m_p \approx 1836 m_e$

1 eV = 1.602×10^{-19} J

A Joule has units of: [J = kg m²/s²]

$h = 6.63 \times 10^{-34}$ J s = 4.14×10^{-15} eV s

$c = 3 \times 10^8$ m/s

$hc = 1240$ eV nm

$m_e c^2 = 0.511$ MeV

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b f(x) |\psi(x)|^2 dx.$$

You can also make use of following definite integral:

$$\int_0^1 x^m (1-x)^n dx = \frac{n! m!}{(n+m+1)!}$$