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The Dynamic Synthesis of an Energy-Efficient Slider-Crank-Mechanism

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Abstract

When a mechanism is operated in its so-called Eigenmotion, the energy input to accelerate and decelerate the links of this mechanism is equal to zero. Therefore only the remaining forces like process forces, friction forces et cetera have to be overcome and the Eigenmotion results to be very energy-efficient. First this paper presents the underlying equations for the calculation of the Eigenmotion of a slider-crank-mechanism. Afterwards the derivation of an equivalent mechanical system of the mechanism is shown. Finally a method to synthesize an energy-efficient slider-crank-mechanism is presented. The dynamic synthesis is therefore formulated as a constrained optimization problem.

Keywords: Dynamic Synthesis, Eigenmotion, Mechanism Synthesis, Dynamic Balancing, Slider-Crank-Mechanism

1. Introduction

Mechanisms form part of many different production machines, e.g. weaving machines, printing machines or packaging machines. The energy-efficiency of their built-in mechanisms is crucial for the profitability of these machines. The following equations hold for plane mechanisms with a rotating input link (crank). The necessary power P_D to drive such a mechanism is the product of the drive torque T_D and the input velocity $\dot{\varphi}$:

$$P_D = T_D \cdot \dot{\varphi}$$

The drive torque can be written as follows [1]:

$$T_D = T_{kin} + T_{pot} + T_{diss} + T_{proc}$$

T_{kin} is the torque which is necessary to overcome the resistances from accelerating and decelerating the links of the mechanism. T_{pot} is the necessary torque to overcome the resistances which result from gravity or springs. T_{diss} and T_{proc} comprise the resistances following from dissipation effects and process forces. The torque T_{kin}, can be derived from the kinetic energy E_{kin} of the mechanism using the Lagrange Equations of 2nd kind [2]:

$$T_{kin} = \frac{d}{dt}\left(\frac{\partial E_{kin}}{\partial \dot{\varphi}}\right) - \frac{\partial E_{kin}}{\partial \varphi}$$

It is evident that the torque T_{kin} vanishes for a constant kinetic energy of the mechanism. The concept of driving a mechanism in its so-called Eigenmotion uses this fact. The Eigenmotion is the specific motion of the crank that results in a constant kinetic energy of the mechanism over the whole period of motion. [1; 3; 4]

The classical dimensional synthesis of mechanisms aims to finding the optimal kinematic parameters for a mechanism driven with a constant input motion [5; 6]. The goal of the methods of dynamic balancing is to find the optimal mass parameters for a mechanism with given kinematic parameters [1; 3]. The dynamic synthesis combines both concepts. It aims to finding the optimal mass and kinematic parameters of a mechanism [7].

The equations show that driving a mechanism in its Eigenmotion can decrease the necessary input torque and can result in lower energy consumption. Bench tests confirmed the effectiveness of driving a mechanism in its Eigenmotion with regard to energy consumption [8].

In this paper the synthesis of an energy-efficient slider-crank-mechanism is presented. First, the underlying equations of the slider-crank-mechanism are shown. The equation of the Eigenmotion of the slider-crank-mechanism is derived. Afterwards an equimomental system of the slider-crank-mechanism is presented. The Eigenmotion of the mechanism is rewritten for this equimomental system. The synthesis of a mechanism with a particular Eigenmotion is formulated as an optimization problem. The parameters of the equimomental system of the slider-crank-mechanism are taken as the design parameters of the optimization. The formulation of the objective function as well as the formulation of the optimization constraints is shown. The optimization results for an exemplarily synthesis are presented.

2. The Slider-Crank-Mechanism

The kinematic parameters and the mass parameters of the slider-crank-mechanism are shown in Fig. 1.

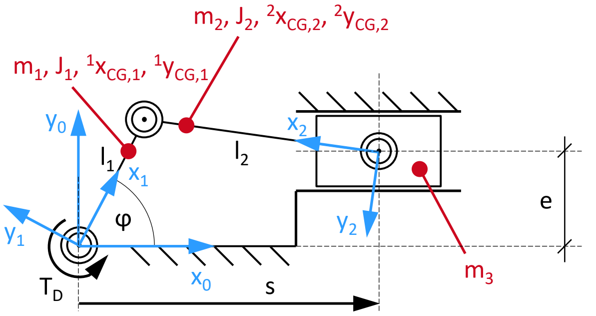

Fig. 1. The slider-crank-mechanism

The slider-crank-mechanism consists of three links. The crank (index ‘1’) is driven by a motor with the drive torque T_D. The input angle is denoted by φ. The output of the mechanism is the stroke s of the slider (index ‘3’). Crank and slider are connected by the coupler (index ‘2’). The offset of the slider is denoted by e. The coordinate systems are highlighted in blue color. The coordinate system ‘0’ is frame fixed. The other two Coordinate systems are body-fixed. The coordinate systems are right-handed, hence the z-axes point out of the image plane. The mass properties of the links are highlighted in red. The position of the center of gravity (CG) of the links is represented in the body-fixed systems. The superscript indicates the corresponding coordinate system. Positions of the center of gravity without superscript are formulated in the coordinate system ‘0’.
The coordinate system ‘2’ is rotated about the angle ψ about the z-axis of the coordinate system ‘0’. The angle ψ can be calculated as follows:

$$\psi = \arcsin\left(\frac{l_1 \sin(\varphi) - e}{l_2}\right)$$

The stroke of the slider s reads:

$$s = l_1 \cos(\varphi) + l_2 \sqrt{1 - \frac{(l_1 \sin(\varphi) - e)^2}{l_2^2}}$$

The derivatives of the angle psi and the stroke s can be calculated as follows:

$$\frac{d\psi}{dt} = \frac{d\psi}{d\varphi}\dot{\varphi} = \frac{l_1 \cos(\varphi)}{l_2 \sqrt{1 - \frac{(l_1 \sin(\varphi) - e)^2}{l_2^2}}}\dot{\varphi}$$

$$\frac{ds}{dt} = \frac{ds}{d\varphi}\dot{\varphi} = -l_1 \sin(\varphi)\,\dot{\varphi} - \frac{(l_1 \sin(\varphi) - e) l_1 \cos(\varphi)}{l_2 \sqrt{1 - \frac{(l_1 \sin(\varphi) - e)^2}{l_2^2}}}\dot{\varphi}$$

In order to calculate the Eigenmotion of the mechanism, the kinetic energy has to be set up. The kinetic energy of the mechanism can be written as the sum of the kinetic energy of its three links:

$$E_{kin} = E_{kin,1} + E_{kin,2} + E_{kin,3}$$

The kinetic energy of a rigid body can be split into a translational (T) and a rotational (R) part:

$$E_{kin,i} = E_{kin,Ti} + E_{kin,Ri}$$

Hence the kinetic energy of the crank can be written as:

$$E_{kin,T1} = \frac{1}{2} m_1 \left({}^1x_{CG,1}^2 + {}^1y_{CG,1}^2 \right) \dot{\varphi}^2$$

$$E_{kin,R1} = \frac{1}{2} J_1 \dot{\varphi}^2$$

The kinetic energy of the coupler is:

$$E_{kin,T2} = \frac{1}{2} m_2 \left(\left(\frac{dx_{CG,2}}{d\varphi}\right)^2 + \left(\frac{dy_{CG,2}}{d\varphi}\right)^2 \right) \dot{\varphi}^2$$

$$E_{kin,R2} = \frac{1}{2} J_2 \left(\frac{d\psi}{d\varphi}\right)^2 \dot{\varphi}^2$$

The slider has only translational kinetic energy due to the translational guiding. Therefore its kinetic energy consists of only one term:

$$E_{kin,T3} = \frac{1}{2} m_3 \left(\frac{ds}{d\varphi}\right)^2 \dot{\varphi}^2$$

The reduced mass moment of inertia is defined as follows:

$$J_{red}(\varphi) = \frac{2 \cdot E_{kin}}{\dot{\varphi}^2}$$

It is the fictive mass moment of inertia of a rotating disk with the same kinetic energy as the mechanism [9]. The reduced mass moment of inertia is a function of the input angle φ. It depends on the kinematic parameters and the mass parameters. It can be written as the sum of the reduced mass moments of inertia of the links:

$$J_{red}(\varphi) = J_{red,1} + J_{red,2}(\varphi) + J_{red,3}(\varphi)$$

These reduced mass moments of inertia of the links are:

$$J_{red,1} = J_1 + m_1\left({}^1x_{CG,1}^2 + {}^1y_{CG,1}^2\right)$$

$$J_{red,2}(\varphi) = m_2\left(\left(\frac{dx_{CG,2}}{d\varphi}\right)^2 + \left(\frac{dy_{CG,2}}{d\varphi}\right)^2\right) + J_2\left(\frac{d\psi}{d\varphi}\right)^2$$

$$J_{red,3}(\varphi) = m_3\left(\frac{ds}{d\varphi}\right)^2$$

The Eigenmotion of a mechanism is defined as its intrinsic motion in the case of constant kinetic energy [1; 9]. It is denoted by the index ‘e’:

$$E_{kin} = \frac{1}{2}J_{red}(\varphi)\dot{\varphi}_e{}^2 = const. = \frac{1}{2}J_{red}(\varphi_0)\dot{\varphi}_0{}^2$$

It is [1; 9]:

$$\dot{\varphi}_e = \frac{\dot{\varphi}_0\sqrt{J_{red}(\varphi_0)}}{\sqrt{J_{red}(\varphi)}} = \frac{C}{\sqrt{J_{red}(\varphi)}}$$

The numerator of the equation is constant and denoted by C. The period time T of the Eigenmotion can be calculated by [9]:

$$T = \frac{1}{C}\int_{\varphi_0}^{\varphi_0+2\pi} \frac{1}{\sqrt{J_{red}(\tilde{\varphi})}}\,d\tilde{\varphi}$$

3. The Equimomental System

The reduced mass moment of inertia depends on the kinematic properties and the mass properties of a mechanism. Using dynamically equivalent systems for the particular links of the mechanism, the reduced mass moment of inertia can be reformulated. In the following, the equivalent systems of the crank and the coupler shall be presented. Afterwards the equimomental system of the complete slider-crank-mechanism is presented. The Eigenmotion of the mechanism is reformulated in terms of the parameters of the equimomental system. Information on equimomental systems can be found in [4; 10].

First, the equimomental system of the crank is defined. The reduced mass moment of inertia of the crank is constant. It depends on four parameters, which can be replaced by a mass m_{A1} as shown in Fig. 2.

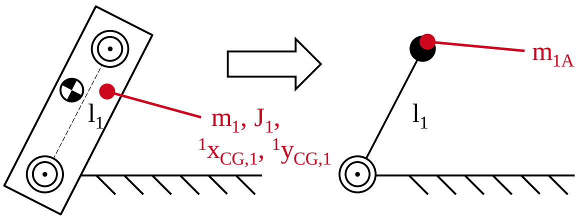

Fig. 2. The equimomental system of the crank

The mass m_{1A} is placed at the position of the connecting joint between the crank and the coupler. It holds:

$$m_{1A} = \frac{J_1 + m_1\left({}^1x_{CG,1}^2 + {}^1y_{CG,1}^2\right)}{l_1^2}$$

Second, the equimomental system of the coupler shall be defined. The assumption is made, that the center of gravity of the coupler lies upon the connecting line between the two joints of the coupler. Hence the equimomental system according to Fig. 3 can be used.

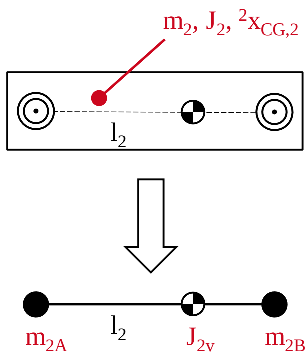

Fig. 3. The equimomental system of the coupler link

The mass parameters m_2, J_2 and ${}^2x_{CG,2}$ are replaced by the parameters m_{2A}, m_{2B} and J_{2v}. The original system and the equimomental system have to have the same mass and the same mass inertia about the center of gravity. Furthermore the position of the center of gravity has to be the same. Taking into account these conditions, the mass parameters of the equimomental system can be derived. It is:

$$m_{2A} = \frac{m_2\left(l_2 - {}^2x_{CG,2}\right)}{l_2}$$

$$m_{2B} = \frac{m_2\,{}^2x_{CG,2}}{l_2}$$

$$J_{2v} = J_2 + m_2\,{}^2x_{CG,2}\left({}^2x_{CG,2} - l_2\right)$$

The slider only possesses translational kinetic energy. Therefore only m_3 has to be taken into account. Fig. 4 finally shows the equimomental system of the complete slider-crank-mechanism.

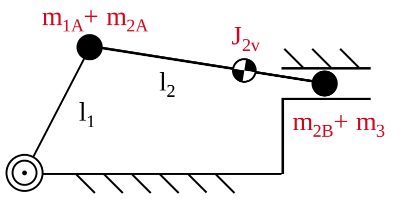

Fig. 4. The equimomental system of the mechanism

The point masses m_{2A} and m_{3A} are located on the same position. They can be replaced by a mass moment of inertia J_{1v}. It holds:

$$J_{1v} = (m_{1A} + m_{2A}) l_1^2$$

The point masses m_{2B} and m_3 also lie on the same position. The can be replaced by the mass m_{3v}:

$$m_{3v} = m_{2B} + m_3$$

Using the mass properties J_{1v}, J_{2v} and m_{3v} of the equimomental system, the Eigenmotion of the slider-crank-mechanism can be written as follows.

$$\dot{\varphi}_e = \frac{C}{\sqrt{J_{1v} + J_{2v}\left(\frac{d\psi}{d\varphi}\right)^2 + m_{3v}\left(\frac{ds}{d\varphi}\right)^2}}$$

In order to reduce the number of parameters the following dimensionless parameters are introduced:

$$\iota_{2v} = \frac{J_{2v}}{J_{1v}}, \quad \mu_{3v} = \frac{m_{3v} l_1^2}{J_{1v}}$$

Using these parameters the Eigenmotion can be rewritten as follows:

$$\dot{\varphi}_e = \frac{\tilde{C}}{\sqrt{1 + \iota_{2v}\left(\frac{d\psi}{d\varphi}\right)^2 + \frac{\mu_{3v}}{l_1^2}\left(\frac{ds}{d\varphi}\right)^2}}$$

The constant $\tilde{C}$ reads:

$$\tilde{C} = \frac{C}{J_{1v}^2}$$

By adjusting $\tilde{C}$, the Eigenmotion can be normalized with respect to a period time T of one second. Normalizing the Eigenmotion is helpful in order to compare the Eigenmotion to other motions. The normalized Eigenmotion is dependent on the following parameters listed within the parameter vector $\mathbf{p}_e$:

$$p_e = (l_1, l_2, e, \iota_{2v}, \mu_{3v})$$

4. The Dynamic Synthesis of the Slider-Crank-Mechanism as an Optimization Problem

The goal of the dynamic synthesis is to find a slider-crank-mechanism which is able to fulfill a desired motion when driven in its Eigenmotion.

In the following the task is formulated as an optimization problem. The goal of an optimization is to find the best set of design parameters $\mathbf{x}$ for a certain task. The task is formulated as an objective function $\mathbf{f}(x)$. The objective function is formulated in such way, that the best combination of parameters minimizes this function. Constraints, i.e. restrictions with respect to the combination of parameters, can also be taken into account. Constraints can be formulated as equality constraint equations h or as inequality constraint equations g and are also dependent on the design parameters. Acceptable combinations of the design parameters have to satisfy these constraint equations. The formal statement of the minimization formulation of an optimization problem is written as follows [11]:

$$\begin{gathered} \text{minimize } f(x) \\ \text{subject to } h(x) = 0, \\ g(x) \leq 0, \\ x \in X \subseteq R^n. \end{gathered}$$

The vector $\mathbf{h}(x)$ contains all equality constraints meanwhile $\mathbf{g}(x)$ contains all inequality constraints. A maximization formulation can be transformed into a minimization formulation by multiplying the objective function by minus one. Algorithms to find solutions of optimization problems are called optimization algorithms. A distinction is made between global and local optimization. Local optimization algorithms seek only local solutions, that is sets of parameters for which the objective function is smaller than at all feasible parameter combinations nearby. Local optimization algorithms do not always find the global minimum. In case of optimization problems, where local optimization algorithms are not suitable to find the global minimum, global optimization algorithms have to be applied. More information on local optimization can be found in [11–13]. Contrary to local optimization algorithms, global optimization algorithms are designed to find a good solution over all input values. A variety of algorithms exists for global optimization. Metaheuristics are procedures to find good (global) solutions for optimization problems. However, the discovery of the globally optimal solution is not guaranteed. This is due to the fact, that these methods do contain some kind of stochastic optimization. Examples for metaheuristics are the Particle Swarm Optimization or the Genetic Algorithm (GA). Information on these metaheuristics can be found in [14–16].

In order to conduct the dynamic synthesis of the slider-crank-mechanism the synthesis was formulated as an optimization problem. The set of design parameters contains the entries of the parameter vector $\mathbf{p}_e$ and is complemented by the initial angle φ_0. It reads:

$$x = (l_1, l_2, e, \iota_{2v}, \mu_{3v}, \varphi_0)$$

The necessary steps within the optimization process are shown in Fig. 5. The process of evaluating the optimization function is depicted within the doted rectangle. Inside of the evaluation of the optimization function the Eigenmotion $\dot{\varphi}_e$ is calculated for a set of design parameters. The Eigenmotion is calculated for the N points of the vector of normalized time $\mathbf{t}$. The Eigenmotion is then integrated in order to achieve the crank angle of the Eigenmotion φ_e over the normalized time. The stroke of the slider-crank-mechanism in Eigenmotion s_e can then be calculated by inserting φ_e in the kinematic equations of the mechanism. In order to compare the stroke of the slider-crank-mechanism s_e in Eigenmotion to the desired ouput stroke s_d the sum of least squares of the difference between both motions is calculated:

$$f = \sum_{i=1}^{N} \left(s_e(i) - s_d(i)\right)^2$$

The vectors $\mathbf{s}_e$ and $\mathbf{s}_d$ contain the values of the Eigenmotion and the desired motion over the normalized time. The optimization process is repeated for different sets of design variables until a stop criterion is reached. A stop criterion could be for example the value of f being under a certain, predefined treshold.

In order to achieve feasible solutions, constraints have to be taken into account. First of all, upper and lower limits (boundaries) of the design parameters have to be determined. The upper and lower boundaries of the design variables are also called box-constraints. Second, the kinematic chain of the slider-crank-mechanism has to be closable for any input angle φ. Further constraints concerning can be implemented. These constraints can concern the dimensions of the mechanism, like for example a relationship between the lengths of different links. Furthermore requirements concerning the output motion could be implemented as constraints.

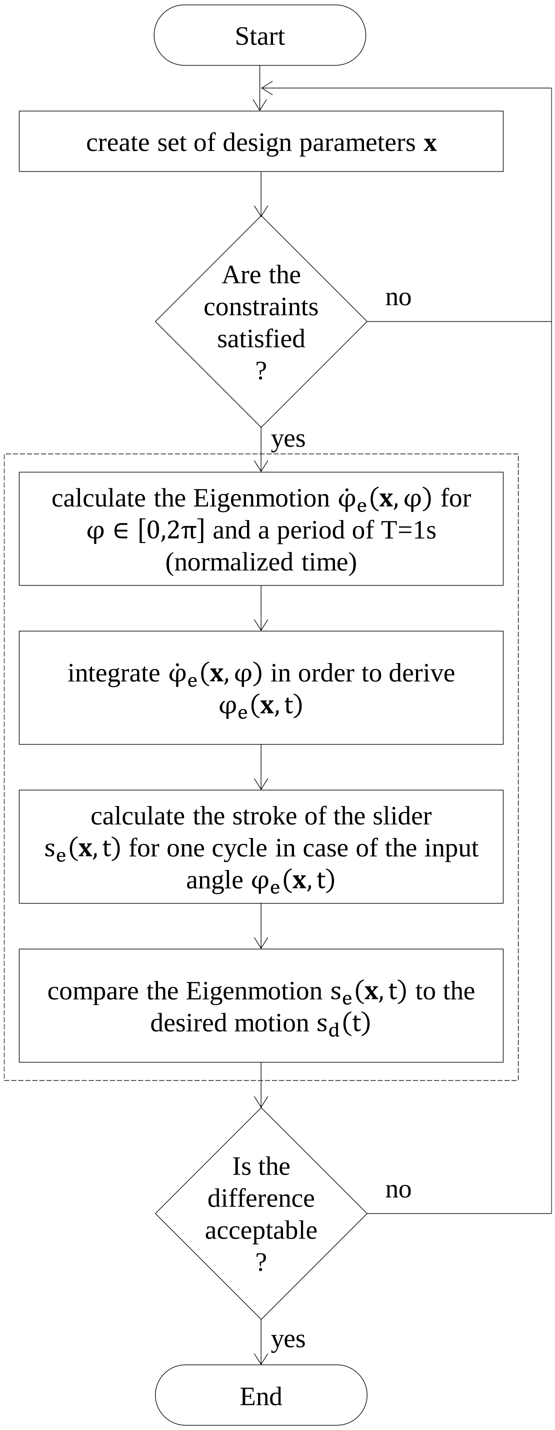

Fig. 5. Flow-chart of the optimization process

5. An Example of the Dynamic Synthesis of the Slider-Crank-Mechanism

In the following an example of the dynamic synthesis of the crank-slider-mechanism is presented. Fig. 6 shows the desired output motion of the crank-slider-mechanism. The output slider should fulfill a descending motion from 0.6 to 0.3 meters with approximately constant velocity between 0.6 and 0.9 seconds of normalized time.

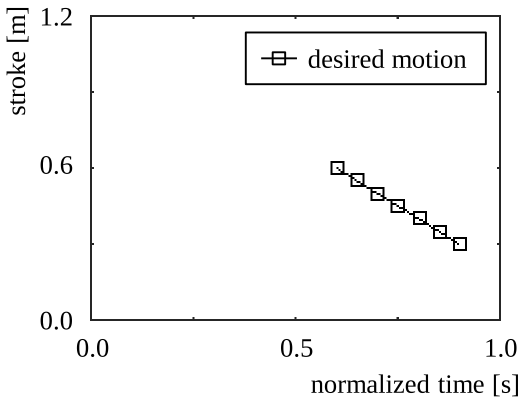

Fig. 6 : The desired output of the slider-crank-mechanism

The upper and lower values of the design parameters were set as listed in Table 1.

Table 1. The boundaries of the design parameters

parameter	*lower boundary*	*upper boundary*
l_1	0.10 m	1.00 m
l_2	0.10 m	1.00 m
e	-0.50 m	-0.50 m
ι_{2v}	-0.05	0.00
μ_{3v}	0.00	0.75
φ_0	0.00	2π

Apart of the box constraints and the closing condition of the linkage no more constraints were implemented.

6. The Results of the Dynamic Synthesis

In order to solve the optimization problem a genetic algorithm was used. The output parameters of this optimization were used as input parameters for a local optimization algorithm. The local optimization was carried out by using a barrier-method.

The result of the optimization is shown in Fig. 7. It can be seen that the output motion of the resulting slider-crank-mechanism in Eigenmotion is close to the desired output motion.

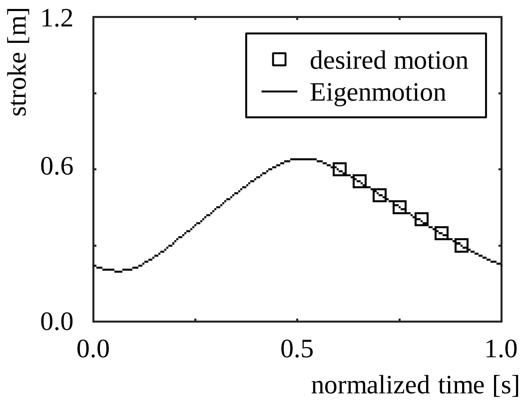

Fig. 7. The result of the dynamic synthesis

The resulting set of parameters of the optimization is listed in Table 2. The parameter φ_0 is the value of the crank angle at the beginning of a cycle. It has no influence on the design of the mechanism.

Table 2. The solution set of parameters

parameter	*value*
l_1	166.7 mm
l_2	557.0 mm
e	333.3 mm
ι_{2v}	-0.05
μ_{3v}	0.75
φ_0	3.6645

The parameters ι_{2v} and μ_{3v} can be used to derive the geometry of the coupler and the crank.

It can be thought of different ways to derive feasible links from these parameters. In the following one approach is shown in order to derive members of the mechanism. First of all, the mass property J_{1v} is set to a preliminary value (denoted by an asterisk):

$$J_{1v}^{*} = 1.0\,\mathrm{kg} \cdot l_1^2$$

Second, the geometry of the coupler link is set to be according to Fig. 8.

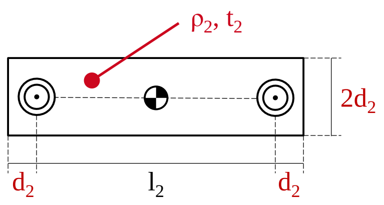

Fig. 8. The predefined geometry of the coupler link

The coupler link is set to be rectangular with its center of mass in the middle of the connecting line between the two joints. The density of the material is denoted ρ_2. Two geometric parameters d_2 and the width of the link t_2 are introduced. The value d_2 and ρ_2 are set to fixed values:

$$d_2 = 40\ \mathrm{mm},\ \ \rho_2 = 2700\,\frac{\mathrm{kg}}{\mathrm{m}^3}$$

Using the preliminary value for J_{1v} the mass of the coupler link can be calculated as follows:

$$m_2^* = \frac{\iota_{2v} \cdot J_{1v}^*}{\left(\frac{1}{12}((l_2 + 2d_2)^2 + 4d_2^2) - \frac{l_2^2}{4}\right)}$$

All values denoted by an asterisk are preliminary values which can be adjusted later. Now the preliminary mass of the slider can be calculated. It is:

$$m_3^* = \mu_{3v}\frac{J_{1v}^*}{l_1^2} - \frac{m_2^*}{2}$$

The mass of the output link is now set to a fixed value:

$$m_3 = 2.5\ \mathrm{kg}$$

A correction factor for the mass parameters is calculated:

$$f^* = \frac{m_3}{m_3^*}$$

Hence:

$$J_{1v} = f^* \cdot J_{1v}^*$$

The mass of the coupler link can then be calculated as follows:

$$m_2 = f^* \cdot m_2^* = 1.5730\ \mathrm{kg}$$

In order to define the coupler geometry the width of the link has to be calculated. It is:

$$t_2 = \frac{m_2}{\rho_2\big((l_2 + 2d_2) \cdot 2d_2\big)} = 11.4\ \mathrm{mm}$$

After defining the coupler geometry, the crank geometry has to be derived. The point mass m_{1A} according to chapter 3 can be calculated as follows:

$$m_{1A} = \frac{J_{1v}}{l_1^2} - \frac{m_2}{2}$$

The crank has to have the same dynamic effect with respect to the reduced mass moment of inertia as the point mass m_{1A}. On the basis of geometry similar to the coupler geometry depicted in Fig. 8 the mass properties of the crank can be derived. The equation to calculate the mass reads:

$$m_1 = \rho_1\big((l_1 + 2d_1) \cdot 2d_1\big) \cdot t_1$$

The mass moment of inertia about the pivoting point in the frame can be set up as follows:

$$m_{1A} \cdot l_1^2 = \frac{m_1}{12}((l_1 + 2d_1)^2 + 4d_1^2) + m_1\frac{l_1^2}{4}$$

In order to derive the mass m_1 from this equation, the value d_1 is set to a certain value:

$$d_1 = 50\ \mathrm{mm}$$

The density of the material of the crank is then set to:

$$\rho_1 = 7870\,\frac{\mathrm{kg}}{\mathrm{m}^3}$$

The width of the crank can then be calculated. It is:

$$t_1 = \frac{m_1}{\rho_1\big((l_1 + 2d_1) \cdot 2d_1\big)} = 27.1\ \mathrm{mm}$$

The presented procedure of using the output values of the optimization in order to build feasible links is based on primitive geometries. Future work can be on the field of contemplating more complex geometries.

7. Conclusion

In this paper the Eigenmotion of the slider-crank-mechanism was presented. Therefore the underlying kinematic and dynamic equations were derived. An equimomental system of the slider-crank-mechanism was introduced in order to simplify the equations.

Subsequently a method for the dynamic synthesis of the crank-slider-mechanism was presented. Therefore the task was formulated as an optimization problem. The optimization problem was solved by the use of a genetic algorithm.

An example of the use of the method was shown. The results showed the suitability of the method to derive feasible mechanisms which can fulfill a desired output motion when moved in the Eigenmotion. In order to conclude the example, an approach of designing the links of the mechanism was presented.

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