PHOT 222: Quantum Photonics

Final exam: example questions

Michaël Barbier, Spring semester (2024-2025)

General information on the exam

Grading: The final exam counts for 60% of your total grade.

Exam type: The exam consists of 6 open questions/problems. The exam is a written exam and all questions can be answered using only pen and paper. Calculators, mobile phones, laptops are not needed, and are not allowed to be used during the exam.

The duration of the exam is 3 hours.

Exam questions

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam.

Please tell if any question is unclear or ambiguous.

Question 1: Excited Electron in a Box

An electron in a 1D infinitely deep square well of width L has wave function solutions $\psi_n(x)$ and E_n given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L}), \text{ with } x \in [0, L], \qquad E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

(1/3) Assume that the electron has the wave function $\psi_3(x)$, what is the probability for the electron to be found in interval $[L/3, L]$?
(2/3) Sketch graphically the probability density for an electron in state $\psi_2(x)$.
(3/3) Is the probability of an electron with wave function $\psi_2(x)$ to be in interval $[0, L/3]$: larger than 1/3, equal to 1/3, or smaller than 1/3?

Question 2: Quantum Hydrogen Atom

A hydrogen atom is in the 2S state: $n = 2$, $l = 0$, $m_l = 0$ (and the spin magnetic quantum number can be either positive or negative: $m_s = \pm 1/2$).

(1/2) What is probability to find the electron within the Bohr radius a_0?
(2/2) Calculate the expectation value of the radius.

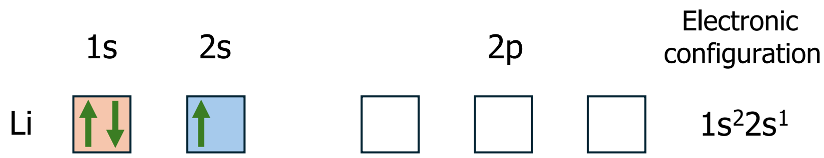
Question 3: Atomic Spectra

The Balmer series for the hydrogen atom represents spectral lines corresponding to transitions from a state $n > 2$ to $n = 2$.

(1/2) Give the formula that determines the wavelengths of the spectral lines in the Balmer series.
(2/2) What is the frequency of the spectral line with the highest energy?

Question 4: Many-electron atoms

The electronic configuration for Lithium can be presented as:



(1/2) Write down the electronic configuration for an oxygen atom in the same manner (oxygen has 8 electrons). you don't have to color the boxes according to the number of electrons.
(2/2) State the exclusion principle and Hund's rule.

Question 5: Molecular spectra

Spectra of molecules are defined by their electronic, rotational, vibrational, and translational energies.

(1/2) Suppose a certain molecule has a rotational quantum number $J = 4$ and transitions to $J = 5$ thereby absorbing a photon with frequency of 2×10^{11} Hz. Calculate the moment of inertia I of this molecule.
(2/2) Afterwards another photon is incident on the same molecule but which has rotational quantum number $J = 3$. What should the frequency of this photon be for stimulated emission to occur?

Question 6: Energy band structure

An electron-hole pair is formed in a pure Silicon at 300 K by an incident photon. Assume that the bandgap of Silicon equals 1.12 eV.

(1/1) What is the maximum wavelength of the photon?

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg, 1 eV $=$ 1.602×10^{-19} J
Mass of a proton: $m_p \approx 1836\ m_e$
Values: $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s, $hc = 1240$ eV nm

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b \psi(x)^* \, f(x) \, \psi(x) dx.$$

The "orbital quantum number" l is also sometimes called the "azimuthal quantum number".