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Stability Analysis of Orthotropic Conical Shells Resting On Winkler Elastic Foundation Based on the FOSDST

Abdullah SOFIYEV[1*], Ali NAJAFOV[2]
[1*]Department of Civil Engineering of Engineering Faculty of Suleyman Demirel University, 32600, Isparta, Turkey, E-mail: abdullahavey@sdu.edu.tr
[2*]Institute for Machine Elements of Azerbaijan Technical University,
E-mail: ali.najafov@rohe.az

Abstract
In this study, the stability of freely supported orthotropic conical shells (OCS) resting on Winkler elastic foundation (WEF) on the basis of the first order shear deformation shell theory (FOSDST) is analyzed by using Donnell-type shell theory and Galerkin method.
KEYWORDS: Stability, Orthotropic Material, Elastic foundation, FOSDST, Critical pressure.

1. INTRODUCTION

The anisotropic constructions are widely used in automotive, marine and aerospace industries, which require a strong, rigid and lightweight construction. The possibility of predicting the reaction of composite orthotropic shells subjected to lateral pressure is of particular interest to researchers in the field of continuous mechanics. Previously, numerous studies have been carried out on the loss of stability of orthotropic conical shells using the classical theory of shells (CST) [1-3]. The development of the theory of shells made it possible to know the important role of shear stresses (or deformations) in the behavior of structures consisting of composite materials. Due to the increasing importance of orthotropic materials in the design of composite structures and their characteristics, the loss of stability, taking into account the effect of transverse shear strains or using shear deformations shell theory (SDST), is vital. In this context, the researchers have published some publications, taking into account the influence of shear deformations on the bending of structural elements [4-6]. In the above studies, the effect of continuous environments on the stability of the orthotropic conical shells is not taken into consideration. In recent years, orthotropic conical shells have been used in various elastic media. The effects of such environments on the stability behavior of the anisotropic conical shells have not yet been investigated based on the SDST [7, 8]. The purpose of this study is to solve the problem of loss of stability of orthotropic conical shells resting on WEF by using FOSDST.

2. METHOD OF SOLUTION

Consider orthotropic truncated conical shell subjected to the uniform lateral pressure, P, resting on the WEF in which the notations are presented in Fig. 1. Let the coordinate system ($Oz\theta r$) be chosen such that, the origin O is at the vertex of the whole cone.

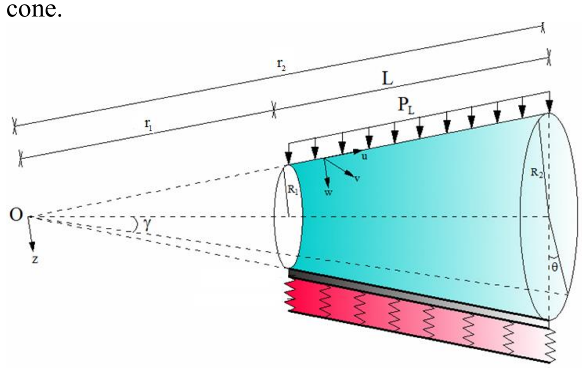

Figure 1. Orthotropic truncated conical shells on the WEF with the coordinate systems and notations

The effect of WEF is modeled as

$$R = K_w w \tag{1}$$

where R is the force per unit area, $K_w (N/m^3)$ is the Winkler foundation stiffness (spring stiffness) and w is the small deflection.

The stress-strain relationships of orthotropic conical shells within the first order shear deformations theory (FOSDST) are obtained as [9]:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{bmatrix}=\begin{bmatrix}\overline{E}_1 & \nu_{12}\overline{E}_2 & 0 & 0 & 0\\ \nu_{12}\overline{E}_1 & \overline{E}_2 & 0 & 0 & 0\\ 0 & 0 & G_{12} & 0 & 0\\ 0 & 0 & 0 & G_{13} & 0\\ 0 & 0 & 0 & 0 & G_{23}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\\ \varepsilon_{13}\\ \varepsilon_{23}\end{bmatrix} \quad (2)$$

where $\sigma_{ij}(i,j=1,2,3)$ and $\varepsilon_{ij}(i,j=1,2,3)$ are the normal and shear components of strains and stresses of orthotropic conical shells, respectively; $\overline{E}_1=E_1/(1-\nu_{12}\nu_{21})$, $\overline{E}_2=E_2/(1-\nu_{12}\nu_{21})$, E_1 and E_2 are Young's moduli of the orthotropic material along r and θ directions, respectively; G_{ij} are shear moduli which characterize angular chances between principal directions r and θ, r and z, θ and z, respectively, ν_{12} and ν_{21} are the Poisson's ratios.

The shear stresses of conical shells resting on WEF within the FOSTSD expressed as [6, 8]:

$$\sigma_{13}=\frac{d\Gamma(z)}{dz}F_1,\ \sigma_{23}=\frac{d\Gamma(z)}{dz}F_2 \quad (3)$$

where F_1 and F_2 are the rotations of normal's to the mid-surface with the respect to θ and r axes, respectively, $\Gamma(z)$ is shear stress or deformation function and z is the thickness coordinate of the conical shell.

The stability and compatibility equations of orthotropic truncated conical shells under lateral pressure and resting on the WEF within the FOSDST are obtained as [8]

$$\begin{bmatrix}L_{11} & L_{12} & L_{13} & L_{14}\\ L_{21} & L_{22} & L_{23} & L_{24}\\ L_{31} & L_{32} & L_{33} & L_{34}\\ L_{41} & L_{42} & L_{43} & L_{44}\end{bmatrix}\begin{bmatrix}\Phi\\ w\\ \varphi\\ \psi\end{bmatrix}=0$$
(4)

where $L_{ij}(i,j=1,2,6)$ are relevant differential operators, depending on the orthotropic conical shell characteristics within FOSDST , L_{42} contains lateral pressure and WEF, here Φ is the Airy stress function [8].

Due to the boundary conditions of the conical shell are freely-supported, the solution of Eq. (4) is sought as [8]:

$$\begin{aligned}\Phi&=\Phi_1 S_2\,\bar{r}^{(\eta+1)}\sin\left[\ln\bar{r}^{m_1}\right]\cos(m_2\phi)\\ w&=w_1\bar{r}^{\eta}\sin\left[\ln\bar{r}^{m_1}\right]\cos(m_2\phi)\\ \varphi&=\varphi_1\bar{r}^{\eta}\cos\left[\ln\bar{r}^{m_1}\right]\cos(m_2\phi)\\ \psi&=\psi_1\bar{r}^{\eta}\sin\left[\ln\bar{r}^{m_1}\right]\sin(m_2\phi)\end{aligned} \quad (5)$$

where $\Phi_1, w_1, \varphi_1, \psi_1$ are unknown functions, η is a parameter that will be obtained from the minimum condition of the critical lateral pressure (CLP), $\bar{r}=r/s_2$, $m_1=\frac{m\pi}{\ln(s_2/s_1)}$, $m_2=\frac{n}{\sin\gamma}$, in which, *m* is the half wave number in meridional direction and *n* is the circumferential wave number.

Substituting (5) into Eq. (4) and employing Galerkin's method to the resulting equations, after some mathematical operations, the following expression is obtained for the CLP of orthotropic truncated conical shell resting on the WEF on the basis of the FOSDST:

$$P_{Lcrw}^{FOSDST}=\frac{U_3(U_2U_7-U_1U_5)-(U_2U_4-U_1U_5)U_6}{U_2U_3L_P}+\frac{L_wK_w}{L_P} \quad (6)$$

where $U_j(i=1,2,...,7)$ are coefficients including the properties of orthotropic conical shell [6], the shear deformation functions and the following definitions apply:

$$\begin{aligned}L_P&=\frac{\left(2m_2^2+1\right)m_1^2(1-e^{2(\eta+1)r_0})\tan\gamma}{r_2\left[4m_1^2+(2\eta+1)^2\right](2\eta+1)},\\ L_w&=-\frac{1}{4}\frac{m_1^2(1-e^{2(\eta+1)r_0})}{\left[m_1^2+(\eta+1)^2\right](\eta+1)}\end{aligned} \quad (7)$$

As the shear deformations are not considered, the expression for the CLP of orthotropic conical shell on the WEF on the basis of the CST is obtained in a special case.

3. CONCLUSIONS AND RESULTS

In this section, the effects of shear deformations and WEF on the CLPs of conical shells within CST and FOSDST are studied. The shear deformation function is $\Gamma(z)=z(1-4z^2/3h^2)$ and $z_1=z/h$ [9]. Table 1 shows the variation of the values of CLP of orthotropic conical shells on the WEF within CST and FOSDST versus the semi-vertex angle γ. The following orthotropic material properties and conical shell parameters are used [9]:

$E_1 = 53.7791\times10^9; E_2 = 17.9264\times10^9;$ $\nu_{12} = 0.25$
$G_{12} = G_{23} = 8.96325\times10^9$, $G_{13} = 3.4474\times10^9$, and $r_1/h = 25$, $L/r_1 = 0.2$. The coefficient of WEF is $K_w = 2\times10^{10}\left(N/m^3\right)$. The values of the CLP for freely supported orthotropic conical shells are determined at $\eta = 2.4$. The values of CLPs for orthotropic conical shells with and without WEF in the framework of the CST and FOSDST decrease with the increasing the semi-vertex angle γ. As the semi-vertex angle γ increases, the difference between the CLPs within CST and FOSDST increases, moreover the shear deformations effect is more pronounced in orthotropic truncated conical shells. The maximum effects of shear deformations are 49.73% and 45.42% for unconstrained conical shell and conical shells resting on the WEF, respectively. As can be seen, considering the soil effect considerably reduces the effect of shear deformations on the CLP.

Table 1. Variation of CLP ($\times10^3$) of orthotropic conical shells on the WEF within CST and FOSDST versus the γ.

γ	P_{1Lcr}^{FOSDST}	P_{1Lcr}^{CST}	P_{1Lcrw}^{FOSDST}	P_{1Lcrw}^{CST}
0º	5.484	7.978	7.015	9.643
15º	4.723	7.506	6.135	9.077
30º	3.787	6.560	5.001	7.938
45º	2.799	5.237	3.755	6.347
60º	1.830	3.640	2.486	4.414

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