A COMPARATIVE ASSESSMENT OF THE PISARENKO HARMONIC DECOMPOSITION TYPE TONE DETECTOR

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ABSTRACT

In this paper, we consider the performance of the Pisarenko harmonic decomposition method which is a special case of MUSIC algorithm for a single tone detection problem in additive white Gaussian noise environment. The detection process consists of comparing Pisarenko harmonic decomposition frequency estimate (PISFE) to the known tone frequency, and declare "tone present" if the two values are sufficiently close. We derive the conditional probability density function of the PISFE under both detection hypotheses and use them to evaluate the performance of the PISFE detector. Simulation and numerical results allow the choice of an optimum threshold for a frequency detection band setting given a false alarm rate P_F. The receiver operating characteristics of the PISFE is obtained computing the corresponding detection probability P_D. It is shown that the PISFE detector is a constant false alarm rate (CFAR) detector with respect to SNR for fixed tone frequency, but not a CFAR detector with respect to tone frequency for the given problem. The performance of the PISFE type detector is compared with both matched filter (MF) and autoregressive (AR) detectors and the analysis is supported by an extensive simulation work. Applications to FSK tone detection problem is discussed briefly.

I. INTRODUCTION

Detecting the presence of sinusoidal tones in noise and estimating their parameters is a problem commonly encountered in such diverse areas as communications systems, geophysics, vibration analysis, acoustics and biomedical applications. Typically, however, the limited number of discrete time observations and the low signal to noise ratio (SNR) handicap the tone detection problem .

The optimal linear processor for the tone detection problem in the presence of additive noise is the well-known matched filter which has been in use for many years [1,2]. For this classical method, the tone is detected by comparing the output of the matched filter at the decision instant with a prespecified threshold. If the matched filter output exceeds the threshold, then the tone is declared to be present; otherwise the decision is the ``noise only" (null hypothesis) case. However the matched filter does not prove to be a robust solution in that the detection performance depends upon the exact knowledge of the waveform and is also sensitive to several effects like interfering signals, impulsive noise, and nonlinear distortion.

As a consequence many alternative techniques to matched filter detection have been proposed in the literature to solve the tone detection problem more efficiently [3,4,5]. Tone detectors based on parametric spectrum estimation have received significant attention mostly due to their robustness features [6-13,25,26]. In this class of methods, the spectrum is estimated through some parametric approach, such as autoregressive method, and the peak of the resulting spectrum is compared against a threshold. Also the parametric expression of the AR spectrum (e.g., the AR polynomial) is solved for the zeroes and the detection is based on testing whether these zeroes fall in certain predetermined intervals. If a peak or (a zero radius) is found to exceed a preselected detection threshold, then a tone is declared to be present ; otherwise the decision is the null hypothesis.

In another technique, applicable also to multitone situations, the detection problem is based on the estimates of the tone frequencies and on testing whether these estimates fall in predetermined intervals [13]. A few of typical parametric frequency estimators that can be used for tone frequency estimation are Maximum Likelihood (ML), Maximum Entropy (ME), Prony and Pisarenko methods [12,14,23-24]. The ML technique is statistically the most efficient method [21]. However in a multi-tone environment (e.g., FSK or multifrequency coding (MFC) detector applications), the ML method becomes computationally burdensome as one has to go through a multidimensional nonlinear optimization. Standard methods for FM demodulation such as limiter-discriminators, zero crossing detectors, and differential or product detectors can also be used to estimate the frequency of noisy sinusoids [22]. Finally frequency estimation from discrete-time observations using autoregressive (AR) models has been studied under the names of linear prediction, maximum entropy estimation, and maximum likelihood whitening filter [12,14, 23-24].

These techniques except the ML, are suboptimal although they are computationally attractive. The merits and difficulties of various estimation techniques are discussed in detail in [12]. Previous studies have indicated that the tone detection performance of parametric spectrum based methods are inferior to matched filter performance and even to that of the periodogram in additive white Gaussian noise [5,6,8-10,13]. However these tone detectors exhibit a robust behaviour. Also it is well known that the periodogram is optimal only for one cisoid in additive white Gaussian noise, otherwise it is suboptimal.

A robust tone detector based upon AR spectrum peak thresholding or pole radius thresholding and AR frequency estimation (ARFE) have been analyzed in [8-11, 13]. In this work we investigate a detector, called PISFE, based upon Pisarenko frequency estimation, where frequency estimates can be obtained by solving for the zeroes of the polynomial whose cofficients are the elements of the minimum eigenvector of the autocorrelation matrix. This PISFE based tone detector is sensitive against constant Doppler shifts like Kay's robust detector but in contrast has the advantage that it can be used for multitone situations, e.g., FSK, MFC or dual tone multifrequency receiver (DTMF) applications.

We limit the analysis to a single tone in additive white noise since for multi-tones the analysis becomes tedious. The properties of the PISFE detector are given in section 2. The performance of the PISFE detector is discussed in sections 3 and 4 using the expressions of the conditional probability density function of the estimated frequency. An FSK detector based on PISFE is considered in section 5.

II. PISFE DETECTOR AND ITS PROPERTIES

In this section, we investigate a tone detector based on the PISFE. The signal under consideration is a single sinusoid sampled uniformly in the presence of additive white Gaussian noise, i.e.,

$$x_k = \sqrt{2A}\cos[\omega\, kT + \phi] + n_k \quad k = 1,2,.......N \qquad (1)$$

where A is a non-random amplitude, ϕ is a random phase angle uniformly distributed on $(-\pi,\pi)$, ω is the tone frequency, T is the sampling period, $\{n_k\}$ is a real white Gaussian noise sample sequence with zero mean and power σ_n^2 and N is the number of data samples.

For a single tone case, the PISFE frequency estimate is obtained in terms of the autocorrelation coefficients by assuming $T=1$ as follows [14, 16]:

$$\hat{\omega} = \arccos(\psi) \qquad (2)$$

where

$$\psi = \frac{r(2) + \sqrt{r^2(2) + 8r^2(1)}}{4r(1)}$$

and $r(k)$ denotes the k'th autocorrelation coefficient of the input samples and is estimated (calculated) as:

$$r(k) = \frac{1}{N}\sum_{i=1}^{N-k} x_i x_{i+k}$$

From Eq. (2), it can be seen that the estimated tone frequency depends on the autocorrelation coefficient at lag 1 and 2 and does not depend on the signal waveform. For a single tone case, this result is very practical related to the computational complexity.

The proposed tone detection scheme consists of obtaining the PISFE estimate via Eq. (2) and testing whether its value lies in a preselected band. In other words, our tone detection by estimation scheme is applicable to dual tone multifrequency situations where tones can occur only at certain predesigned values. The performance of the PISFE detector is parameterized by the number of discrete samples N, the signal to noise ratio, and the detection band size and the tone frequency that determine both the probability of detection P_D and false alarm P_F. The hypothesis testing of "tone present" (H_1) versus "noise only" (H_0) based on a PISFE frequency estimate is as follows;

$$\begin{aligned} |\omega - \hat{\omega}| &\le \delta\omega \quad \textit{Decide for } H_1 \\ |\omega - \hat{\omega}| &> \delta\omega \quad \textit{Decide for } H_0 \end{aligned} \qquad (3)$$

where $\delta\omega$ is the preselected detection band and the tone frequency ω is one of the allowed multifrequencies.

For large number of data samples N, it can be shown that the performance of the PISFE detector is [8,13]

- dependent on the random phase angle ϕ in the tone and the tone frequency ω, but; asymptotically independent of the random phase angle ϕ i.e., as $N \to \infty$, the PISFE detector does not depend on it asymptotically.
- independent of the noise power σ_n^2 under H_0, i.e., it is a constant false alarm rate (CFAR) detector;
- independent of the signal form, provided that the signal autocorrelation function (ACF) is unchanged.

The proofs of the first and third properties of the PISFE detector given above are similar to those given by Kay in [8]. The second property, i.e. the CFAR property, follows from the fact that, in the ``noise only" case, the elements of the minimum eigenvector are independent of the ACF at lag zero and hence of the noise power. In fact, this can be offered by any parametric method based on measuring the distance between a spectral peak and a reference frequency, since the probability density function of the spectral components will not change with the noise level. Additionally, in the literature there are other standard noise level control techniques (note that we are speaking about CFAR with respect to noise level, and not with respect to noise distribution) such as using automatic gain control loops or variable thresholds with classical procedures (as the periodogram).

III. BEHAVIOR OF THE PISFE PDFS

The conditional probability density functions (pdf's) of the PISFE tone frequency $\hat{\omega}$ for both of the hypotheses are obtained from the pdf of the intermediate random variable ψ under the corresponding hypothesis either H_0 or H_1 using the transformation

$$p(\hat{\omega} \mid H_i) = \sqrt{1-\cos^2(\hat{\omega})}\, p_{\psi \mid H_i}(\cos(\hat{\omega})) \qquad (4)$$

where $i=0,1$ denotes the hypothesis number. The conditional pdf's of ψ are derived in the Appendix. The derivations are based on the Central Limit Theorem (CLT) so the pdf expressions are expected to be valid only for large N (e.g. $N > 50$). The pdf's are given for "the noise only" hypothesis, H_0, as :

$$p(\psi \mid H_0) = \frac{4K_1}{(1+A_1(\psi))A_2(\psi)}u(\psi) + \frac{4K_1}{(1-A_1(\psi))A_2(\psi)}u(-\psi) \qquad (5)$$

while for the "signal present" hypothesis H_1 one obtains :

$$p(\psi \mid H_1) = \frac{4K_1 e^{-A_4/2}}{A_2(\psi)}\left[\frac{B_1(\psi)}{(1+A_1(\psi))}u(\psi) + \frac{B_2(\psi)}{(1-A_1(\psi))}u(-\psi)\right] \qquad (6)$$

where *u(.)* represents the unit step function. The remaining definitions are given as:

$$K_1 = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}$$

$$A_0(\psi) = \frac{2\psi^2 - 1}{\psi}$$

$$A_1(\psi) = \frac{A_0(\psi)}{\sqrt{A_0^2(\psi)+8}}$$

$$A_2(\psi) = \frac{1}{(1-\rho^2)}\left[\left(\frac{A_0(\psi)}{\sigma_1}\right)^2 + \frac{1}{\sigma_2^2} - \frac{2\rho A_0(\psi)}{\sigma_1\sigma_2}\right] \quad (7)$$

$$A_3(\psi) = \frac{2\sqrt{A}}{(1-\rho^2)}\left[\frac{\rho}{\sigma_1\sigma_2}\left(\cos(2\omega) + A_0(\psi)\cos(\omega)\right) - \frac{\cos(\omega)}{\sigma_2^2} - \frac{A_0(\psi)\cos(2\omega)}{\sigma_1^2}\right]$$

$$A_4 = \frac{A}{(1-\rho^2)}\left[\left(\frac{\cos(2\omega)}{\sigma_1}\right)^2 + \left(\frac{\cos(\omega)}{\sigma_2}\right)^2 - \frac{2\rho\cos(\omega)\cos(2\omega)}{\sigma_1\sigma_2}\right]$$

$$B_1(\psi) = \left[1 + \frac{A_3(\psi)}{4}\sqrt{\frac{2\pi}{A_2(\psi)}}\exp\left(\frac{A_3^2(\psi)}{8A_2(\psi)}\right)\left(erf\left(\frac{A_3(\psi)}{\sqrt{8A_2(\psi)}}\right) - 1\right)\right]$$

$$B_2(\psi) = \left[1 + \frac{A_3(\psi)}{4}\sqrt{\frac{2\pi}{A_2(\psi)}}\exp\left(\frac{A_3^2(\psi)}{8A_2(\psi)}\right)\left(erf\left(\frac{A_3(\psi)}{\sqrt{8A_2(\psi)}}\right) + 1\right)\right]$$

with σ_1^2, σ_2^2 and ρ representing the variances and the correlation coefficient of the autocorrelation lags *r(1)* and *r(2),* respectively and erf(.) denotes the error function.

Simulations were carried out on the $p(\hat{\omega} \mid H_1)$ to verify the agreement with Eq. (6). Fig. 1 shows a plot of the $p(\hat{\omega} \mid H_1)$ and the histogram obtained from the simulations. This histogram is generated using 100.000 realizations of *x{k}*, as in Eq. (1) with *N*=50. Each frequency estimate is obtained by estimating *r(1)* and *r(2)* and then inserting these into Eq. (2). The figure depicts that the analytic expression for the pdf in Eq(6) closely matches the simulation results.

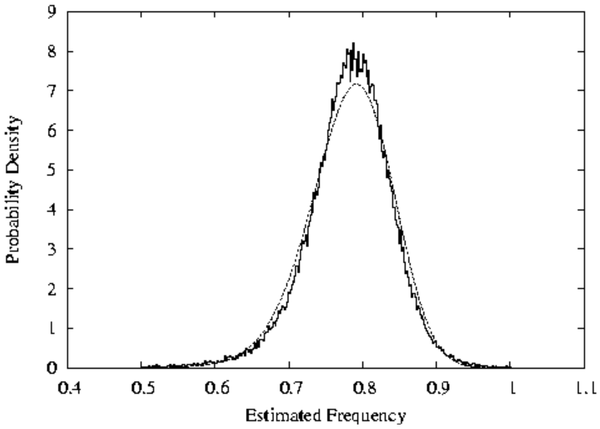

Figure 1. Pdf of the PHD frequency estimate for the H_1 hypothesis versus frequency, solid line : analytic computation, histogram lines: histogram obtained by simulation ($\omega_1=\pi/4$, N=50, μ=0dB, 100.000 noise realizations).

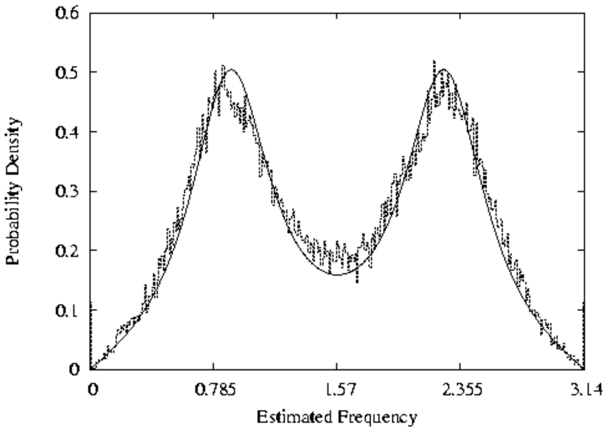

Figure 2. Pdf of the PHD frequency estimate for the H_0 hypothesis versus frequency, solid line : analytic computation, histogram lines: histogram obtained by simulation (N=50, 100.000 noise realizations)

In order to make a meaningful comparison of simulation results and Eq. (5), in addition to ``signal plus noise" case, ``noise only" case derived pdf has also been tested using simulations, as illustrated in Fig. 2. The normalized histogram of frequency estimates agrees with Eq. (5) once again.

From equations (5) and (6), one may observe the following :

- $p(\hat{\omega} \mid H_0)$ does not depend upon the noise power σ_n^2.
- $p(\hat{\omega} \mid H_1)$ does not depend upon the signal waveform but only upon its first two autocorrelation coefficients (see Appendix).

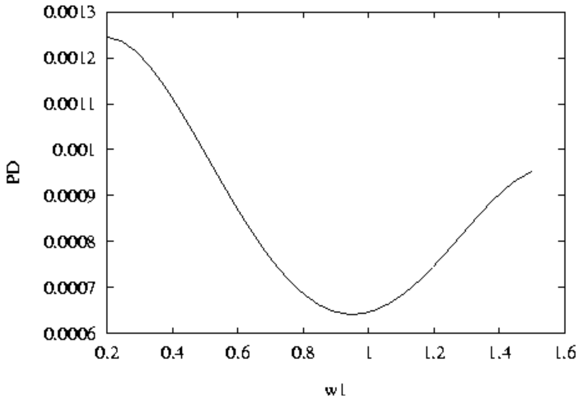

Figure 3. Variations of the tone detection performance of the PISFE detector versus tone frequency for constant probability of false alarm (P_F=1E-4, N=100, μ = 0 dB).

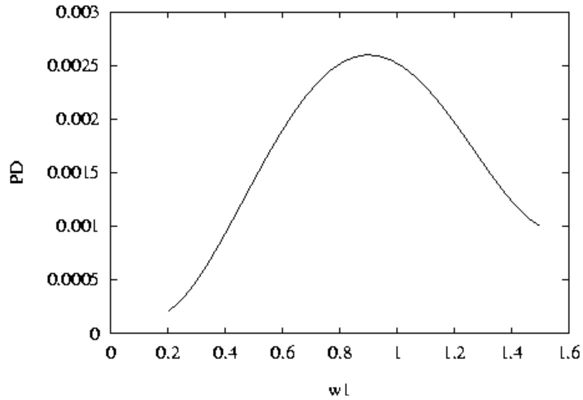

Figure 4. Variations of the tone detection performance of the PISFE detector versus tone frequency for constant detection band ($\delta\omega$=1.6E-4, N=100, μ = 0 dB).

IV. ANALYSIS OF THE TONE DETECTION PERFORMANCE OF THE PISFE DETECTOR

Like any other detector, the performance of the PISFE tone detector can also be investigated through ROCs, that is by plotting the P_D versus P_F for varying values of *N* and μ.

The probability of false alarm P_F, corresponding to the case where a tone is declared to be present when only noise is received, is a function of the tone frequency and the interval size $\delta\omega$ and hence P_F is simply given as:

$$P_F = \int_{\omega-\delta\omega}^{\omega+\delta\omega} p(\hat{\omega} \mid H_0) d\hat{\omega} \tag{8}$$

while one obtains for P_D

$$P_D = \int_{\omega-\delta\omega}^{\omega+\delta\omega} p(\hat{\omega} \mid H_1) d\hat{\omega} \tag{9}$$

Hence, it can be expected that the detection performance will depend on the tone frequency. The detection interval $\delta\omega$ can be solved for a given probability of false alarm P_F and is found to be independent of the background noise power σ_n^2 using Eq. (5).

Recall that since $p(\hat{\omega} \mid H_0)$ is not a uniformly distributed pdf, it can be seen that for each constant $\delta\omega$, a different value of P_F is obtained for different tone frequency ω. Also it can be seen that decreasing $\delta\omega$ is equivalent to decreasing P_F and also the region where one declares H_1 or "tone is present". Hence $\delta\omega$ is decreased until one obtains the smallest possible value of P_F.

The detection performance of the PISFE detector is evaluated for varying ω in two distinct cases :

- P_F is held constant; hence the detection band is computed for each ω.
- $\delta\omega$ is held constant; hence P_F is different for each tone frequency ω.

A. P_F constant = 10^{-4}

Since $p(\hat{\omega} \mid H_0)$ is not constant over 0 to 2π, the bandwidth $\delta\omega$ which will result in $P_F = 10^{-4}$ depends on ω. Also since the $p(\hat{\omega} \mid H_0)$ reaches their peak value near $\omega T=\pi/4$, one can expect that $\delta\omega$ is smaller for frequencies around $\omega T=\pi/4$. The bands that give $P_F = 10^{-4}$ are given in Table 1 for SNR= 0 dB and *N*=100. The plots of P_D for different values of ω are given in Fig. 3. It can be seen that the detection performance reaches its lowest value near $\omega T=\pi/4$. At first sight, this result may appear as suprising since the detection performance is expected to be high near $\omega T=\pi/4$ (as it will be shown later, this is true only when the detection band $\delta\omega$ is held constant). The reason behind this observation is that $\delta\omega$ is smaller for frequencies near to $\omega T=\pi/4$ because the $p(\hat{\omega} \mid H_0)$ reaches its peak value near this frequency ; hence the smaller the $\delta\omega$ is, the lower the probability of detection P_D.

B. Detection band $\delta\omega$ is held constant

The detection performance for a constant preselected $\delta\omega$ = 1.6 x 10^{-4} rad/sec is given for various values of ω in Fig. 4 for SNR= 0 dB and N= 100.

Note that P_D reaches its peak near $\omega T=\pi/4$. This is expected because of the fact that, the variance of the estimate takes its minimum value at this frequency [16] ; hence for a preselected $\delta\omega$, the hypothesis testing results in a higher P_D.

The detection probabilities of the PISFE, AR and the periodogram type detectors are plotted versus SNR, with *N* as the parameter, in Figures 5 and 6 for $P_F = 10^{-4}$ and $\omega T=\pi/4$. As in [8,13,25,26], the bank of matched filters detector is referred to as the periodogram detector. The simulation results indicate that the performance of the PISFE detector still remains poor compared to the AR and periodogram detectors. Like the other detectors PISFE detector consistently provides a higher detection probability for higher SNR values. Note that the performance of the PISFE detector seems to catch up with those of the AR and periodogram detectors with increasing number of data samples, *N*.

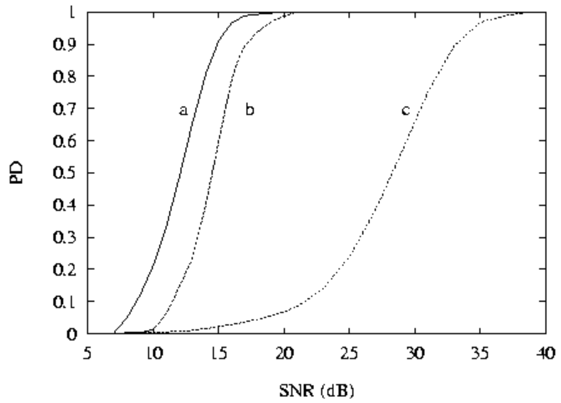

Figure 5. Comparison of the tone detection performance of PISFE, AR and periodogram type detectors, a: Periodogram b: AR c: PISFE ($\omega = \pi/4$, P_F = 1E-4, N=100, 500 noise realizations).

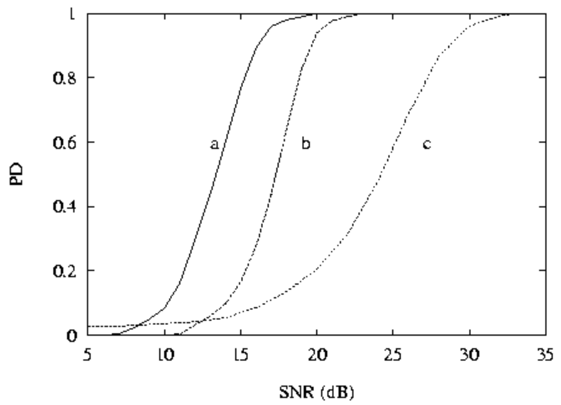

Figure 6. Comparison of the tone detection performance of PISFE, AR and periodogram type detectors, a: Periodogram b: AR c: PISFE ($\omega = \pi/4$, P_F = 1E-4, N=1000, 500 noise realizations).

V. ANALYSIS OF FSK TONE DETECTION

The PISFE detection scheme can be extended in a straightforward manner to multitone cases, e.g.,to non-coherent FSK (NCFSK) detection. The observed sequence can be expressed as

$$x_k = \begin{cases} \sqrt{2}\, A_1 \cos[\omega_1 kT + \phi] + n_k & for\, H_0 \\ \sqrt{2}\, A_2 \cos[\omega_2 kT + \phi] + n_k & for\, H_1 \quad k = 1,2,....N \end{cases} \tag{10}$$

where it is assumed, without loss of generality, $\omega_1 > \omega_2$ and ϕ is an arbitrary phase defined earlier. The PISFE detector for FSK type modulated signals with non-equal powers uses the decision strategy:

$$\hat{\omega} \geq \frac{A_2\omega_2 + A_1\omega_1}{2\sqrt{A_1 A_2}} \quad Decide\ for\ H_1 \tag{11}$$

$$\hat{\omega} \prec \frac{A_2\omega_2 + A_1\omega_1}{2\sqrt{A_1 A_2}} \quad Decide\ for\ H_0$$

When the powers are equal, i.e., $A_1 = A_2$ the decision threshold becomes simply the average of the two frequencies ω_1 and ω_2. The conventional approach to NCFSK detection is to process the input signal with a pair of quadrature matched filters, tuned to the tone frequencies ω_1 and ω_2, respectively, and compare the envelopes. If the matched filter is implemented via a Fast Fourier Transform (FFT), μ_0, the SNR at the output of a DFT bin, can be found as $\mu_0 = \mu N/2$ [19, 20]. Therefore with this correction, the error probability expression of the matched filter based NCFSK receiver can be written as [18] :

$$P_{e,FSK} = \frac{1}{2} e^{-\frac{1}{2}\mu_0} \tag{12}$$

For the case of PISFE based FSK detector, the performance can be computed in terms of the total probability of error as ;

$$P_{e,FSK} = \frac{1}{2}\int_0^{\omega_0} p(\hat{\omega} \mid H_1)d\hat{\omega} + \frac{1}{2}\int_{\omega_0}^{\pi} p(\hat{\omega} \mid H_0)d\hat{\omega} \tag{13}$$

where

$$\omega_0 = \frac{\omega_1 + \omega_2}{2}$$

and the conditional probability densities $p(\hat{\omega} \mid H_1)$ and $p(\hat{\omega} \mid H_0)$ are defined as in Eq. (6) by setting $\hat{\omega} = \omega_1$ and $\hat{\omega} = \omega_2$, respectively.

In Figures 7 and 8, the matched filter-based NCFSK error performance has been compared with PISFE schemes. The PISFE detector is inferior to matched filter detection and this discrepancy increases with increasing N. For example the degradation, is about 13-14 dB for N=100.

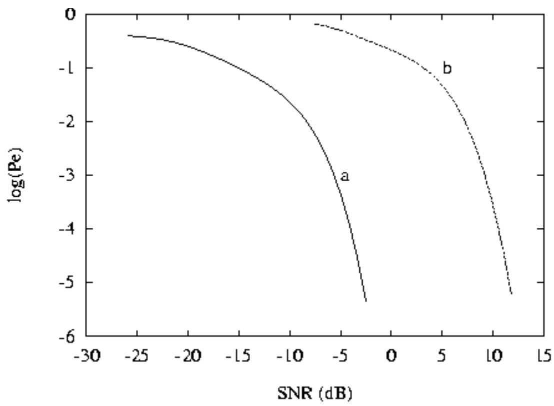

Figure 7. Comparison of the error probabilities of PISFE and matched filter type detectors, a: NCFSK, b: PISFE ($\delta\omega = \pi / 100$, N=100).

VI. CONCLUSIONS

The performance of the PISFE detector using the frequency estimate for the tone signals in white noise has been evaluated, considering in particular the expressions of the probability of false alarm and the probability of detection as functions of the SNR, the number of discrete observations and the tone frequency.It has been justified that the proposed PISFE detector is a CFAR with respect to SNR for fixed tone frequency, but not CFAR with respect to tone frequency for a single tone in white Gaussian noise and that detection performance can be improved for the cases of unknown signal form and/or noise level .

APPENDIX

To derive the conditional pdf's $p(\hat{\omega} \mid H_1)$ and $p(\hat{\omega} \mid H_0)$, we start with finding the conditional pdf's $p(\psi \mid H_1)$ and $p(\psi \mid H_0)$. These pdf's can be obtained by using the statistics of r(1) and r(2) [16].

For H_o :

Using the variable transformation $z_1=\psi$ and z_2=r(1) one can obtain the conditional pdf of ψ under H_o

$$p(\psi \mid H_0) = \int_{-\infty}^{\infty} \frac{p_{\psi,z_2}(z_2 A_0(\psi), z_2)}{\left| \frac{1}{4z_2}(1+\mathrm{sgn}(z_2)A_1(\psi)) \right|} dz_2$$

Perfoming the integration one obtains the expression in Eq. (5).

For H_1 :

Using the same variable transformation one obtains the pdf as the solution of the integral

$$p(\psi \mid H_1) = K_1 \int_{-\infty}^{\infty} \exp\left\{ -\frac{1}{2}\left[A_2(\psi) z_2^2 + A_3(\psi) z_2 + A_4\right]\right\} dz_2$$

Solving the integration the exression in Eq. (6) is obtained.

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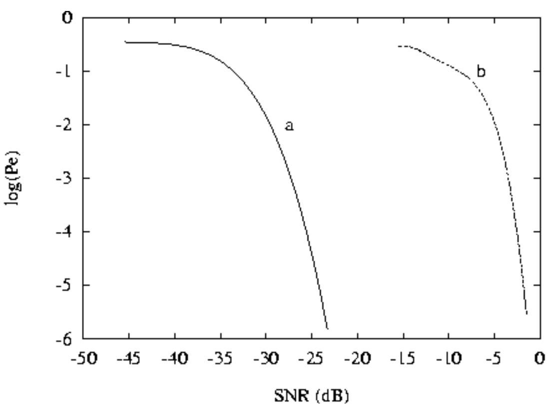

Figure 8. Comparison of the error probabilities of PISFE and matched filter type detectors, a: NCFSK, b: PISFE ($\delta\omega = \pi / 100$, N=1000).