

Grading: The final exam will count for 60% of your total grade.

Exam type: Closed-book, all questions can be answered **using only pen and paper**. Calculators, mobile phones, etc. are not allowed to be used during the exam.

The duration of the final exam is 3 hours.

Please fill in all questions listed below. Each of the questions is valued equally in the score calculation of the exam. Please tell if any question is unclear or ambiguous.

Question 1: Photoelectric effect

When we shine monochromatic light onto a metallic plate, photoelectrons can be emitted from the surface if the frequency of the light is high enough. The table below shows some metals with their corresponding (approximate) workfunction ϕ .

Metal	ϕ (in eV)
Cesium	2.0
Barium	2.5
Lithium	3.0
Silver	4.0
Nickel	5.0

(a) Suppose the target metal plate is Cesium, and blue light with a wavelength of 400 nm, what is the maximum kinetic energy K_{max} that the emitted photoelectrons can acquire?

(b) Consider the table of metals with their workfunctions ϕ . If we shine blue light with a wavelength of 400 nm, which of the above metals in the table would exhibit photoemission?

Question 2: Quantum Harmonic Oscillator

Consider a quantum harmonic oscillator. The eigenenergies E_n for the eigenstates ψ_n are given by $E_n = (n + \frac{1}{2}) \hbar\omega$ with $n = 0, 1, 2, 3, \dots$

(a) If the ground state has energy $E_0 = 1$ eV, what is ω ?

(b) Sketch the potential $V(x)$ and the wave functions $\psi_0(x)$, $\psi_1(x)$, and $\psi_2(x)$.

Question 3: Infinite square well

Assume an electron is in an infinite well with width L . The ground state is given by $\psi_1(x) = A \sin(\frac{\pi x}{L})$

(a) Derive an expression for the normalization constant A .

(b) What is the expectation value $\langle x^2 \rangle$ for the ground state?

Question 4: Bohr model

A hydrogen atom in the ground state $n = 1$ absorbs a photon of wavelength λ_1 and goes to energy level $n = 3$. It afterwards emits a photon with wavelength λ_2 and goes to level $n = 2$.

- (a) What are the wavelengths of the photon λ_1 and λ_2 ?
- (b) Calculate the velocity of the electron and the orbit radius in the second excited state, that is $n = 3$.

Question 5: Normal Zeeman effect

Assume a hydrogen atom is placed in a magnetic field with $\vec{B}$ along the z-axis. This results in an extra term in the energy $E_n \rightarrow E_n + m_l \mu_B B$ where the Bohr magneton $\mu_B \approx 5.8 \times 10^{-5}$ eV/Tesla, and m_l is the magnetic orbital quantum number (to simplify the case here we ignore spin).

- (a) What is the wavelength corresponding to the $2p$ to $1s$ transition if $B = 0$ Tesla?
- (b) If we now turn on the magnetic field: in how many lines will the spectral line split? If the splitting between two adjacent lines is about 1.16×10^{-6} eV, what is the required magnitude B of the magnetic field?

Question 6: Rotational energy-levels in a diatomic molecule

Assume you have a CO molecule consisting of a carbon and an oxygen atom. The rotational energy levels E_J with rotational quantum number $J = 0, 1, 2, 3, \dots$ are given by

$$E_J = \frac{\hbar^2}{2I} J(J+1),$$

with $I = \mu r^2$ the moment of inertia, where r is the inter-atomic distance and $\mu = \frac{m_1 m_2}{m_1 + m_2}$ the reduced mass.

- (a) Suppose that a CO molecule with rotational energy E_2 absorbs a photon and transitions to E_3 . If the photon's frequency $f \approx 3.5 \times 10^{11}$ Hz, what is then the moment of inertia I of the CO molecule?
- (b) Derive the inter-atomic distance r (distance between the C and the O atom) from the value of I . Remember that carbon has an atomic mass of approximately 12 and oxygen 16 atomic mass units (or Dalton, see formulas/values).

Values and formulas:

Mass of an electron: $m_e = 9.11 \times 10^{-31}$ kg

Mass of a proton: $m_p \approx 1836 m_e$

1 eV = 1.602×10^{-19} J

An atomic mass unit: 1 Dalton = 1.66×10^{-27} kg $\approx \frac{5}{3} \times 10^{-27}$ kg

Bohr magneton: $\mu_B = 9.3 \times 10^{-24}$ J/T = 5.8×10^{-5} eV/T

Joule in SI units: [J = kg m²/s²]

$h = 6.63 \times 10^{-34}$ J s = 4.14×10^{-15} eV s

$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}$ J s = 6.582×10^{-16} eV s

$c = 3 \times 10^8$ m/s

$hc = 1240$ eV nm

Rydberg energy unit: Ry = 13.6 eV

Rydberg constant for hydrogen: $R_H = 1.0968 \times 10^7 \text{m}^{-1} \approx 1.1 \times 10^7 \text{m}^{-1}$

Rydberg constant for heavy atoms: $R_\infty = 1.0974 \times 10^7 \text{m}^{-1} \approx 1.1 \times 10^7 \text{m}^{-1}$

$m_e c^2 = 0.511$ MeV

For a wave function $\psi(x)$ with $x \in [a, b]$, the expectation value of a function $f(x)$ is:

$$\langle f(x) \rangle = \int_a^b f(x) |\psi(x)|^2 dx.$$

Definite integrals:

$$\begin{aligned}\int_0^1 \sin^2(\pi x) dx &= \frac{1}{2} \\ \int_0^1 x \sin^2(\pi x) dx &= \frac{1}{4} \\ \int_0^1 x^2 \sin^2(\pi x) dx &= \frac{1}{6} - \frac{1}{4\pi^2} \\ \int_0^1 x^3 \sin^2(\pi x) dx &= \frac{1}{8} - \frac{3}{8\pi^2}\end{aligned}$$