PHOT 110: Introduction to programming

Exercises 12: Numpy arrays

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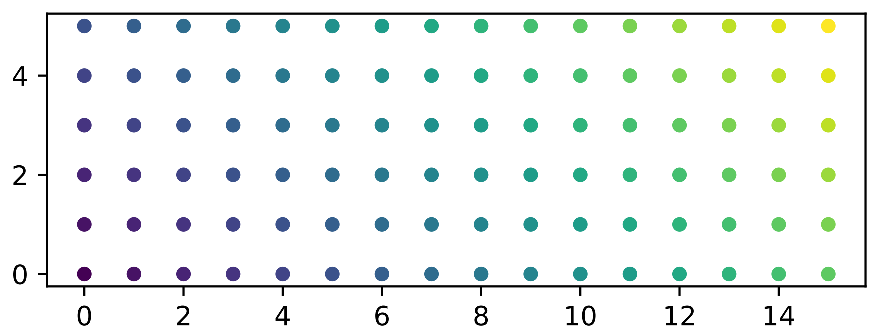
1. 2D domains

First use Numpy's `arange(start, stop, step)` function to create an array with following numbers: $[2, 2.1, 2.2, 2.3, 2.4, 2.5]$. Then try to get the same array using Numpy's `linspace(start, stop, n_points)` function.

Afterwards, use Numpy's `arange()` function to create two intervals containing integers with e.g. $x \in [0, 15]$ and $y \in [0, 5]$, and make a 2D domain of them using the `xx, yy = np.meshgrid(x, y)` function. Plot the coordinates thus created using a scatter plot (`plt.scatter(xx, yy)`).

The plot should similar to the following:



2. plotting 2D arrays/functions

Use a 2D domain/interval using the `np.meshgrid` command such as in exercise 1, but using `np.linspace()` for the intervals with $x \in [-1, 1]$ and $y \in [-1, 1]$ at a finer grid, and plot the function for that domain:

$$z = f(x, y) = y \sin(2\pi\, x)$$

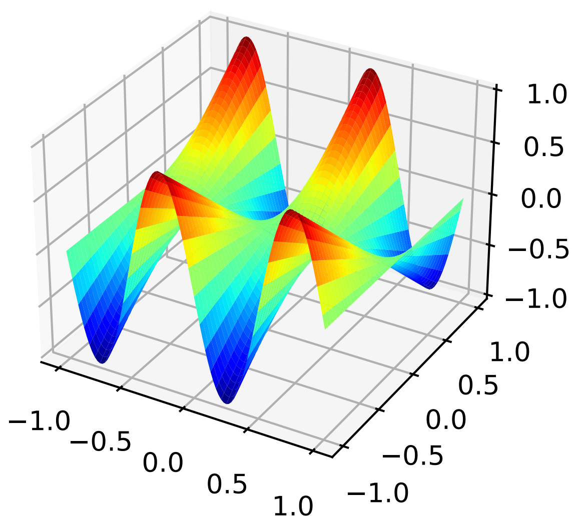
For the plot use the `plot_surface()` from Matplotlib:

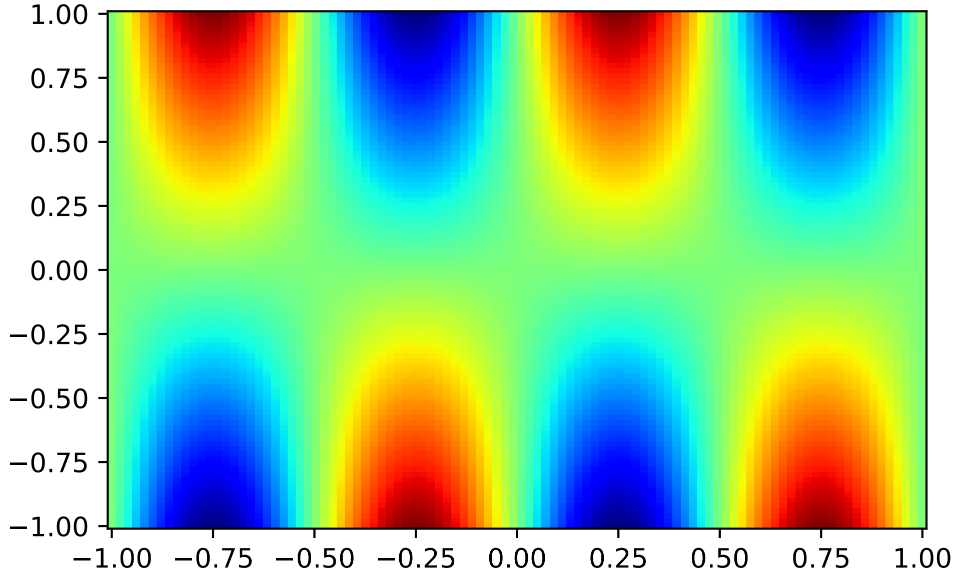
```
fig, ax = plt.subplots(subplot_kw={"projection": "3d"})
ax.plot_surface(xx, yy, zz, cmap=cm.jet)
plt.show()
```

Now try to plot the same data but using a density plot, you can make a density plot as follows:

```
fig, ax = plt.subplots()
ax.pcolormesh(xx, yy, zz, cmap=cm.jet)
# If you want to have equal aspect ratio:
ax.set_aspect('equal')
plt.show()
```

The output for the plots should be similar as below:



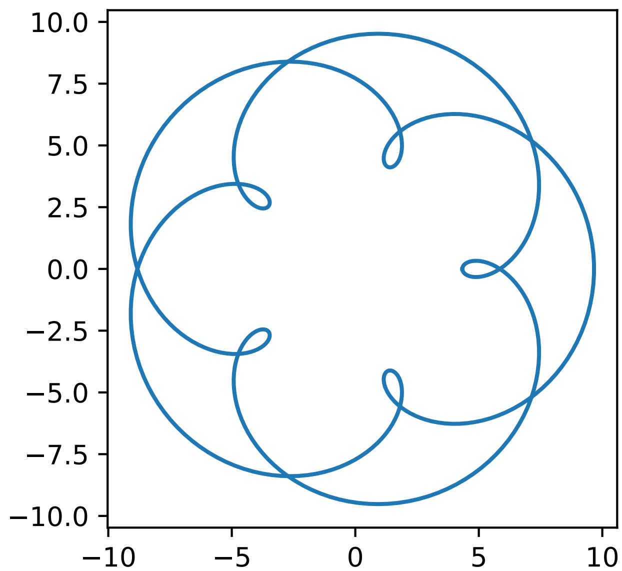


3. Plotting a parameteric curve in 2D

For the data of a parametric curve in 2D we can use an array with 2 columns for the x, and y coordinates (or two separate arrays for the two different coordinates). The (x,y) coordinates can be function of a single parameter t. Try to plot the following epitrochoid curve:

$$(x,\, y) = \begin{cases} x = (R + r)\cos(t) - d\,\cos\left(\left(\frac{R}{r} + 1\right)\, t\right) \\ y = (R + r)\sin(t) - d\,\sin\left(\left(\frac{R}{r} + 1\right)\, t\right) \end{cases}$$

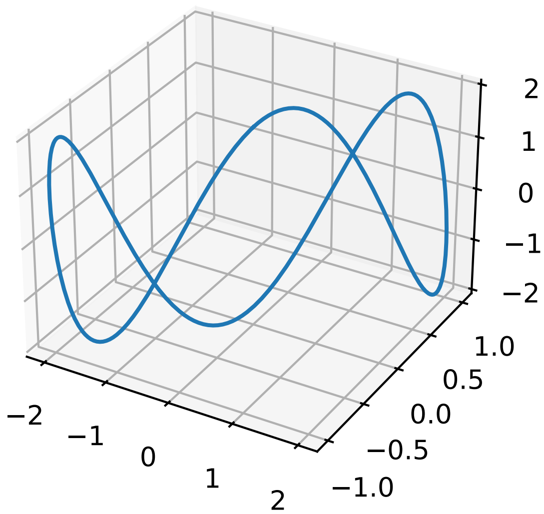
An epitrochoid curve is defined by a disc with radius r rolling over another circular disc with radius R, and tracing a point attached on the first disc at distance d from its center, see also the Wikipedia entry on epitrochoid curves. Try to plot an epitrochoid with $R = 5$, $r = 2$, and $d = 8/3$. What happens if you change the radii? How should you choose your interval for t?



4. Plotting a parameteric curve in 3D

For the data of a parametric curve in 3D we can use an array with 3 columns for the x, y, and z coordinates (or three separate arrays for the three different coordinates). The (x,y,z) coordinates can be function of a single parameter t. Try to plot the following curve:

$$(x, y, z) = (2\sin(2\pi t),\ \sin(2\pi t),\ 2\sin(6\pi t + \pi/3))$$



5: Solve a system of equations

Solve the following system of equations in x, y, and z:

$$\begin{cases} x + y = 0 \\ x + y + z = 5 \\ 2x - z = -2 \end{cases}$$

by converting it to a matrix equation:

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$$

And then multiplying both sides of the equation by the inverse matrix from the left.

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$$

Thereby calculate and print the values for x, y, and z. Verify by hand whether this is indeed a solution.