PHOT 222: Quantum Photonics

Final exam: questions (version A)

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General information on the exam

Grading: The final exam counts for 60% of your total grade.

Exam type: The exam consists of 6 open questions/problems. The exam is a written exam and all questions can be answered using only pen and paper. Calculators, mobile phones, laptops are not needed, and are not allowed to be used during the exam.

The duration of the exam is 3 hours.

Exam questions

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam.

Please tell if any question is unclear or ambiguous.

Question 1: Bohr model

A hydrogen atom is in state $n = 2$ and gets ionized by absorbing a photon.

(a) What is the minimum energy that the photon must have?
(b) What is the wavelength of the photon for that energy?

Question 2: Quantum Hydrogen Atom

The radial wave function of a hydrogen atom in the $2s$ state is given by:

$$R_{2,0}(r) = \frac{2}{\sqrt{2a_0^3}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

(a) Calculate the most likely radius for the particle to be found, i.e. where $|R_{2,0}(r)|^2 4\pi r^2$ attains its maximum value.
(b) Calculate the expectation value: $\langle r \rangle$. *Hint*: Integrating in spherical coordinates the volume element $dV = 4\pi r^2 dr$.

Question 3: Angular momentum

A hydrogen atom has an angular momentum $L = 2.583 \times 10^{-34}$ Joule.

(a) What is the orbital quantum number l?
(b) What are the possible values for the magnetic quantum number m_l?

Question 4: Shielding and effective nuclear charge

A Lithium atom in the ground state has configuration $1s^2\,2s^1$. Assume that the $Z_{\text{eff},2s} \approx 1.28$ and $Z_{\text{eff},2p} \approx 1$. Then suppose that the atom absorbs a photon and the electron in the $1s$ orbital gets excited to the $2p$ orbital.

(a) What is the wavelength of the photon assuming that the effective nuclear charges are good approximations.
(b) The real wavelength for the excitation from 2s to 2p is actually 670 nm. Assuming that our approximation for $Z_{\text{eff},2p} \approx 1$ is to blame, what would be correct energy level of the $2p$ orbital be (to compensate for the error in the wavelength)?

Question 5: Normal Zeeman effect

A hydrogen atom H is put in a magnetic field $\vec{B} = B\vec{e}_z$. Ignore spin and assume that the only contribution to the hydrogen atom energy levels is $U = m_l \mu_B B$ with μ_B the Bohr magneton.

(a) Calculate the energy levels of the $3d$ orbital if $B = 0.1$ Tesla.
(b) How strong does the magnetic field have to be for the $2p$ and $3d$ energy levels to overlap?

Question 6: Molecular spectra

Assume you have a NO molecule existing of an oxygen and a nitrogen atom with a separation distance $r_0 = 0.115$ nm.

(a) Calculate the reduced mass μ and the moment of inertia I of the molecule. Remember that nitrogen has an atomic mass of approximately 14 and oxygen 16 atomic mass units (or Dalton, see formulas/values).
(b) Suppose the molecule is in state with rotational quantum number $l = 3$, and transitions to $l = 4$ thereby absorbing a photon. Calculate the wavelength of the photon.

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg
Mass of a proton: $m_p \approx 1836\ m_e$
1 eV $= 1.602 \times 10^{-19}$ J
An atomic mass unit: $1\,\text{Dalton} = 1.66 \times 10^{-27}\,\text{kg} = \frac{5}{3} \times 10^{-27}\,\text{kg}$
Bohr magneton: $\mu_B = 9.3 \times 10^{-24}$ J/T $= 5.8 \times 10^{-5}$ eV/T
Joule in SI units: [J = kg m^2/s^2]
$h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s
$c = 3 \times 10^8$ m/s
$hc = 1240$ eV nm
$m_e c^2 = 0.511$ MeV

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b f(x)\, |\psi(x)|^2\, dx.$$

You can also make use of following definite integrals:

Definite integrals:

$$\int_0^\infty x^n e^{-ax}\, dx = \frac{n!}{a^{n+1}}$$
$$\int_0^\infty e^{-ax^2}\, dx = \frac{\sqrt{\pi}}{2\sqrt{a}}$$
$$\int_0^\infty x^2 e^{-ax^2}\, dx = \frac{\sqrt{\pi}}{4a^{3/2}}$$
$$\int_0^\infty x^4 e^{-ax^2}\, dx = \frac{3\sqrt{\pi}}{8a^{5/2}}$$