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Forward and Inverse Kinematics Analysis of Denso Robot

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Abstract
A forward and inverse kinematic analysis of 6 axis DENSO robot with closed form solution is performed in this paper. Robotics toolbox provides a great simplicity to us dealing with kinematics of robots with the ready functions on it. However, making calculations in traditional way is important to dominate the kinematics which is one of the main topics of robotics. Robotic toolbox in Matlab® is used to model Denso robot system. GUI studies including Robotic Toolbox are given with simulation examples.
Keywords: Robot Kinematics, Simulation, Denso Robot, Robotic Toolbox, GUI

1. Introduction

Robot kinematics specifies the analytical study of the motion of a robot manipulator. Formulating the reasonable kinematics models for a robot mechanism is very important in order to investigate the behaviour of industrial manipulators. There are two different spaces used in kinematics modelling of manipulators. They are called Cartesian space and Quaternion space. The transformation between two Cartesian coordinate systems can be decayed into a rotation and a translation. There are a lot of approaches to represent rotation like Euler angles, Gibbs vector, Cayley-Klein parameters, orthonormal matrices etc. Homogenous transformations based on 4x4 real matrices have been used dominantly in robotics [1].

Robotics Toolbox contains a lot of functions that are demanded in robotics and addresses fields such as kinematics, dynamics, and trajectory generation. The Toolbox is convenient for simulation besides analyzing results from experiments with real robots. It is also an effective tool for education. The Toolbox is organized on a very common method of representing the kinematics and dynamics of serial-link manipulators. It is described by the matrices. These include, in the basic case, the Denavit and Hartenberg parameters of the robot [2]. Any serial-link manipulator can be configured by the user. A number of examples are provided for well-known robots such as the Puma 560 and the Stanford arm [3]. Constantin et al. used Robotic Toolbox in forward kinematics analysis of an industrial robot [4].

This study includes kinematics of robot arm which is available Gaziantep University, Mechanical Engineering Department, Mechatronics Lab. Forward and Inverse kinematics analysis are performed. Robotics Toolbox is also applied to model Denso robot system. A GUI is built for practical use of robotic system.

2. Robot Arm Kinematics

The robot kinematics can be categorized into two main parts; forward and inverse kinematics. Forward kinematics problem is not difficult to perform and there is no complexity in deriving the equations in contrast to the inverse kinematics. Especially nonlinear equations make the inverse kinematics problem complex. They may also be coupled and have not got unique solutions. Thus solutions obtained mathematically may not solve the problem physically [5]. Liu et al. applied geometric approach for inverse kinematics analysis of a 6 dof robot [6]. Qiao et al. used double quaternions to get solution for inverse kinematics problem [7]. Nubiola and Bonev offered a simple and efficient way to solve inverse kinematics problem for 6R robots [8]. It is noticed that, Artificial Intelligence (AI) methods are frequently used in inverse kinematics problem [9, 10, 11] in recent years.

2.1 Forward Kinematics Analysis

The forward kinematics problem is related between the individual joints of the robot manipulator and the position and orientation of the tool or end-effector. The joint variables are the angles between the links for revolute or prismatic joints, and the link extension in the prismatic or sliding joints [12]. A systematic way of describing the geometry of a serial chain of links and joints was proposed by Denavit and Hartenberg and is known today as Denavit-Hartenberg (DH) notation [2]. The matrix A representing four movements is found by postmultiplying the four matrices giving four movements to reach frame {j-1} to frame {j} in Figure 1.

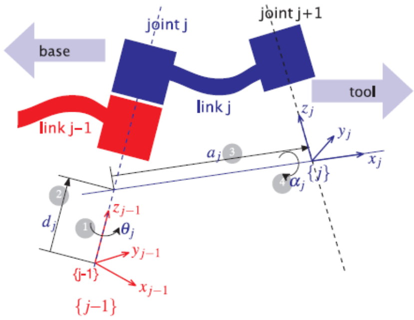
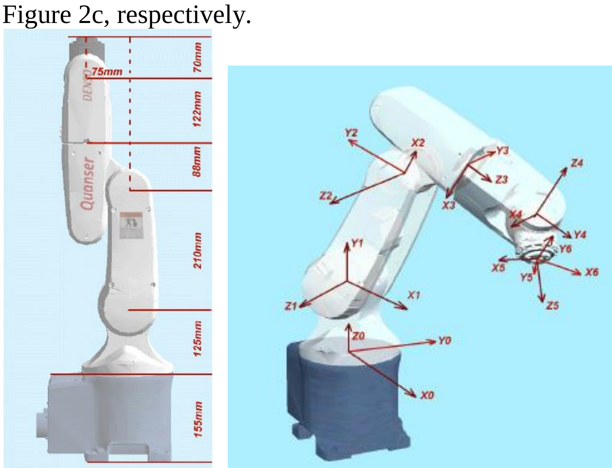


Fig. 1. DH representation of a general purpose joint-link combination

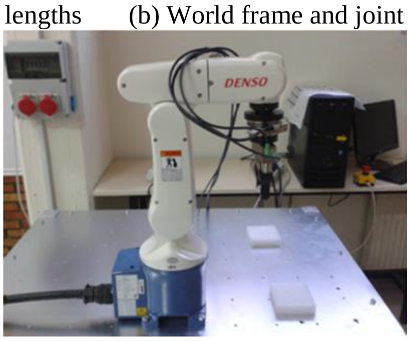
Transformation between two joints in a generic form [3] is given in Eq. (1).

$$
{}^{j-1}A_j = \begin{bmatrix} \cos\theta_j & -\sin\theta_j\cos\alpha_j & \sin\theta_j\sin\alpha_j & a_j\cos\theta_j \\ \sin\theta_j & \cos\theta_j\cos\alpha_j & -\cos\theta_j\sin\alpha_j & a_j\sin\theta_j \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

Denso Robot is a 6 Degrees-of-Freedom (DOF) robotic manipulator. The link lengths are given in Figure 2(a). World frame and joint frames used in calculations and home position of Denso robot are shown in Figure 2b and Figure 2c, respectively.



(a) Link lengths (b) World frame and joint frames



(c) Home position

Fig.2. General overview of DENSO

The following table shows DH parameters of the Denso robotic arm necessary to derive the kinematics of the robot. Gripper is not included in the analysis.

Table 1. DH Parameters of the Denso Robotic Arm

Joint i	θ_i	d_i	a_i	α_i	Joint Limits (degrees)
1	q_1	d_1	0	$\pi/2$	–160, 160
2	q_2	0	a_2	0	–120, 120
3	q_3	0	a_3	$-\pi/2$	20, 160
4	q_4	d_4	0	$\pi/2$	–160, 160
5	q_5	0	0	$-\pi/2$	–120, 120
6	q_6	d_6	0	0	–360, 360

where d_1 = 0.125m, a_2 =0.21m, a_3 =-0.075m, d_4 = 0.21m and d_6 = 0.07m.

Transformation matrix for each joint can be obtained by using Eq. (1). The parameters given in Table 1 are substituted into Eq. (1) to find each of them. Six transformation matrices are presented in Eq. (2).

$$
A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
A_3 = \begin{bmatrix} C_3 & 0 & -S_3 & a_3C_3 \\ S_3 & 0 & C_3 & a_3S_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
A_5 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
$$

where Cos and Sin are abbreviated to C and S, respectively. The total transformation between the base of the robot and the hand is;

$$
{}^{R}T_H = A_1A_2A_3A_4A_5A_6 \tag{3}
$$

Transformation matrices for six axes given in Eq. (2) are postmultiplied in an order which is given in Eq. (3). This equality is shown in Eq. (4).

$$\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_1A_2A_3A_4A_5A_6 \tag{4}$$

where n (normal), o (orientation), a (approach) elements are for orientation and P (position) elements are position elements relative to the reference frame [12]. The elements of the matrix shown in left hand side of Eq. (4) are given in Eqs. (5, 6, 7 and 8).

$$\begin{aligned} n_x &= -S_6(C_4S_1 + S_4(C_1C_2C_3 - C_1S_2S_3)) - C_6(C_5(S_1S_4 - C_4(C_1C_2C_3 - \\ &\quad C_1S_2S_3)) + S_5(C_1C_2S_3 + C_1C_3S_2)) \\ n_y &= S_6(C_1C_4 + S_4(S_1S_2S_3 - C_2C_3S_1)) + C_6(C_5(C_1S_4 - C_4(S_1S_2S_3 - \\ &\quad C_2C_3S_1)) - S_5(C_2S_1S_3 + C_3S_1S_2)) \\ n_z &= C_6(S_5(C_2C_3 - S_2S_3) + C_4C_5(C_2S_3 + C_3S_2)) - S_4S_6(C_2S_3 + C_3S_2) \end{aligned} \tag{5}$$

$$\begin{aligned} o_x &= S_6(C_5(S_1S_4 - C_4(C_1C_2C_3 - C_1S_2S_3)) + S_5(C_1C_2S_3 + C_1C_3S_2)) - \\ &\quad C_6(C_4S_1 + S_4(C_1C_2C_3 - C_1S_2S_3)) \\ o_y &= C_6(C_1C_4 + S_4(S_1S_2S_3 - C_2C_3S_1)) - S_6(C_5(C_1S_4 - C_4(S_1S_2S_3 - \\ &\quad C_2C_3S_1)) - S_5(C_2S_1S_3 + C_3S_1S_2)) \\ o_z &= -S_6(S_5(C_2C_3 - S_2S_3) + C_4C_5(C_2S_3 + C_3S_2)) - C_6S_4(C_2S_3 + C_3S_2) \end{aligned} \tag{6}$$

$$\begin{aligned} a_x &= S_5(S_1S_4 - C_4(C_1C_2C_3 - C_1S_2S_3)) - C_5(C_1C_2S_3 + C_1C_3S_2) \\ a_y &= -S_5(C_1S_4 - C_4(S_1S_2S_3 - C_2C_3S_1)) - C_5(C_2S_1S_3 + C_3S_1S_2) \\ a_z &= C_5(C_2C_3 - S_2S_3) - C_4S_5(C_2S_3 + C_3S_2) \end{aligned} \tag{7}$$

$$\begin{aligned} P_x &= d_6(S_5(S_1S_4 - C_4(C_1C_2C_3 - C_1S_2S_3)) - C_5(C_1C_2S_3 + C_1C_3S_2)) - d_4(C_1C_2S_3 \\ &\quad + C_1C_3S_2) + a_2C_1C_2 + a_3C_1C_2C_3 - a_3C_1S_2S_3 \\ P_y &= a_2C_2S_1 - d_6(S_5(C_1S_4 - C_4(S_1S_2S_3 - C_2C_3S_1)) + C_5(C_2S_1S_3 + C_3S_1S_2)) - \\ &\quad d_4(C_2S_1S_3 + C_3S_1S_2) + a_3C_2C_3S_1 - a_3S_1S_2S_3 \\ P_z &= d_1 + d_4(C_2C_3 - S_2S_3) + d_6(C_5(C_2C_3 - S_2S_3) - C_4S_5(C_2S_3 + \\ &\quad C_3S_2)) + a_2S_2 + a_3C_2S_3 + a_3C_3S_2 \end{aligned} \tag{8}$$

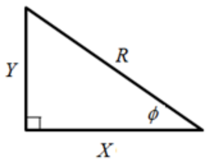
2.2 Inverse Kinematics Analysis

The transformation process of the position and orientation of an end-effector from Cartesian space to joint space is defined as inverse kinematics problem. There are three solutions approaches; analytical, numerical and semi analytical [5]. Analytical approach is used herein.

To find the inverse kinematics solution for the 1st joint θ_1 as a function of the known elements, the 6th link transformation inverse is postmultiplied as follows in Eq. (9).

$$A_1A_2A_3A_4A_5A_6A_6^{-1} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix} xA_6^{-1} \tag{9}$$

where $A_6A_6^{-1} = I$. I is identity matrix. In this case, the above equation is resulted in Eq. (10).



$$A_1A_2A_3A_4A_5 = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix} xA_6^{-1} \tag{10}$$

The required multiplication in Eq. (10) is carried out and it yields as Eq. (11).

$$\begin{bmatrix} '' & '' & '' & C_1 \\ '' & '' & '' & S_1 \\ '' & '' & '' & '' \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_xC_6 - o_xS_6 & o_xC_6 + n_xS_6 & a_x & P_x - a_xd_6 \\ n_yC_6 - o_yS_6 & o_yC_6 + n_yS_6 & a_y & P_y - a_yd_6 \\ n_zC_6 - o_zS_6 & o_zC_6 + n_zS_6 & a_z & P_z - a_zd_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

It is noticed that the elements located in 1st row & 4th column (abbreviated to (1, 4)) and 2nd row & 4th column (abbreviated to (2, 4)) can be used in defining θ_1 (these abbreviations are also used in the remaining part of the inverse kinematics analysis). All elements in the left-hand side of the Eq. (11) are known. However, all of them are not used in calculation of θ_1 because of limitations of space. Due to these reasons, they are not written. The symbol ‘ " ’ is used instead of them. From (1, 4) and (2, 4) elements, θ_1 is found in Eq. (12).

$$\theta_1 = \tan^{-1}\left(\frac{P_y - a_yd_6}{P_x - a_xd_6}\right) \pm pi \tag{12}$$

The 1st link inverse transformation matrix is premultiplied by Eq. (10) to find the inverse kinematics solution for the 3rd joint (θ_3) as a function of the known elements. It is given in Eq. (13).

$$A_1^{-1}A_1A_2A_3A_4A_5 = A_1^{-1}x\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix} xA_6^{-1} \tag{13}$$

where $A_1A_1^{-1} = I$. I is identity matrix. In this case the above equation is resulted in Eq. (14).

$$A_2A_3A_4A_5 = A_1^{-1}x\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix}xA_6^{-1} \qquad (14)$$

The required multiplication in Eq. (14) is carried out and it yields as Eq. (15).

$$\begin{bmatrix} " & " & " & a_3C_{23}-d_4S_{23}+a_2C_2 \\ " & " & " & d_4C_{23}+a_3S_{23}+a_2S_2 \\ " & " & " & " \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} " & " & " & C_1(P_x-a_xd_6)+S_1(P_y-l_5a_y) \\ " & " & " & P_z-d_1-l_5a_z \\ " & " & " & " \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

S_{23} and C_{23} refer to $\text{Sin}(\theta_2+\theta_3)$ and $\text{Cos}(\theta_2+\theta_3)$, respectively. From (1, 4) and (2, 4) elements of the equation,

$$\begin{aligned} a_3C_{23}-d_4S_{23}+a_2C_2 &= C_1(P_x-a_xd_6)+S_1(P_y-l_5a_y) \\ d_4C_{23}+a_3S_{23}+a_2S_2 &= P_z-d_1-l_5a_z \end{aligned} \qquad (16)$$

Right hand side of the Eq. (16) is known. They are recalled as;

$$\begin{aligned} A &= C_1(P_x-a_xd_6)+S_1(P_y-l_5a_y) \\ B &= P_z-d_1-l_5a_z \end{aligned} \qquad (17)$$

Eq. (16) can be rewritten as below.

$$\begin{aligned} a_3C_{23}-d_4S_{23}+a_2C_2 &= A \\ d_4C_{23}+a_3S_{23}+a_2S_2 &= B \end{aligned} \qquad (18)$$

Having taken the squares of these expressions, they are added to each other and it yields as;

$$a_2^{\,2}+a_3^{\,2}+d_4^{\,2}+2a_2a_3\underbrace{(C_2C_{23}+S_2S_{23})}_{\text{Cos}(-\theta_3)}+2a_2d_4\underbrace{(S_2C_{23}-C_2S_{23})}_{\text{Sin}(-\theta_3)} = A^2+B^2 \qquad (19)$$

The known parts are taken to the right hand side as shown in Eq. (20),

$$2a_2a_3\,\text{Cos}(-\theta_3)+2a_2d_4\,\text{Sin}(-\theta_3) = A^2+B^2-a_2^{\,2}-a_3^{\,2}-d_4^{\,2} \qquad (20)$$

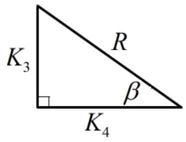
Eq. (20) is then simplified and rewritten as Eq. (21),

$$X\,\text{Cos}(-\theta_3)+Y\,\text{Sin}(-\theta_3) = Z \qquad (21)$$

where

$$\begin{aligned} X &= 2a_2a_3 \\ Y &= 2a_2d_4 \\ Z &= A^2+B^2-a_2^{\,2}-a_3^{\,2}-d_4^{\,2} \end{aligned} \qquad (22)$$

A triangle including X and Y can be formed to solve Eq. (21) as an auxiliary technique.



where $X = R\,\text{Cos}\,\phi$ $\quad Y = R\,\text{Sin}\,\phi$

$R = \sqrt{X^2+Y^2}$ $\quad \phi = \text{Tan}^{-1}\left(\frac{Y}{X}\right)$

X and Y expressions are substituted in Eq. (21), it yields,

$$R\,\text{Cos}\,\phi\,\text{Cos}(-\theta_3)+R\,\text{Sin}\,\phi\,\text{Sin}(-\theta_3) = Z \qquad (23)$$

Eq. (24) is obtained by using trigonometric addition formula in Eq. (23),

$$\text{Cos}(\phi+\theta_3) = Z/R \qquad (24)$$

with the inverse cosine operation, $\phi+\theta_3 = \pm\text{Cos}^{-1}(Z/R)$ equality is obtained. Then θ_3 is found as given in Eq. (25).

$$\theta_3 = \pm\text{Cos}^{-1}(Z/R)-\phi \qquad (25)$$

The 2nd link inverse transformation matrix is premultiplied by Eq. (14) to find the inverse kinematics solution for the 2nd joint (θ_2) as a function of the known elements in Eq. (26).

$$A_2^{-1}A_2A_3A_4A_5 = A_2^{-1}A_1^{-1}x\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix}xA_6^{-1} \qquad (26)$$

where $A_2A_2^{-1} = I$. I is identity matrix. In this case the above equation is resulted in Eq. (27).

$$A_3A_4A_5 = A_2^{-1}A_1^{-1}x\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix}xA_6^{-1} \qquad (27)$$

The required multiplication in Eq. (27) is carried out, and then yields Eq. (28).

$$\begin{bmatrix} " & " & " & a_3C_3-d_4S_3 \\ " & " & " & d_4C_3+a_3S_3 \\ " & " & " & " \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} " & " & " & C_2(C_1(P_x-a_xd_6)+S_1(P_y-a_y-d_6))-a_2-S_2(d_1-P_z+a_zd_6) \\ " & " & " & -S_2(C_1(P_x-a_xd_6)+S_1(P_y-a_yd_6))-C_2(d_1-P_z+a_zd_6) \\ " & " & " & " \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (28)$$

From (1, 4) element of the equation above,

$$a_3C_3-d_4S_3 = C_2(C_1(P_x-a_xd_6)+S_1(P_y-a_y-d_6))-a_2-S_2(d_1-P_z+a_zd_6) \qquad (29)$$

Eq. (29) can be rewritten as,

$$C_2K_1+S_2(-K_2) = D \qquad (30)$$

where

$$K_1 = (C_1(P_x - a_x d_6) + S_1(P_y - a_y - d_6))$$
$$K_2 = (d_1 - P_z + a_z d_6)$$
$$D = a_3 C_3 - d_4 S_3 + a_2$$

A triangle including K_1 and K_2 can be formed to solve Eq. (30) as an auxiliary technique. The procedure applied in determination of θ_3 is carried out. So, all steps are not explained.

where $\quad S = \sqrt{K_1^{\,2} + K_2^{\,2}} \qquad \gamma = \mathrm{Tan}^{-1}\left(\dfrac{-K_2}{K_1}\right)$

Then, θ_2 is found as given in Eq. (31)

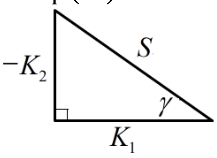
$$\theta_2 = \gamma \pm \mathrm{Cos}^{-1}(D/S) \tag{31}$$

θ_2 expression can also be obtained from (2,4) elements of Eq. (28),

$$d_4 C_3 + a_3 S_3 = -S_2(C_1(P_x - a_x d_6) + S_1(P_y - a_y d_6)) - C_2(d_1 - P_z + a_z d_6) \tag{32}$$

Eq. (32) can be rewritten as,

$$S_2 K_3 + C_2 K_4 = E \tag{33}$$



where

$$K_3 = (C_1(P_x - a_x d_6) + S_1(P_y - a_y d_6))$$
$$K_4 = (d_1 - P_z + a_z d_6)$$
$$E = -d_4 C_3 - a_3 S_3$$

A triangle including K_3 and K_4 can be formed to solve Eq. (33) as done in the previous steps.

where $\quad R = \sqrt{K_3^{\,2} + K_4^{\,2}} \qquad \beta = \mathrm{Tan}^{-1}\left(\dfrac{K_3}{K_4}\right)$

Then, θ_2 is found as given in Eq. (34).

$$\theta_2 = \beta \pm \mathrm{Cos}^{-1}(E/R) \tag{34}$$

The inverse transformation matrices of 1st, 2nd and 3rd joints are premultiplied by Eq. (4) to find the inverse kinematics solution for the 5th joint (θ_5). It is given in Eq. (35).

$$(A_1A_2A_3)^{-1}A_1A_2A_3A_4A_5A_6 = (A_1A_2A_3)^{-1}x\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}$$

where $(A_1A_2A_3)^{-1}A_1A_2A_3 = I$. I is identity matrix. The above equation is resulted in Eq. (36).

$$A_4A_5A_6 = (A_1A_2A_3)^{-1}x\begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_x \\ n_z & o_z & a_z & P_x \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{36}$$

The required multiplication in Eq. (36) is carried out, and it is given in Eq. (37).

$$\begin{bmatrix} '' & '' & '' & '' \\ '' & '' & '' & '' \\ '' & '' & C_5 & '' \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} '' & '' & '' & '' \\ '' & '' & '' & '' \\ '' & '' & '' & a_zC_{23} - a_xS_{23}C_1 - a_yS_{23}S_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{37}$$

From (3, 3) elements of the equation above,

$$C_5 = a_zC_{23} - a_xS_{23}C_1 - a_yS_{23}S_1 \tag{38}$$

Then, θ_5 is found as given in Eq. (39).

$$\theta_5 = \pm\mathrm{Cos}^{-1}(a_zC_{23} - a_xS_{23}C_1 - a_yS_{23}S_1) \tag{39}$$

The inverse transformation matrices of 1st, 2nd and 3rd joints are premultiplied by Eq. (4) to find the inverse kinematics solution for the 4th joint (θ_4) as a function of the known elements. Multiplied matrix used in previous step is also used in determination of θ_4 angle. It is given in Eq. (40). The elements (1, 3) and (2, 3) of the equation are preferred in Eq. (42).

$$\begin{bmatrix} '' & '' & -C_4S_5 & '' \\ '' & '' & -S_4S_5 & '' \\ '' & '' & '' & '' \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} '' & '' & a_zS_{23} + a_xC_{23}C_1 + a_yC_{23}S_1 & '' \\ '' & '' & a_yC_1 - a_xS_1 & '' \\ '' & '' & '' & '' \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{40}$$

Then, θ_4 is found as in Eq. (41).

$$\theta_4 = \tan^{-1}\left(\frac{a_yC_1 - a_xS_1}{a_zS_{23} + a_xC_{23}C_1 + a_yC_{23}S_1}\right) \tag{41}$$

The inverse transformation matrices of 1st, 2nd and 3rd joints are premultiplied by Eq. (4) to find the inverse kinematics solution for the 6th joint (θ_6) as a function of the known elements. Multiplied matrix used in previous two steps (θ_5 and θ_4) is also used in determination of θ_6 angle. It is given in Eq. (35). The elements (3, 1) and (3, 2) of the equation are given in Eq. (42),

$$\begin{bmatrix} " & " & " & " \\ " & " & " & " \\ C_6S_5 & -S_5S_6 & " & " \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} " & " & " & " \\ " & " & " & " \\ n_zC_{23}-n_xS_{23}C_1-n_yS_{23}S_1 & o_zC_{23}-o_xS_{23}C_1-o_yS_{23}S_1 & " & " \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{42}$$

Then, θ_6 is found as given in Eq. (43).

$$\theta_6 = \tan^{-1}\left(\frac{-(o_zC_{23}-o_xS_{23}C_1-o_yS_{23}S_1)}{n_zC_{23}-n_xS_{23}C_1-n_yS_{23}S_1}\right) \tag{43}$$

Inverse problem solution does not always give one solution; the same end effector pose can be reached in many different configurations [13]. Previous positions of the motors are fed to the program in each step in order to offer a solution to this problem in Denso robot system. The difference between calculated position and previous position are obtained. The solution having the minimum difference is selected. So the robot will not try to jump to far positions; it will go to the reach the nearest solution. Each solution obtained by the inverse kinematics analysis should be tested in order to determine whether or not they bring the end-effector to the desired position.

3. Robotics Toolbox in Matlab®

The Toolbox performs many functions for analyzing and simulation of arm type robotics in fields of kinematics, dynamics, and trajectory generation. The Toolbox is based on a very general method of representing the kinematics and dynamics of serial-link manipulators. These parameters are included in MATLAB® objects [3]. Designed model by robotics toolbox is given in Figure 3.

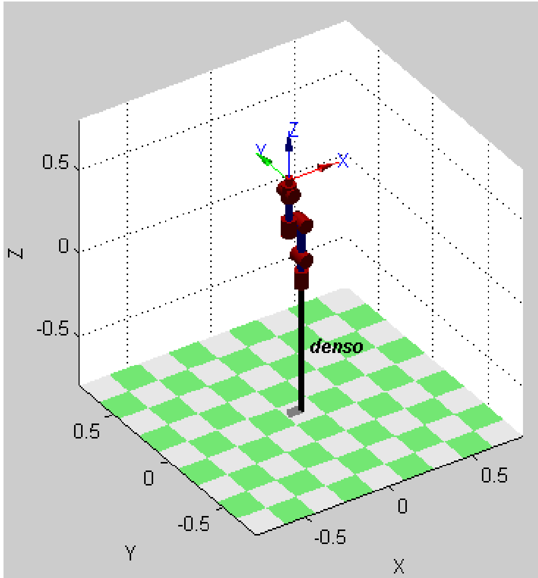


Fig. 3. Robotics toolbox with Denso

The joint space trajectories can be calculated by inverse kinematics and a simulation for robot can be done to move robot from initial position to final position in Cartesian space.

A straight line whose initial and final coordinates in X, Y, Z coordinates are [0.35 0 0.4] m. and [0.2 0.1 0.5] m. respectively is given in Figure 4. It is desired to plan the path composed of N number of points; N= 50 in this example.

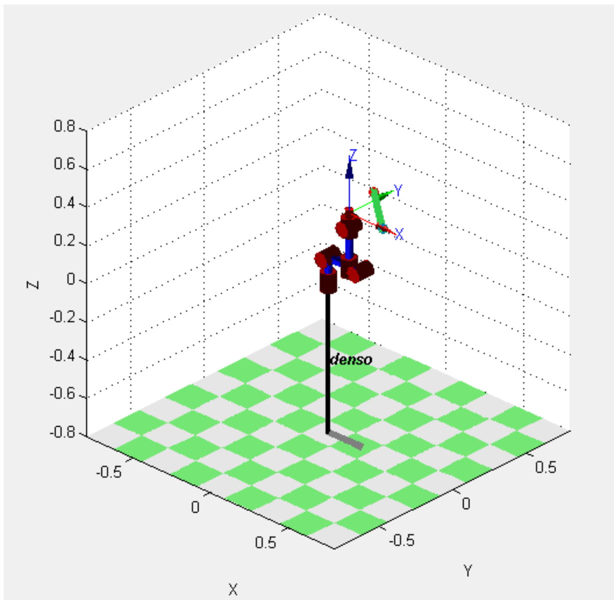


Fig. 4. Planned path

The Cartesian coordinates (X, Y and Z) of the path is given in Figure 5.

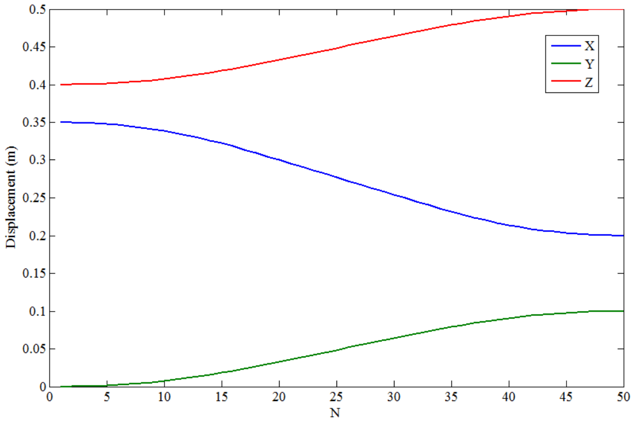


Fig. 5. Coordinates of the path

Transformation matrices are calculated by using the path data. Inverse kinematics analysis is applied. The angular displacements of the links of the robot are obtained as in Figure 6.

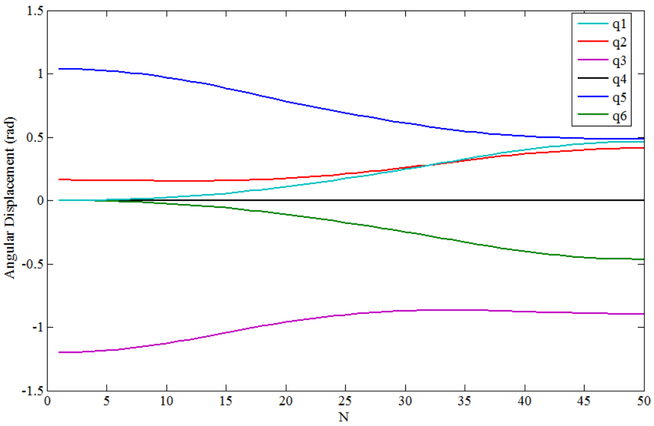
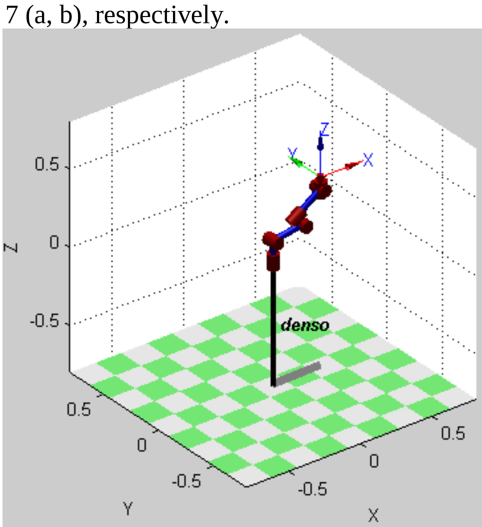
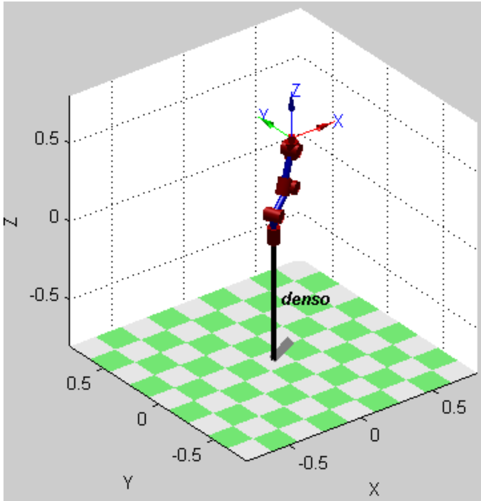


Fig. 6. Angular displacements of the robot links

Initial and final configurations of the robot are shown in Figure 7 (a, b), respectively.



(a)



(b)

Fig. 7. Initial and final configurations of the robot

4. Guide User Interface (GUI)

Graphical User Interface development environment offers a set of tools in order to generate graphical user interfaces (GUIs) [14]. They greatly facilitate the operation of designing and building GUIs. A GUI example has been prepared for Denso robot including the forward kinematics. GUI is given in Figure 8. Push buttons, sliders, axes etc. can be added on it. Additions are being visible on the m. file simultaneously as a function. Robotics Toolbox is embedded to GUI. The results are compared with the expressions obtained in the analytical solution. It is proved that same results are obtained by the robotic toolbox and the analytical solution.

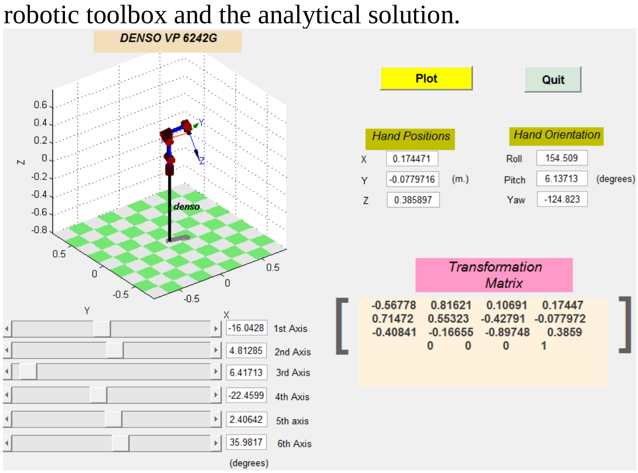
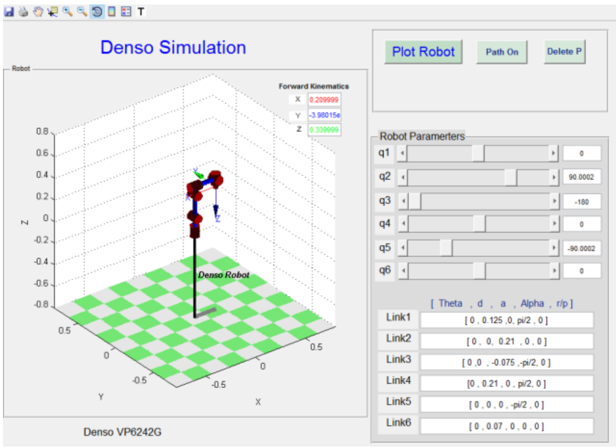
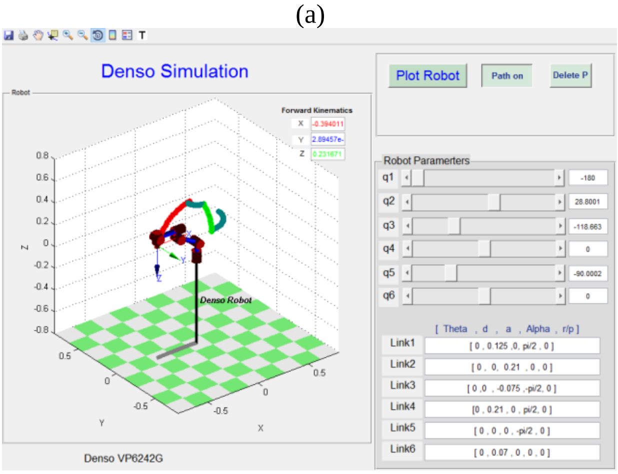


Fig. 8. Designed GUI example I

Figure 9 shows the designed simulation program by MATLAB/GUIDE to create serial link robot (Link 1-6) and control the joint angles (q_1, q_2, q_3, q_4, q_5 and q_6). DH parameters can directly be changed. Figure 9(a) and Figure 9(b) show the initial condition of the robot and the path generated with the given coordinates, respectively.



(a)



(b)

Fig. 9. Designed GUI example II

5. Conclusion

In this study, it is focused on determining the analytical solution of forward & inverse kinematics of the Denso 6 axis robot which is available in the laboratory. The equations obtained are reported.

Robotics toolbox is provided a great simplicity to us about kinematics of robots with the ready functions on it. However, making calculations in traditional way is important in order to control the robot and to form a background for further studies. Toolbox is then used to verify the results obtained by analytical way. The results are the same. User interfaces are the effective tools to show many works in a compact way. Due to this reason, Guide User Interface is performed. It is possible to reach simultaneous transformation matrix, position & orientation of robot hand when angular displacements of the motors are changed one by one.

This study is contained a part of theoretical and numerical kinematics analysis of Denso robot. It is performed to build a background study for rehabilitation robotics issues. Studies on adapting Denso robot for upper extremity rehabilitation are going on in Mechatronics Laboratory at Gaziantep University.

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