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Sliding Mode Based Self-Tuning PID Controller for Second Order Systems

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Abstract
In this paper, a sliding mode based self-tuning PID controller is proposed for second order systems. While developing the controller, it is assumed that the system model has a part which contains nonlinear terms similar to PID structure. The controller and update rules for PID parameters are obtained from Lyapunov stability analysis. Numerical simulations are conducted on a Twin-Rotor Multi-Input Multi-Output System (TRMS) model to show the performance of the proposed controller.
Keywords: Sliding Mode Controller, Self-Tuning PID.

1. Introduction

PID control is the most preferred control technique in industrial applications due to its simple structure and convenience in implementation [1]. However, the effectiveness of the PID controller is based on the accurate selection of its parameters. Despite the good performance results in linear systems, the selection of the parameters might be very difficult and time wasting with the rise of nonlinearities of the system. To deal with this problem many approaches of self-tuning PID controllers have been presented till today. These approaches can be divided into two main categories: i) model based approaches and ii) rule-based approaches. In model based approaches, the tuning mechanism is based on the knowledge of the system model [2]. In rule based approaches, the tuning is based on some optimization or estimation rules without model knowledge, which basically mimics an experienced operator's behavior [2]. A good survey can be found in [2] on this topic.

In the literature, many studies can be found on self-tuning PID controller and its applications. In [3], An *et. al.* presented a self-tuning method for PID controllers based on the theory of adaptive interaction for the quadrotor system. In [4], a self-tuning PID control scheme based on support vector machine (SVM) and particle swarm optimization (PSO) were presented. Jiang and Jiang proposed a fuzzy based self-tuning PID controller for temperature control [5]. Zheng *et. al.* used fuzzy module to tune PID controller parameters according to the error and change in error [6]. In [7] and [8], genetic algorithm was utilized to tune the PID parameters. Na presented a study on water level control of a nuclear steam generator with PID controller of which parameters were tuned by model predictive control (MPC) [9]. In [1], least squares support vector machine identifier was utilized to tune parameters of PID controller. Fan *et. al.* used neural network to tune PID controller for position tracking of a pneumatic artificial muscle [10]. Gundogdu and Komurgaz presented a self-tuning algorithm for PID controller based on adaptive interaction approach [11]. In [12], Howell and Best used continuous action reinforcement learning automata (CARLA) method to tune the PID controller parameters while controlling engine idle-speed. In [13], Shih and Tseng designed a self-tuning PID controller by using integral of time-weighted absolute error (ITAE) optimal control principle and the pole-placement approach to control position of a servo-cylinder. Dong and Mo presented model reference adaptive PID controller for motor control system with backlash [14]. In [15], Chamsai *et. al.* presented an adaptive PID controller combined with sliding mode controller for uncertain nonlinear systems. Chang and Yan proposed an adaptive PID controller based on sliding mode controller for uncertain chaotic systems [16]. Kuo *et. al.* presented an adaptive sliding mode controller with PID tuning method for a class of uncertain systems [17].

In this paper, a sliding mode based self-tuning PID controller is proposed for uncertain second order systems. Different from the literature, it is assumed that the model contain nonlinear terms similar to PID structure. The controller and update rules for PID parameters are obtained from Lyapunov stability analysis. Numerical simulations are conducted on a Twin-Rotor Multi-Input Multi-Output System (TRMS) model to test the performance of controller and parameter update rule.

The rest of the paper is organized as follows; the system model is presented in Section 2. Control and parameter update rule design are presented in Section 3. Numerical simulation results are given in Section 4.

Finally concluding remarks are presented in Section 5.

2. System Model

The following second order system is considered in this paper,

$$\dot{x}_1(t) = x_2 \tag{1}$$
$$\dot{x}_2(t) = f(\mathrm{x}) + u(t) \tag{2}$$

where x(t) = $[x_1(t), x_2(t)]^T$ is state vector, $u(t) \in R$ is control signal. The function $f(\cdot):R^2 \to R$ is assumed in the form of

$$f(x) = g(x) + k_p x_1(t) + k_d x_2(t) + k_i \int x_1(t). \tag{3}$$

where $g(\cdot):R^2 \to R$ is unknown function, k_p, k_d and k_i are unknown system parameters.

Assumption 1: It is assumed that the function $g(\cdot)$ is bounded as

$$|g(x)| \le \rho \tag{4}$$

where ρ is known.

Assumption 2: It is assumed that the system parameters, k_p, k_d and k_i are in known bounded regions.

Assumption 3: It is assumed that x(t) is available and continuous.

3. Control and Parameter Update Rule Design

The objective of the controller is to utilize that $x_1(t)$ track a desired trajectory while updating PID parameters. To achieve this objective, the error system is designed as follows,

$$\tilde{x}_1(t) = x_{d1} - x_1 \tag{5}$$
$$\tilde{x}_2(t) = x_{d2} - x_2 \tag{6}$$

where x_{d1} and x_{d2} are desired trajectories. To construct sliding mode controller, the filtered error signal is designed as

$$s = \tilde{x}_2 + 2\lambda\tilde{x}_1 + \lambda^2 \int \tilde{x}_1. \tag{7}$$

The derivative of (7), which will be utilized later, is

$$\begin{aligned}\dot{s} &= \dot{\tilde{x}}_2 + 2\lambda\dot{\tilde{x}}_1 + \lambda^2\tilde{x}_1 \\ &= \dot{x}_{d2} - g - k_p x_1 - k_d \dot{x}_1 - k_i \int x_1 - u \\ &\quad + 2\lambda\dot{x}_{d1} - 1\lambda^2 \dot{x}_1 + \lambda^2\tilde{x}_1.\end{aligned} \tag{8}$$

The control input is designed as

$$u = u_{PID} + u_R \tag{9}$$

where u_R is sliding part of the controller and

$$u_{PID} = \hat{k}_p \tilde{x}_1 + \hat{k}_d \dot{\tilde{x}}_1 + \hat{k}_i \int \tilde{x}_1 \tag{10}$$

where $\hat{k}_p$, $\hat{k}_d$ and $\hat{k}_i$ are estimates of k_p, k_d ve k_i, respectively.

By substituting the (9) and (10) in (8), it is obtained as

$$\begin{aligned}\dot{s} &= \dot{x}_{d2} - g - \tilde{k}_p x_1 - \tilde{k}_d \dot{x}_1 - \tilde{k}_i \int x_1 \\ &\quad - \hat{k}_p x_{d1} - \hat{k}_d \dot{x}_{d1} - \hat{k}_i \int x_{d1} - u_R \\ &\quad + 2\lambda\dot{x}_{d1} - 2\lambda\dot{x}_1 + \lambda^2\tilde{x}_1\end{aligned} \tag{11}$$

where

$$\tilde{k}_p = k_p - \hat{k}_p,\ \tilde{k}_d = k_d - \hat{k}_d,\ \tilde{k}_i = k_i - \hat{k}_i. \tag{12}$$

The Lyapunov function in (13) is utilized to construct update rules for PID gains and design u_R

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{k}_p^2 + \frac{1}{2}\tilde{k}_d^2 + \frac{1}{2}\tilde{k}_i^2. \tag{13}$$

The derivative of (13) is obtained as

$$\begin{aligned}\dot{V} &= s\dot{s} + \tilde{k}_p\dot{\tilde{k}}_p + \tilde{k}_d\dot{\tilde{k}}_d + \tilde{k}_i\dot{\tilde{k}}_i \\ &= s(\dot{x}_{d2} - g - \hat{k}_p x_{d1} - \hat{k}_d \dot{x}_{d1} - \hat{k}_i \int x_{d1} - u_R \\ &\quad + 2\lambda\dot{x}_{d1} - 2\lambda\dot{x}_1 + \lambda^2\tilde{x}_1) - \tilde{k}_p(s x_1 + \dot{\hat{k}}_p) \\ &\quad - \tilde{k}_p(s\dot{x}_1 + \dot{\hat{k}}_d) - \tilde{k}_i(s\int x_1 + \dot{\hat{k}}_i)\end{aligned} \tag{14}$$

From (14), the update rules of $\hat{k}_p$, $\hat{k}_d$ and $\hat{k}_i$ are selected as in (15) to eliminate the terms with gain errors.

$$\dot{\hat{k}}_p = -s x_1,\ \dot{\hat{k}}_d = -s\dot{x}_1,\ \dot{\hat{k}}_i = -s\int x_1 \tag{15}$$

After substitution of (15) in (14), $\dot{V}$ is obtained as

$$\begin{aligned}\dot{V} &= s(\dot{x}_{d2} + 2\lambda\dot{\tilde{x}}_1 + \lambda^2\tilde{x}_1) - sg \\ &\quad - s(\hat{k}_p x_{d1} + \hat{k}_d \dot{x}_{d1} + \hat{k}_i \int x_{d1}) - s u_R\end{aligned} \tag{16}$$

The input signal u_R should be designed to make $\dot{V}$ negative. To achieve this purpose, u_R will be investigated by separating into three terms as

$$u_R = u_1 + u_2 + u_3. \tag{17}$$

u_1, is designed as to eliminate first two terms in (16) as

$$u_1 = \dot{x}_{d2} + 2\lambda\dot{\tilde{x}}_1 + \lambda^2\tilde{x}_1 + k\,\mathrm{sgn}(s),\ k \in R^+ \tag{18}$$

To eliminate the term sg in (16), the condition in assumption 1 can be utilized. From (4) the following inequality can be obtained

$$-sg \le |s|\rho,\ \rho \in R^+ \tag{19}$$

By using (19), u_2 is designed as follows

$$u_2 = \frac{|s|}{s}\rho . \qquad (20)$$

By substituting (18) and (20) in (16), $\dot{V}$ is obtained as,

$$\dot{V} \le -k|s| - sL - su_3 \qquad (21)$$

where

$$L = \hat{k}_p x_{d1} + \hat{k}_d \dot{x}_{d1} + \hat{k}_i \int x_{d1} . \qquad (22)$$

An upper bound for L can be defined as

$$L_m > |\bar{k}_p||x_{d1}| + |\bar{k}_d||\dot{x}_{d1}| + |\bar{k}_i|\left|\int |x_{d1}|\right| . \qquad (23)$$

where $\bar{k}_p$, $\bar{k}_d$ and $\bar{k}_i$ are upper bounds of k_p, k_d and k_i, respectively.

Hence, the following inequality can be written

$$-sL < |s|L_m \qquad (24)$$

Remark 1: In (23), L_m may go to infinity for $x_{d1} \neq 0$ since integral term. But it should be kept in mind that the main interested term is $|s|L_m$. So if it can be proven that s(t) converge to 0, fast enough, then, it can be assumed that the term $|s|L_m$ stays bounded. So L_m can be accepted as bounded.

In the rest of the paper, it will be proven that s(t) converge to 0 with a tunable rate.

From (24),

$$\dot{V} = -k|s| + |s|L_m - su_3 . \qquad (25)$$

So, u_3 can be obtained as

$$u_3 = \frac{|s|}{s} L_m \qquad (26)$$

This leads

$$\dot{V} < -k|s| . \qquad (27)$$

From (27), it can be said that s, $\hat{k}_p$, $\hat{k}_d$ and $\hat{k}_i$ are bounded. To show that s(t) goes to zero with respect to time, s(t) should be investigated in deep by taking the time derivative of s^2 as

$$\frac{1}{2}\frac{d}{dt}s^2 = s\dot{s}$$

$$= (\dot{x}_{d2} - g - k_p x_1 - k_d x_1 - k_i \int x_1 - u)$$

$$+ 2\lambda\dot{x}_{d1} - 2\lambda x_1 + \lambda^2 x_1)s . \qquad (28)$$

By substituting (9) and (15) in (28), it is obtained as,

$$\frac{1}{2}\frac{d}{dt}s^2 < -k|s| + (-\tilde{k}_p x_1 - \tilde{k}_p \dot{x}_1 - \tilde{k}_i \int x_1)s . \qquad (29)$$

If k is selected as

$$k > \left|\breve{k}_p x_1 + \breve{k}_d \dot{x}_1 + \breve{k}_i \int x_1\right| + \eta \qquad (30)$$

where

$$\breve{k}_p = \bar{k}_p - \underline{k}_p \qquad (31)$$

$$\breve{k}_d = \bar{k}_d - \underline{k}_d \qquad (32)$$

$$\breve{k}_i = \bar{k}_i - \underline{k}_i , \qquad (33)$$

where $\underline{k}_p$, $\underline{k}_d$ and $\underline{k}_i$ are lower bounds of k_p, k_d and k_i, respectively, (29) is obtained as

$$\frac{1}{2}\frac{d}{dt}s^2 < -\eta|s| \qquad (34)$$

which leads

$$\dot{s} < -\eta|s| . \qquad (35)$$

From (35), it is seen that starting from any initial condition, the state trajectory reaches to the surface in a finite time smaller than $|s(t=0)|/\eta$ and then converges to $x_d(t)$ exponentially with a time constant equal to 1/λ [18].

4. Numerical Simulations

The performance of the control law in (9) and update rule in (15) were evaluated by conducting numerical simulation by using the dynamic model of a 2-DOF helicopter which is known as TRMS.

During the simulation, the parameter values of input signal were selected as λ=diag(30,30), k=diag(1,1), $\rho=[1\ 1]^T$, $L_m = [1\ 1]^T$. The initial values of gain estimates were set to $k_p=[5\ 5]^T$, $k_d=[5\ 5]^T$ and $k_i=[5\ 5]^T$. The initial positions of the axes were $x(0)=[0.5\ 0.5]^T$ in radian and the desired positions were selected as $x_d=[0.4\ 0.3]^T$ in radian.

In the numerical simulations, it was observed that the control law performed satisfactorily. The position errors and the control inputs of yaw and pitch axes are presented in Figures 1, 2, 3 and 4, respectively. The PID gain estimates are given in Figures 5, 6 and 7. As can be seen in the figures, both the yaw and the pitch errors are driven to the vicinity of zero.

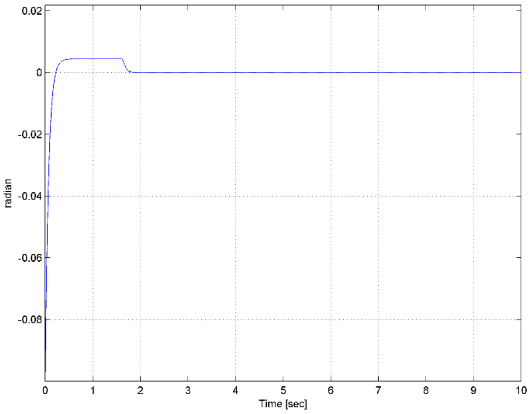

Fig. 1. Yaw axis position error

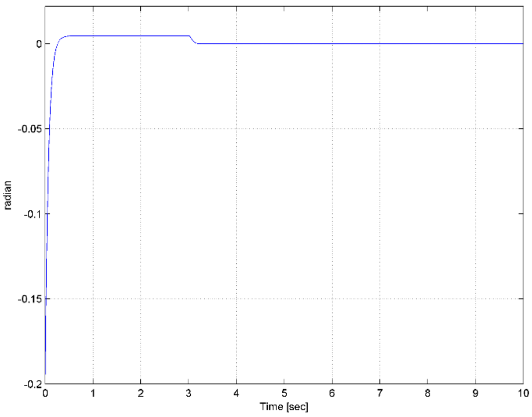

Fig. 2. Pitch axis position error

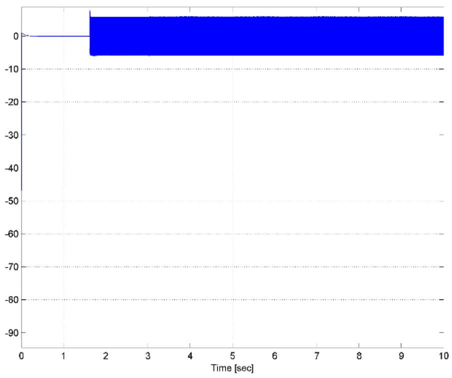

Fig. 3. Input signal for yaw axis

5. Conclusions

In this paper, a sliding mode based self-tuning PID controller was designed for second order systems. While designing the controller, it was assumed that the system model contain nonlinear terms similar to PID structure. The controller and update rule for PID parameters were obtained from Lyapunov stability analysis. The effectiveness of the controller and update rule were evaluated by conducting numerical simulation and achieved satisfactory results.

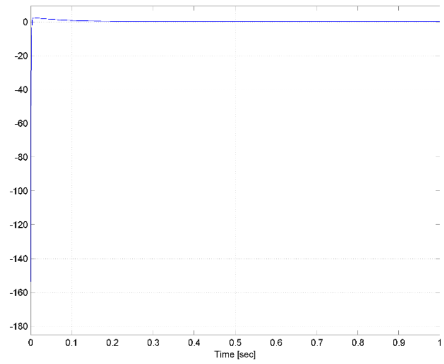

Fig. 4. Input signal for pitch axis

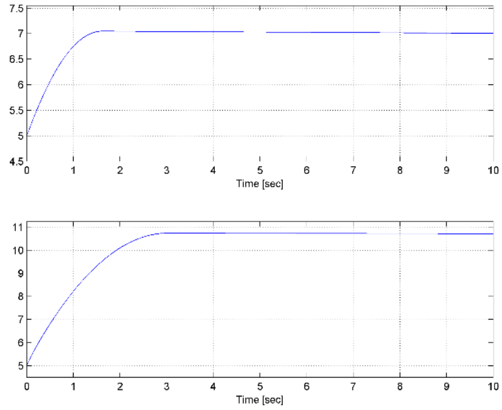

Fig. 5. K_p estimates for yaw axis (top) and pitch axis (bottom)

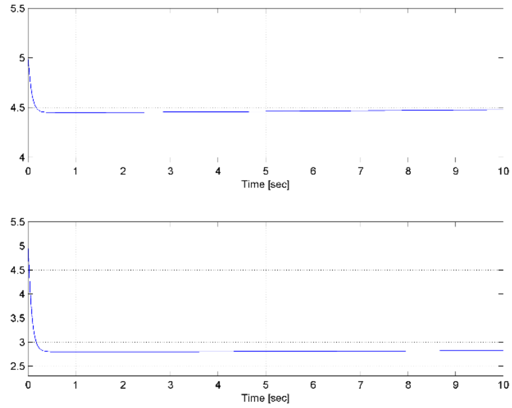

Fig. 6. K_d estimates for yaw axis (top) and pitch axis (bottom)

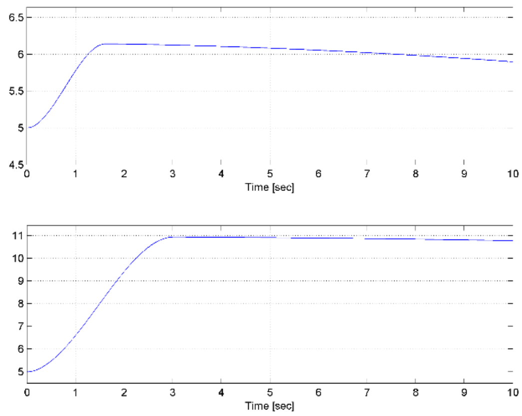

Fig. 7. K_i estimates for yaw axis (top) and pitch axis (bottom)

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