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The Vibration of Non-Homogenous Nano-Micro Elements of The Euler-Bernulli Beam Theory According to The Nonlocal Theory

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Abstract
In this article, the vibration analysis of non-homogeneous nano and micro elements has been investigated with the applying of nonlocal elasticity theory. Beam has been chosen as the structural model type and also Euler-Bernoulli beam theories have been used for beam theories. The motion equations of Euler-Bernoulli beam theories were obtained by utilizing the nonlocal elasticity theory, which was proposed by Eringen. According to the different boundary conditions, the vibration equations of micro-nano beam has been generated. Analysis has been performed over the carbon nanotube and microtubules in order to observe the effect of nonlocal behavior and results have been compared with the classical theory.
Keywords: Nonlocal elasticity theory, nano-micro elements, non-homogeneous, Euler-Bernoulli beam theories, vibration, free vibration frequency.

1. Introduction

The vast majority of strength and stability of various beam layered structural theories are studied in detail in scientific literatues which is made different isotropic non-homogeneous elastic materials [1-3]. İn recent times, regarding the rapid advancement of artificial and composite material technologies, in most cases the structures are made with those materials. İn this case, isotropic properties are formed in various materials and it is necessary to take into consideration these factors to solve concrete problems. These factors are considerably important especially in the stability and vibration problems.

Nowadays, it is essential to utilize accurate hypotheses and theories while trying to solve numerous problems. One of these theories is the non-local elasticity theory which has proposed by Eringen. In number [4], bending problem of isotropic beams was investigated using these theories

In number [4-5] - The issue was isotropic nano – micro elements, stability and strength problems based on Eringen theory.

In numbers [6-7] - generally beams theory has established on the basis of this theory and on the basis of various speculation, stability and vibration that have been investigated.

The [8] - Regarding the non-local continuum theory the ortotrop nano plates model was realised as polymer plate then stability problems have been investigated.

The [9] - nano beams prepared from functional graded materials on the basis of non-local Timoshenko theory that vibration problem had been modeled and various problems had been solved.

The [10] - On the basis of thermo elastic non-local model called functional graded nano beams' vibration problem has been reviewed under sinusoidal pressure.

The [11] - symmetric stability problems of Nano beams on the vertical condition were investigated according to non-local discrete and continuum theories.

The [12] - generally main equations of beams' theories were obtained on the basis of non-local elasticity theory of Eringen and bending problems was investigated in detailed.

The [13] - on the basis of Timoshenko beam model of nano and microstructures, non-linear vibration problems were researched.

The [14] - non-linear equations of fluid flowing through the tubes were obtained and its stability condition was investigated.

The [15] - symmetric for layered beams due to tension theory the model was structured and deflection equation was obtained.

The [16] - the stability problems of non-linear beam were investigated on the basis of gradient theory.

The [17] - the vibration problems of micro plates were investigated with modified tensions theory.

The [18] - while solving the theory of displacement deformations of functional graded plates general condition and Karmans non-linear theory or finite elements method were analysed.

The [19] - the vibration problems was investigated in the elastic condition due to anisotrop layered beams theories.

Sizes effects have significantly importance on construction elements in nano and micro measured elements, nowadays, mostly elasticity theory (in scientific literature sometimes it is also called non-local elasticity theory) is used which has been proposed by Eringen. In classic elasticity theory which equilibrium equations are considered the same all parts of the object. This candition actually is true if dimensions of part would be large. However, if dimensions of part decreases, there would require to take into consideration inner structure of the body, so there should also take into account the interaction of the particles which close to point in the examined equations. Tensions at a point in calculating the amount which was proposed Eringen with the theory of elasticity, displacement and deformation at the point where you just know it is not enough and that is offered to them in the form of the dependence of all the points are a function of displacement.

For the reasons discussed article that bending and stability of heterogeneous beams' issues are studied based on the theory of Eringen.

MAIN PHYSICAL RELATIONS

Cauchy equations of motion in non-local theory of elasticity for the homogenous and isotropic body are:

$$\tau_{kl,l} + \rho\left(f_i - \frac{\partial^2 u_i}{\partial t^2}\right) = 0, \quad (1)$$

For general case, Eringen non-local elasticity theory, the physical equations of body are:

$$\tau_{kl}(x) = \int_v \varepsilon_{klmn}(x - x')\,\varepsilon_{klmn} dv(x'), \quad (2)$$

Here is τ_{kl} stress tensor, ρ density of body, f intensity of force for the body, u- displacement vector, v - the volume occupied by the body, t – time, ε_{kl} deformations and are defined below:

$$\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right). \quad (3)$$

ε_{klmn}, is the function of $x - x'$vector from where it can be seen that stresses in x point are dependent on the displacements and deformations in point x'. The stresses and deformations in point x' are:

$$\tau(x') = \lambda\varepsilon_{mn}(x')\delta_{ke} + 2\mu\varepsilon_{ke}(x'), \quad (4)$$

$$\varepsilon_{kl}(x') = \frac{1}{2}\left(\frac{\partial u_k(x')}{\partial x'_l} + \frac{\partial u_l(x')}{\partial x'_k}\right),$$

In those equations, $\tau(x')$ is the classic (Cauchy) or local stress tensor of the body in point x'. $\varepsilon_{kl}(x')$ is the linear deformation of particle in the x' point.

After some algebra, the equations of Eringen non-local elasticity theory are:

$$[1 - (l_0 a)^2\nabla^2]\sigma_{kl} = \tau_{kl,} \quad (5)$$

$$[1 - (l_0 a)^2\nabla^2]\tau_{kl} = \lambda\varepsilon_{kl}\delta_{kl} + 22\mu\varepsilon_{kl}.$$

a – inner characteristic length, l_0 is the constant coefficient for the material.

Hereby, the equalities can be written as follows;

$$\left[1 - (l_0 a)^2\frac{\partial^2}{\partial x^2}\right]\sigma_{xx} = E\,\varepsilon_{xx,} \quad (6)$$

$$\left[1 - (l_0 a)^2\frac{\partial^2}{\partial x^2}\right]\tau_{xx} = 2G\,\varepsilon_{xz,}$$

Assume that material is non-uniform like $E = E(z)$ (modulus of elasticity is the continuous function of thikness coordinate.

According to Euler Bernoulli beams theory

$$\varepsilon_{kl} = E\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right) \quad (7)$$

expression is achieved. Here u is the displacement of the beam, w is the bending.

Force and moment are calculated as follows

$$P = \int_A \sigma_{xx} dA \quad N = \int_A \tau_{xz} dA,\ M = \int_A \sigma_{xx} z dA \quad (8)$$

After some algebra, the moment is calculated as:

$$M = -KI\frac{d^2 w}{dx^2}\ . \quad (9)$$

KI is the generalized hardness characteristic of the beam.

If $E = E_0\left[1 + \gamma\frac{z^2}{h^2}\right]$, then in that case,

$$KI = E_0 I\left[1 + \gamma\frac{z^3}{20}\right], \quad (10)$$

where $E_0 I$ is the hardness of the homogenous beam.

The result from (6) is shown below:

$$M\left[1 - (e_0 a)^2\frac{\partial^2}{\partial x^2}\right] = -KI\frac{\partial^2}{\partial x^2}w. \quad (11)$$

FORMULATION OF THE VIBRATION PROBLEM

Now the vibration problem of the beam will be analysed. The equation of motion for the beam is as :

$$\frac{\partial P}{\partial x} + f = m_0\frac{\partial^2 U}{\partial t^2} \quad (12)$$

$$\frac{\partial^2 M}{\partial x} + q - \frac{\partial}{\partial x}\left(P\frac{\partial w}{\partial x}\right) = m_0\frac{\partial^2 w}{\partial t^2} - m_2\frac{\partial^2 w}{\partial x^2\partial t^2} \quad (13)$$

$$m_0 = \int_s \rho ds = \rho s;\ m_2 = \int_s z^2 ds = \rho s\frac{h^2}{12} \quad (14)$$

The expressions for moment and force are:

$$P = Ks\frac{\partial U}{\partial x} + \mu\frac{\partial}{\partial x}\left(m_0\frac{\partial^2 U}{\partial t^2} - f\right) \quad (15)$$

$$M = -KI\frac{\partial^2 w}{\partial x^2} + \mu\left[\frac{\partial}{\partial x}\left(P\frac{\partial w}{\partial x}\right) - q - m_0\frac{\partial^2 w}{\partial t^2} - m_2\frac{\partial^2 w}{\partial t^2\partial x^2}\right], \quad (16)$$

In here, $M = (e_0 a)^2$ and after substituting the formula of moment in the equation above, the result will be:

$$\frac{\partial^2}{\partial x^2}\left(-KI\frac{\partial^2 w}{\partial x^2}\right) + \mu\frac{\partial^2}{\partial x^2}\left[\frac{\partial}{\partial x}\left(P\frac{\partial w}{\partial x}\right) - q + m_0\frac{\partial^2 w}{\partial t^2} - m_2\frac{\partial^4 w}{\partial t^2\partial x^2}\right] + q - \frac{\partial}{\partial x}\left(P\frac{\partial w}{\partial x}\right) = m_0\frac{\partial^2 w}{\partial t^2} - -m_2\frac{\partial^4 w}{\partial t^2\partial x^2} \quad (17)$$

If boundry conditions is assumed to add into this equation, it could be obtained from the general statement of the problem.

SOLUTION OF THE PROBLEM

Generally the solution of the problem depends on various difficulties. In this case, it is examined in different certain conditions. Assume that, force is axially applied to the beam (in 17th equation, P=const, f=0, q=0 is accepted). After some calculations, specific vibration equations are obtained as below:

$$\frac{\partial^2}{\partial x^2}\left(-KI\frac{\partial^2 w}{\partial x^2}\right) + \mu\frac{\partial^2}{\partial x^2}\left[\frac{\partial}{\partial x}\left(P\frac{\partial w}{\partial x}\right) + m_0\frac{\partial^2 w}{\partial t^2} - m_2\frac{\partial^4 w}{\partial t^2\partial x^2}\right] - \frac{\partial}{\partial x}\left(P\frac{\partial w}{\partial x}\right) = m_0\frac{\partial^2 w}{\partial t^2} - m_2\frac{\partial^4 w}{\partial t^2\partial x^2} \quad (18)$$

We can accept 18th equations periodic solutions as below

$$w(x,t) = w_1(x)e^{i\omega t}. \quad (19)$$

We put (19) expression to (18) one in order to obtain following equation:

$$d_1\frac{d^4w_1}{dx^4}+d_2\frac{d^2w_1}{dx^2}-rw_1=0 \qquad (20)$$

From here it can be write;

$$d_1=KI-\mu P-\mu m_2\omega^2$$
$$d_2=m_2\omega^2+P+\mu m_0\omega^2,\quad r=m_0\omega^2 \qquad (21)$$

ω – is the vibration frequencey.

General solution of the (20) – equation is given below;

$$w_1=c_1\sin\alpha x+c_2\cos\alpha x+c_3\sin h\beta x+c_4\cos h\beta x \qquad (22)$$

Here,

$$\alpha^2=\frac{1}{2d_1}\left(d_2+\sqrt{d_2^2+4d_1r}\right),\ \beta^2=\frac{1}{2d_1}\left(-d_2+\sqrt{d_2^2+4d_1r}\right) \qquad (23)$$

c1, c2, c3, c4 are the integral constants with the accepted boundary conditions. In special condition, if a=b, P=0, m_2=0, μ=0 accepted, after classical elasticity theory the given problem coluld be solved.

Utilizing (22) expression with some calculations, it can be obtained as below:

$$\frac{dw_1}{dx}=\alpha(c_1\cos\alpha x-c_2\sin\alpha x)+\beta(c_3\cos h\,\beta x+c_4\cos h\beta x) \qquad (24)$$

$$M=-d_1\frac{d^2w}{dx^2}-\mu rw_1=(d_1\alpha^2-\mu r)(c_1\sin\alpha x+c_2\cos\alpha x)-(d_1\beta^2+\mu r)(c_3\sin h\beta x+c_4\cos h\beta x) \qquad (25)$$

$$N=-d_1\frac{d^3w_1}{dx^3}-d_2\frac{dw_1}{dx}=\alpha(d_1\alpha^2-d_2)(c_1\cos\alpha x-c_2\sin\alpha x)-\beta(d_1\beta^2+d_2)(c_3\cos h\beta x+c_4\sin h\beta x) \qquad (26)$$

We obtained from (23) equation:

$$(2d_1\alpha-d_2)^2=d_2^2+4d_1r\quad or\quad d_1\alpha^4-d_2\alpha^2-r=0 \qquad (27)$$

After some algebric operations:

$$(KI-\mu P-\mu m_2\omega^2)\alpha^4-(m_2\omega^2+P+\mu m_0\omega^2)\alpha^2-m_0\omega^2=0 \qquad (28)$$

For general vibration equation:

$$\omega^2=\alpha^2\frac{KI\alpha^2-(1+\mu\alpha^2)P}{(m_0+m_2\alpha^2)(1+\mu\alpha^2)}. \qquad (29)$$

If is looked at free vibration (P=0) it can be obtained for vibration frequency:

$$\omega=\alpha^2\sqrt{\frac{KI}{(m_0+m_2\alpha^2)(1+\mu\alpha^2)}} \qquad (30)$$

While α is defined within various boundary conditions given edges of the beam which exclusive vibration frequencey (30) calculated.

Let's examine both edges of the beam on the slider fixed condition. In this case, if x=0 and x=a, following conditions should be accounted:

$$w_1=0\ and\ M=-d_1\frac{d^2w_1}{dx^2}-\mu m_0\omega^2w_1=0 \qquad (31)$$

$$\text{or}\ \ w_1=0\ \text{and}\ \frac{d^2w_1}{dx^2}=0 \qquad (32)$$

conditions must be to take into consideration.

$P\neq 0\ or\ \alpha^2\neq\beta^2$ we can get boundary condition such as

$$c_2=0\ and c_4=0,$$
$$c_1\sin\alpha a-c_3\sin h\beta a=0,$$
$$c_1\sin\alpha a(d_1\alpha^2-r\mu)-c_3\sin h\beta a(d_1\alpha^2+r\mu)=0. \qquad (33)$$

Equations are obtained. Afterthat, in order to differentiate bending from 0, (33) systems expressions determinant would be 0;

$$\sin\alpha a=0\ or\ \alpha_n=\frac{n\pi}{a}. \qquad (34)$$

In this case, it can be obtained exclusive vibration frequency from (29):

$$\omega_n=\left(\frac{n\pi}{a}\right)^2\sqrt{\frac{KI\left(\frac{n\pi}{a}\right)^2-\left[1-\mu\left(\frac{n\pi}{a}\right)^2\right]}{\left[m_0+m_2\left(\frac{n\pi}{a}\right)^2\right]\left[1+\mu\left(\frac{n\pi}{a}\right)^2\right]}}. \qquad (35)$$

If loading is 0, we can obtain from (35) for vibration frequency:

$$\omega_n=\left(\frac{n\pi}{a}\right)^2\sqrt{\frac{KI}{\left[m_0+m_2\left(\frac{n\pi}{a}\right)^2\right]\left[1+\mu\left(\frac{n\pi}{a}\right)^2\right]}}. \qquad (36)$$

Looking at the above candition (36), it can be get beams homogenious frequency of vibration:

$$\omega_n=\omega_n^R\sqrt{1+\frac{3}{20}\gamma}. \qquad (37)$$

In here, ω_n^R is the vibration frequency which was obtained by J.Reddy [6] and calculated with following equation:

$$\omega_n=\left(\frac{n\pi}{a}\right)^2\sqrt{\frac{E_0I}{\left[m_0+m_2\left(\frac{n\pi}{a}\right)^2\right]\left[1+\mu\left(\frac{n\pi}{a}\right)^2\right]}}. \qquad (38)$$

2. Now assume that both edges of the beam are clamped. In this case, the boundary canditions are as below:

$$x=0\ and\ x=a,\ W=0,\qquad \frac{dW}{dx}=0.$$

(22), (24) expressions while writing in this conditions and after some calculations, transendental equations are:

$$-2+2\cos\alpha a\cos h\beta a+\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)\sin\alpha a\sin h\beta a=0. \qquad (39)$$

After solving (39) equation, the results of α together with (29) equation give the the natural frequency of the clamped Bernoulli beam.

For numerical calculations the following values of the parameters are utilized:

$$\rho=2300\,kg/m^3,$$
$$E=1000\ GPa, v=0.19, G=420\ GPa,$$
$$d=1.0\times10^{-9}m, I=\frac{\pi d^4}{64}=4.91\times10^{-38}m^4,$$
$$A=7.85\times10^{-19}m^2$$
$$K_s=0.877, \Omega_0=1.7\times10^{-3}m^2,$$
$$l_i=1.5\times10^{-9}m.$$

The results of calculations are presented in Table 1 and Figure 1. Here, dashed lines represent the solution of analogical homogeneous problem.

Table 1. Dimensionless frequency of vibration non-local e

e_0	γ	2	4	6	8	10
$e_0 = 0.33$	$\gamma = 0$	0.940	0.810	0.676	0.567	0.483
	$\gamma = 1$	1.008	0.868	0.725	0.608	0.518
	$\gamma = 2$	1.072	0.923	0.771	0.646	0.551
	$\gamma = -1$	0.865	0.745	0.622	0.522	0.444
	$\gamma = -2$	0.790	0.680	0.568	0.476	0.406
$e_0 = 0.67$	$\gamma = 0$	0.888	0.695	0.541	0.435	0.361
	$\gamma = 1$	0.952	0.745	0.580	0.466	0.387
	$\gamma = 2$	1.012	0.792	0.617	0.496	0.412
	$\gamma = -1$	0.817	0.639	0.498	0.400	0.332
	$\gamma = -2$	0.746	0.584	0.454	0.365	0.303
$e_0 = 1$	$\gamma = 0$	0.845	0.620	0.466	0.368	0.302
	$\gamma = 1$	0.906	0.665	0.499	0.394	0.324
	$\gamma = 2$	0.963	0.707	0.531	0.420	0.344
	$\gamma = -1$	0.777	0.570	0.429	0.339	0.278
	$\gamma = -2$	0.710	0.521	0.391	0.304	0.254

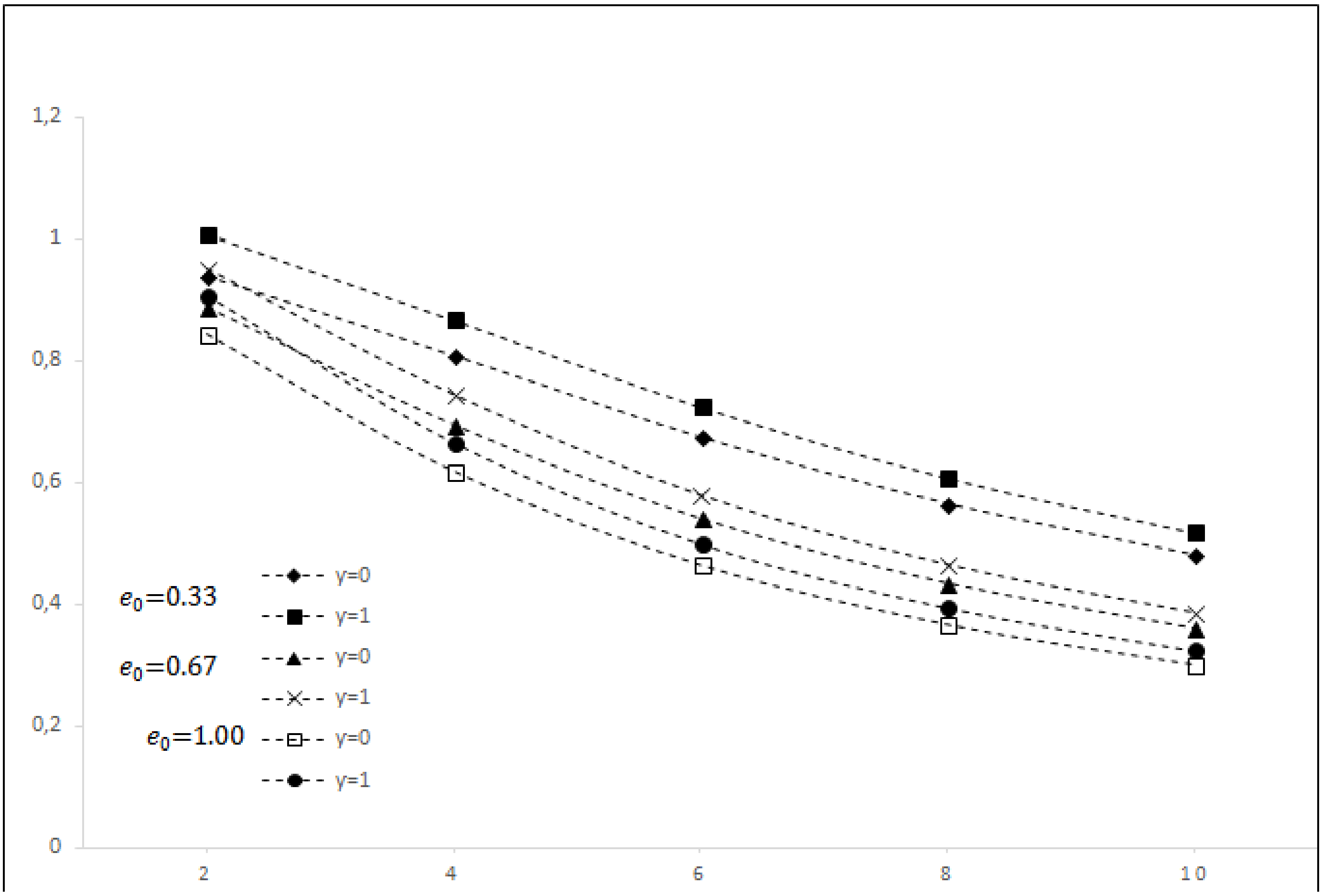

Fig 1. Dimensionless vibration equation from non-local theory (e_0) and homogeneous parameters (γ) dependence. Calculations analysis illustrate that consideration of homogeneousness could influence on the results

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