AF15 Flow Around a Bend Apparatus
Introduction

The engineer is frequently presented with problems of flow contained within tubes and ducts. Such flows may be classified as internal flows to distinguish them from flows over bodies such as aerofoils, called external flows. It is sometimes necessary to shape a duct in such a way that particular requirements are met. For example, it may be necessary to change the shape of cross-section from square to rectangular with a small loss of total pressure, or it may be required to form a bend in such a way that the distribution of velocity at the exit is as nearly uniform as it can be made.

Due to the presence of boundary layers along the duct walls, the fluid mechanics of such flows are sometimes extremely complicated. Separation may be produced where the pressure rises in the direction of flow, as illustrated in Figure 8.1(a).

![Figure 8.1(a) Schematic Representation of Separating and Reattaching Flow in a Duct](image)

This shows a duct of increasing cross-sectional area in which the flow decelerates with an accompanying rise of pressure. Separation of flow from one wall is shown, followed by a region of severe turbulence in which there is mixing between the main flow and the region of recirculating flow (often called the separation bubble). The turbulent mixing leads to loss of total pressure, the size of this loss depending on the
extent of the separation. It should be emphasised that the flow shown in the figure is schematic only.

The separation line is rarely steady. The size of the separated zone often fluctuates violently, and in some cases the separation is intermittent. Separation might occur over more than one surface and would not normally take place uniformly over one side as shown for illustrative purposes in the diagram. A further complication arises from secondary flow which is again due to boundary layer effects.

Figure 8.1(b) Formation of Secondary Flow in a Bend of a Duct

Figure 8.1(b) shows one example of the formation of a secondary flow in a gently-curving duct of rectangular cross-section. The curvature of the flow is accompanied by a pressure gradient which rises across the section from the inner to the outer wall. The pressure gradient extends over the whole section, so that the boundary layers on the upper and lower walls are subjected to the same pressure gradient as the main flow. But because the streaming velocity in the boundary layer is less than in the main part of the flow, the curvature of the streamlines in the boundary layer is more severe, as indicated. This gives rise to a net inward-directed flow adjacent to the upper and lower walls, which sets up a secondary flow in the form of a double rotation, superimposed on the main stream. The motion emerging from the curve in the duct is therefore a pair of contra-rotating spirals, the strength of which depends on the amount of curvature and on the thickness of the boundary layer.
In this experiment we investigate the flow around a $90^\circ$ bend in a duct of rectangular section, using pressure tappings along the walls to establish the pressure distributions. Figure 8.2 indicates flow approaching a bend with a uniform velocity $U$. Within the bend we shall assume a free vortex distribution of velocity, given by

$$u = \frac{C}{r}$$

(8-1)

where $u$ is the streaming velocity at radius $r$ from the centre of curvature of the bend. Separation and secondary flow will be neglected. The constant $C$ may be found by applying the equation of continuity as follows:

$$Q = Ub(r_2 - r_1) = b \int_{r_1}^{r_2} u \, dr$$

(8-2)

where $b$ is the width of the section of the duct. Substituting for $u$ from Equation (8-1) and performing the integration leads to the result
\[ C = U \frac{r_2 - r_1}{\ln (r_2 / r_1)} \]  

(8-3)

so the velocity distribution is, in dimensionless form,

\[ \frac{u}{U} = \frac{r_2 - r_1}{r \ln (r_2 / r_1)} \]  

(8-4)

The corresponding pressure distribution may be found by assuming that Bernoulli’s equation may be applied between the upstream section and a section within the bend as follows:

\[ p_o + \frac{1}{2} \rho U^2 = p + \frac{1}{2} \rho u^2 \]  

(8-5)

where \( p_o \) is the static pressure upstream and \( p \) is the pressure at radius \( r \) in the bend. It is convenient to express \( p \) in the form of a dimensionless pressure coefficient \( c_p \) where

\[ c_p = \frac{p - p_o}{\frac{1}{2} \rho U^2} \]  

(8-6)

From Equation (8-5) this may be written

\[ c_p = 1 - \frac{u^2}{U^2} \]  

(8-7)

which may be evaluated for any radius \( r \) by substituting the appropriate value of \( u/U \) obtained from Equation (8-4). A comparison with measured values of \( c_p \) may be made as shown in Table 8.2.

**Description of Apparatus**

Figure 8.3 shows the dimensions of the bend and the positions of the pressure tappings. There is a reference pressure tapping 0 on the side face near the entry, and three sets of tappings; one set of 10 along the outer curved wall, one set of 10 along
the inner curved wall and a set of 9 along a radius of the bend. Air from the contraction section is blown along the duct and is exhausted to atmosphere.

![Figure 8.3 Dimensions of the Bend and the Positions of Pressure Tappings](image)

**Experimental Procedure and Results**

The pressure tappings along the outer wall, the reference tapping 0 and the pressure tapping in the airbox are all connected to the manometer. The air speed is adjusted to a value slightly below the maximum, as indicated by the airbox pressure, and the pressures are recorded. (The setting of air speed slightly below the maximum is to ensure that the same setting may be repeated in later tests). The tappings on the inner wall are then connected in place of the ones on the outer wall. The airbox pressure is adjusted to the previous value and a further set of readings are recorded. Finally the procedure is repeated with the third set of pressure tappings. In Table 8.1 the pressures $p$ are recorded relative to an atmospheric datum and the pressure coefficients $c_p$ are calculated from Equation (8-6).
Airbox pressure \( P = 630 \text{ N/m}^2 \)

Reference tapping pressure \( p_o = 80 \text{ N/m}^2 \)

Velocity pressure of uniform flow along duct \( P - p_o = \frac{1}{2} \rho U^2 = 550 \text{ N/m}^2 \)

<table>
<thead>
<tr>
<th>Tapping No.</th>
<th>Outer Wall</th>
<th>Inner Wall</th>
<th>Radial Section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p ) (N/m²)</td>
<td>( c_p )</td>
<td>( p ) (N/m²)</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
<td>0.02</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>145</td>
<td>0.12</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>205</td>
<td>0.23</td>
<td>-240</td>
</tr>
<tr>
<td>4</td>
<td>325</td>
<td>0.45</td>
<td>-415</td>
</tr>
<tr>
<td>5</td>
<td>330</td>
<td>0.45</td>
<td>-440</td>
</tr>
<tr>
<td>6</td>
<td>320</td>
<td>0.44</td>
<td>-395</td>
</tr>
<tr>
<td>7</td>
<td>240</td>
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<td>-255</td>
</tr>
<tr>
<td>8</td>
<td>85</td>
<td>0.01</td>
<td>-25</td>
</tr>
<tr>
<td>9</td>
<td>35</td>
<td>-0.08</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>-0.15</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 8.1 Measured Pressures and Pressure Coefficients*

From Figure 8.3 the inner and outer surfaces of the bend have radii

\[
r_1 = 50 \text{ mm}
\]

\[
r_2 = 100 \text{ mm}
\]

From Equation (8-4) the velocity distribution across the section according to the free vortex assumption is therefore

\[
\frac{u}{U} = \frac{50}{r \ln(2)} = \frac{72.1}{r}
\]

where \( r \) is expressed in mm. In Table 8.2 we calculate this ratio and the corresponding value of \( c_p \) from Equation (8-7) for a number of values of \( r \).
Table 8.2  Calculated Pressure Coefficients

<table>
<thead>
<tr>
<th>r (mm)</th>
<th>( \frac{u}{U} )</th>
<th>( c_p )</th>
</tr>
</thead>
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<tr>
<td>50</td>
<td>1.443</td>
<td>-1.081</td>
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<tr>
<td>55</td>
<td>1.312</td>
<td>-0.720</td>
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<tr>
<td>60</td>
<td>1.202</td>
<td>-0.445</td>
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<tr>
<td>65</td>
<td>1.110</td>
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<td>1.030</td>
<td>-0.062</td>
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<tr>
<td>75</td>
<td>0.962</td>
<td>0.075</td>
</tr>
<tr>
<td>80</td>
<td>0.902</td>
<td>0.187</td>
</tr>
<tr>
<td>85</td>
<td>0.849</td>
<td>0.280</td>
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<tr>
<td>90</td>
<td>0.801</td>
<td>0.358</td>
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<tr>
<td>95</td>
<td>0.759</td>
<td>0.423</td>
</tr>
<tr>
<td>100</td>
<td>0.721</td>
<td>0.480</td>
</tr>
</tbody>
</table>

Figure 8.4 shows the distribution of measured pressure coefficient over the curved walls and compares the measured and calculated values across the radial section. It may be seen that the pressure across the inlet section is nearly uniform. As the flow approaches the bend, the pressure on the inner wall falls rapidly and on the outer wall...
rises rapidly to values which remain substantially constant round most of the curve. This indicates that the curvature of the flow is also likely to be reasonably constant. The distribution of \( c_p \) over the radial section follows the calculated curve quite closely, indicating that the assumption of a free vortex velocity distribution made in Equation (8-1), together with the assumption that Bernoulli’s equation applies to the flow, give a fairly accurate distribution of the pressure field. The measured pressure distribution varies rather less steeply than calculated, indicating a vortex strength \( C \) somewhat less than that given by Equation (8-3).

Downstream of the bend, the wall pressures readjust until at the duct exit the pressure is constant across the section. It is, however, a little lower than the reference pressure at the inlet, and this difference represents a pressure loss round the bend. It is convenient to express this loss \( \Delta p \) in terms of the velocity pressure \( \frac{1}{2} \rho U^2 \) in the uniform approaching flow by the expression

\[
K = \frac{\Delta p}{\frac{1}{2} \rho U^2}
\]

where \( K \) is the dimensionless loss coefficient. In this case we find, from the change in \( c_p \) from the inlet to the outlet sections, the value:

\[
K = 0.15
\]

Conclusion

The distribution of pressure over the curved walls of a 90° bend of rectangular section has been established by pressure plotting. The pressure coefficient is negative and almost constant round the inner wall, and positive and almost constant round the outer wall. Across the 45° cross-section the pressure distribution may be predicted with reasonable accuracy by assuming free-vortex velocity distribution over the section. The value of loss coefficient \( K \) is 0.15 for this bend.