FINITE ELEMENT ANALYSIS OF SHEAR-CRITICAL REINFORCED CONCRETE BEAMS UNDER IMPACT LOADING

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Abstract

The analysis and design of reinforced concrete (RC) members for impact loads have long been of interest to many researchers, mostly due to military applications such as the fortification against projectiles. Numerous studies on flexure-critical members have been conducted, and reliable analytical methods have been developed for the analysis and design of such members for impacts. However, due to a lack of complete understanding of shear mechanisms under high dynamic conditions, research on shear-critical members heavily relies on experimental studies. These, in turn, are expensive to conduct for increasingly popular, civil applications. In this study, the finite element method is used to simulate the impact loading on shear-critical members, which provides an alternative to experimental approaches. The computer program developed for this purpose is based on the modified compression field theory, with a smeared-crack approach for modeling reinforced concrete, and high strain rate effects in concrete and reinforcing steel are incorporated by using dynamic increase factors as recommended by Comité Euro-International du Béton (CEB). The results of impact tests conducted by Kishi et al. (2002) on simply supported shear-critical RC beams are used to provide a comparison with the computer results. The results show that the computer analyses, with a reasonable level of accuracy, successfully predict the peak mid-span displacements and the time when the peak displacements occur during the loading, as well as the residual displacements, the damage state, and the crack pattern after the impact. The effects of various analyses parameters, such as mass distribution, mesh size, time-step length, and damping ratio are investigated and recommendations are made for selecting these parameters. This study shows that the finite element method, using modified compression field theory approach, can be used satisfactorily for predicting the behaviour of shear-critical reinforced concrete beams under impact loading.

Keywords: impact loads, shear-critical reinforced concrete beams, finite element modeling.

1. Introduction

The analysis and design of reinforced concrete (RC) members for impact loads have long been of interest to many researchers. Depending on the nature of the impact, the impacted structure may respond in several ways: 1) it may suffer local damage only, dissipating the majority of the impact energy at or around the impact zone; 2) it may respond to the impact globally through the bending and deformation of the whole reinforced concrete member; 3) it may respond in such a way that it suffers a combination of both local and global damage. Both the military and the nuclear power industry have been mostly interested in the local response of the reinforced concrete structures, due to the types of design impact loads on the structures they were dealing with. Therefore, there is a substantial amount of experimental and numerical work dealing with the local
damage of reinforced concrete members due to impact loads [1]. Research in global response, on the other hand, has progressed slower. Several researchers have carried out impact tests on simply supported beams and developed numerical and analytical methods to model the basic behaviour of these members under impact loading [2-7]. However, methods developed in most of these studies are either strictly restricted to the type and conditions of the problem investigated, or they require the use of complex and highly computationally demanding numerical methods. There has been some success in predicting the response of flexural-critical members with these methods. However, more complicated problems, such those involving shear-critical members, still heavily rely on experimental studies due to a lack of complete understanding of shear mechanisms under high dynamic conditions. Experimental studies are expensive and time-consuming to conduct for increasingly common civil applications. Therefore, reliable tools are needed for the analysis and design of such members.

In this study, an attempt has been made to model shear-critical reinforced concrete members under impact loading, using a finite element method (FEM). This proposed method is applied to the shear-critical beams tested by Kishi et al. [7]. Analysis results of these beams are presented and compared with the reported test results. It should be noted that the methodology presented here is still under development and therefore, the results are preliminary.

2. Finite element program

The finite element program VecTor2, used in this study [8], was developed at the University of Toronto for the static analysis of reinforced concrete members. The program uses two-dimensional, 4-node rectangular or 3-node triangular elements for modeling reinforced concrete. Reinforcement is modeled either as smeared in concrete or with truss bar elements. VecTor2 uses modified compression field theory (MCFT) with a rotating smeared-crack approach for modeling reinforced concrete. Constitutive relations and details of MCFT formulations are given elsewhere [9]. Dynamic increase factors as recommended by Comité Euro-International du Béton (CEB) [10] are incorporated with these constitutive relations to account for the high strain rate effects in concrete and reinforcement.

VecTor2 was modified and dynamic analysis algorithms were added for this study. Newmark’s implicit constant acceleration method [11] was implemented for numerical integration of equation of motion. In the analysis algorithm developed, the masses of elements are distributed equally to the nodes as lumped masses, leading to a diagonal mass matrix. Damping in the system is introduced with Rayleigh’s method [12]. In order to implement Rayleigh’s method, an eigen-solver algorithm was added, which calculates the periods of the dynamic modes of the structure. The initial stiffness matrix of the structure is used to calculate the damping matrix.

3. Test of shear-critical RC beams by Kishi et al.

Kishi et al. [7] carried out a test program to establish a rational impact-resistant design procedure for shear-critical RC beams. They tested 27 simply supported rectangular RC beams containing no shear reinforcement. The cross-sectional dimensions of the beams were kept constant at 150 x 250 mm, whereas the span length and main rebar’s diameter were varied as shown in Figure 1. The average concrete compressive strength was 33 MPa, and the reinforcement yield strength was 393 MPa.

![Figure 1. Dimensions of test beams [7]](image)

Impact loads were applied at the mid-spans of the beams by dropping a free-falling 300 kg steel weight. The impact velocity of the weight was varied as 1 m/s, 3 m/s, 4 m/s and 5 m/s. During the test, the impact force, the reaction force, and the mid-span displacement were measured and recorded. Crack patterns of the beams after the impact were also sketched and reported.

In this study, beam B36 is considered for the FEM analysis. This beam had a 1500 mm clear...
span and a 0.80% longitudinal reinforcement ratio.

4. Finite element model and analyses

The beam was modeled with VecTor2. 580 rectangular plane stress elements were used to represent the concrete and the drop-weight, and 48 truss elements are used for modeling the reinforcement (Figure 2). The model included 646 nodes. Taking advantage of the symmetry, only half of the beam was modeled. All nodes at the centerline of the beam were restrained against translations in the x-direction, whereas the node at the support was restrained against y-direction displacement. Two strong elements, with higher stiffness compared to the concrete elements, were placed at the support point to represent the support bearing plate.

![Figure 2. Finite element model of B36](image)

The drop-weight was modeled using two rigid rectangular elements (Figure 3). These elements were connected to the structure by three compression-only truss bars, so that when the drop-weight bounced back, it would not pull the beam up.

No information was given about the mass of the beam, therefore the reinforced concrete density was assumed as 2400 kg/m³ and the mass of the concrete elements was distributed equally to the four nodes of each element. Half of the weight of the drop-weight was assigned to the six nodes of the elements representing it. Impact loading was introduced by assigning the impact velocity of the drop-weight to its nodes as initial velocity.

![Figure 3. Modeling the drop-weight](image)

As mentioned earlier, the Rayleigh method was used to introduce damping. In this method, the user assigns damping as percentage of the critical damping to two selected modes of vibration. For this particular specimen, since nonlinearities in the structure represent the dominant damping behaviour, the first mode of vibration was assigned a very small damping factor (0.1% of critical), whereas the second mode was assigned 10% of critical damping. Damping for the remaining modes were calculated according to this selection, resulting in over-damping for most of the higher modes and thus ensuring numerical stability of the solution.

The time-step length for numerical integration was selected as 0.0001 seconds. To ensure accuracy of the solutions, the time-step length was halved and the analyses were repeated.
Comparison of the two solutions revealed that there were no differences. Therefore, it was concluded that the time-step chosen was sufficiently short for an accurate solution.

5. Finite element analyses results

Analyses were repeated for the impact velocities of 1 m/s, 3 m/s and 4 m/s. The 5 m/s case is not included in the analyses, since the test beam was reported to have split into three parts due to the development of very wide cracks, and VecTor2 was not capable of modeling post-failure response. Mid-span displacements, as measured during the test and found with FEM analyses, are compared in Figure 4. Figure 5 presents the same comparison for the support reactions. Note that only the first 0.040 seconds of the response is included in the comparison for support reactions, since the remaining part is virtually constant. Crack patterns are also presented and compared in Figure 6.

When displacements obtained from the analyses and the tests are compared, in general, the agreement is good. In all impact velocities, VecTor2 successfully estimated the initial stiffness of the beams and the time at which the peak displacements occurred. However, except for 1 m/s case, the peak displacement was underestimated by about 20%. For 1 m/s case, where the beam remained almost undamaged, the peak displacement was estimated with almost perfect accuracy. On the other hand, the analysis showed a vibration in the beam after the peak displacement, which was not observed in test data. The residual displacement for 3 m/s case was underestimated by 22%. But for 4 m/s case, VecTor2 estimated the residual displacement accurately. Considering that the analyses predicted the pre-peak response accurately and that the general shape of the displacement histories as observed in the tests and in the analyses match well, discrepancies at the peak displacements and the subsequent response can be partially attributed to the flexibility in the test setup, such as the support conditions. The support of the beam was modeled as rigid since the details were unknown to the authors, but a support with finite stiffness would increase the observed displacements on the beam. In addition, the period of vibration found in the analyses was also shorter than the one observed in the tests, whereas a flexible support would lengthen it in the analyses, bringing the test results and the analyses results to a closer match.

For the reactions at the support, in general there was good agreement between the test and analysis results. In all cases, the VecTor2 analyses reached the peak reaction forces earlier than in the test data, therefore showing a steeper climb before the peak. The peak values of the reaction forces were estimated to within 3% error on average. Only in the 1 m/s case was the peak force significantly underestimated. For the shape of the reaction force history, the analysis results showed a plateau of sustained load similar to that observed in the test. However, this plateau was shorter in the analyses and it started to decline earlier. This behaviour can be attributed to the actual test conditions, such as the stiffness of the drop-weight and local damage under the impact point. Since the details of the drop-weight are not known to the authors, it was modeled as rigid, whereas a flexible drop-weight or penetration of the drop-weight in concrete would cause a longer plateau of sustained load at the support.

The VecTor2 analyses estimated the crack patterns with reasonable accuracy as well. For the 1 m/s case, the analysis indicated flexural cracks around the mid-span, similar to the reported pattern. On the other hand, some minor diagonal shear cracks observed in the analysis were not observed in the test. For the 3 m/s case, larger flexural cracks at the mid-span and some inclined cracks close to the support were successfully predicted. For 4 m/s case, VecTor2 successfully predicted the large flexural and shear cracks at the mid-span, as well as the minor flexural cracks occurring above the support point.

In order to investigate the effects of damping, the analyses were repeated by using several different parameters for the modes of vibration selected and the damping factors assigned to these modes. The results of these analyses are not presented here for brevity. In general, it was found that the damping factor assigned to the first mode affects the results most. However, different damping factors assigned to the first mode did not result in any significant change in the response as long as it was kept under 1% of critical. Damping factors higher than 1% of critical reduced the displacement response and thus increased the difference between the test and the analysis results. On the other hand, it was necessary to assign a high damping ratio for the higher modes of vibration since low damping ratios for the higher modes caused numerical instability. 10% of critical damping assigned to the second mode was found to be sufficient in this particular case to ensure numerical stability, and values higher than 10% did not result any significant change in the response.
Figure 4. Comparison of analysis results to test results for mid-span displacement
Figure 5. Comparison of analysis results to test results for support reaction
Figure 6. Comparison of analysis results to test results for crack patterns
One of the main advantages of the method found was the shortness of the time required to complete an analysis. Each analysis carried out for this study took less than 2 hours on average, with an Intel Pentium IV 2GHz computer. Similar analyses with complex three dimensional FEM models are known to take much longer time to complete. Although it increased the analysis time, a reduction in the length of the time-step did not seem to have a severe adverse effect on the time spent, since the iterations converged much faster with a shorter time-step.

6. Conclusion

A two-dimensional finite element model was developed for the analysis of shear-critical RC beams under impact loading. Computer simulations were subsequently carried out for beams tested by Kishi et al. Although the method is still under development, comparisons of the mid-span displacements and support reactions between the analysis results and test results showed good agreement. Modeling of the impact weight with compression-only truss bars was found to be a simple and accurate method, eliminating the need for an impact force history or solution of complex contact problems for modeling impact on structures. The Rayleigh method, as used with the parameters recommended, also proved to be an effective way to introduce damping in the structural model. The proposed method is also computationally efficient compared to more complex three-dimensional finite element models. Therefore, it is a promising tool for the analysis and design of shear-critical RC beams.

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References