Proceedings of the International Symposium of Mechanism and Machine Science, 2017
AzC IFToMM – Azerbaijan Technical University
11-14 September 2017, Baku, Azerbaijan

Topics 3. Dynamics of machines

About application of methods of direct linearization for calculation of interaction of nonlinear oscillatory systems with energy sources

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Abstract.
Use of methods of direct linearization for calculation of the nonlinear oscillatory systems interacting with energy sources of limited power is described. As application of procedure the compelled fluctuations of nonlinear system with limited excitement are considered.
Keywords: method, direct linearization, oscillatory system, interaction, energy source.

1. Introduction

Now the question of economy of energy has moved to the forefront, including in all branches of equipment, at design and calculation of various cars, mechanisms, devices, etc. In this context the theory of oscillatory systems with limited excitement (or interactions of oscillatory systems with energy sources of limited power) which basis was the known physical effect of – Sommerfeld's effect found by A.Sommerfeld at the beginning of the last century (1902) in the analysis of controllability of electric motors deserves attention. Systematic studying of this effect was carried out by V.O.Kononenko whose result was his fundamental monograph which has appeared in 1964 [1]. Development and a condition of this theory has found further in books [2-5, 21] and a set of articles.

In the theory of oscillatory systems with limited excitement the main method of the analysis is the method of averaging of nonlinear mechanics which use is connected by considerable labor and time expenditure depending on a type of the nonlinear characteristic. Such expenses are inherent also in another, described in many works (for example, [2,6-9]), to the known methods of nonlinear mechanics: consecutive approximations, harmonious linearization, power balance, etc. The method of direct linearization (MDL) described in the monograph [11] and a number of articles [12-20] which became further improvement of the method [2,9] offered in 1952 G.Panovko essentially differs from these methods. Indisputable advantage of MDL to carrying out calculations of various technical systems in practice is caused by its properties: simplicity of application; lack of labor-consuming and difficult approximations of various orders applied in the known methods of nonlinear mechanics; possibility of receiving final settlement ratios irrespective of a concrete type and degree of nonlinearity; rather small expenses of work and time (is several orders less in comparison with methods of nonlinear mechanics). Comparison of a number of the results received by the known methods of nonlinear mechanics and MDL stated above contains in [11] and some other works given below in the list of references. It is carried out in case of system with an ideal power source and shows their coincidence: qualitative (full) and quantitative (depending on accuracy parameter: from full coincidence to several percent of discrepancy).

In tasks of the theory of oscillatory systems with limited excitement, at least, two equations, one of which describes actually oscillatory system, another - to loudspeaker of a power source as which in the majority of tasks the electromechanical activator [2] is are considered. As the purpose of the real work is procedure of application of MDL in tasks of the theory of oscillatory systems with limited excitement, will consider her on the basis of the one-mass oscillatory system described by the differential equations of a general view

$$\ddot{x}+kx+cx=F(x,\dot{x},\varphi,\dot{\varphi},\ddot{\varphi}), \qquad (1)$$
$$\ddot{\varphi}=M(\dot{\varphi})+H(\varphi,\dot{\varphi},x,\dot{x},\ddot{x}).$$

The first equation in (1) describes the movement of oscillatory system, the second – an energy source (an electric motor rotor). The $F(x,\dot{x},\varphi,\dot{\varphi},\ddot{\varphi})$, $M(\dot{\varphi})$ and $H(\varphi,\dot{\varphi},x,\dot{x},\ddot{x})$ functions generally nonlinear can also have different concrete types. The last of them reflect respectively a driving force of an energy source and loading (including from oscillatory system) on him.

2. Procedure of replacement of the nonlinear characteristic power source of linear

The nonlinear $G(z)$ or $G(z,\dot{z})$ function depending on some variable of z is replaced by a method of direct linearization [11] with linear function

$$G_*(z) = B + kz\,, \qquad (2)$$

where B and k – the linearization coefficients depending on the linearization accuracy parameter method determining accuracy.

The real characteristics of forces which are, as a rule, nonlinear are approximated in practice in most cases by polynomial function which we will write down in a look

$$G(z) = \sum_n b_n z^n\,,\ b_n = const\,,n = 0,\ 1,\ 2,\ 3,\ 4,\ \ldots$$

For this function coefficients of linearization are defined according to [11] expressions

$$B = \sum_n b_n B_n\,,\ B_n = N_n \varsigma^n\,,\ n = 0, 2, 4, \ldots\ (n - \text{even}),\ (3)$$

$$k = \sum_n b_n k_n\,,\ k_n = \bar{N}_n \varsigma^n\,,\ n = 1, 3, 5, \ldots\ \ (n - \text{odd}),$$

$$N_n = (2r+1)/(2r+n+1)\,,$$

$$\bar{N}_n = (2r+3)/(2r+n+2)\,,\ \varsigma = \max|z|\,,$$

r – linearization accuracy parameter.

Polynomial function it is possible to describe also nonlinear characteristic of an energy source

$$M(\dot{\varphi}) = \sum_i \alpha_i \dot{\varphi}^i\,,\ \alpha_i = const\,,\ i = 0,\ 1,\ 2,\ 3,\ \ldots,\quad (4)$$

which it is replaceable a linear form

$$M_*(\dot{\varphi}) = B_M + k_M \dot{\varphi}\,. \qquad (5)$$

The power source equation taking into account (5) takes a form

$$\ddot{\varphi} = B_M + k_M \dot{\varphi} + H(\varphi, \dot{\varphi}, x, \dot{x}, \ddot{x})\,. \qquad (6)$$

Coefficients of B_M and k_M are defined by expressions of a look (3) on condition of replacement of ς by $\Omega = \max|\dot{\varphi}|$ giving the maximum size of the average value of speed Ω of a power source considered below.

3. About the decision of the linearized system of the equations

Under conditional names of "a linear form" and "replacement of variables with averaging" in work [11] two methods for the decision of the linearized system of the equations are offered. We will consider here only use of a method of replacement of variables with averaging for the energy source equation because for oscillatory system it is described in [11]. In this method ratios are used

$$x = \upsilon p_\circ^{-1} \cos\psi\,,\ \dot{x} = -\upsilon \sin\psi\,,\ \psi = p_\circ t + \xi\,, \qquad (7)$$

from where expression of $\upsilon = ap_\circ$, where a and $p_\circ$ – respectively amplitude and frequency of fluctuations follows.

We will apply procedure of averaging for the period to the equation (6), believing $\dot{\varphi} = \theta$ therefore we have

$$\frac{d\Omega}{dt} = B_M + k_M \Omega + H(\cdots)\,, \qquad (8)$$

$$H(\cdots) = \frac{1}{2\pi}\int_0^{2\pi} H(\varphi, \dot{\varphi}, x, \dot{x}, \ddot{x})\, d\psi\,.$$

The equation (8) allows to define change in time of average value Ω speeds θ of an energy source. From (7) at $\dot{\Omega} = 0$ we will receive the equation determining parameters of stationary movements

$$B_M + k_M \Omega + H(\cdots) = 0\,. \qquad (9)$$

The equation (9) establishes dependence between amplitude and speed of Ω because generally $H(\cdots) \Rightarrow H(a, \Omega)$.

4. Example

System shown in fig.1 which it is representable the equations of a general view can be the example described by the equations of a look (1)

$$m\ddot{x} + F(\dot{x}) + f(x) = mr\dot{\varphi}^2 \cos\varphi + mr\ddot{\varphi}\sin\varphi\,, \qquad (10)$$

$$J\ddot{\varphi} = M(\dot{\varphi}) + m\ddot{x}r\sin\varphi + mgr\sin\varphi\,,$$

where m – the mass of the unbalanced body fixed at r distance from an engine rotor shaft axis with the total moment of inertia of the I rotating parts, $f(x)$ – the nonlinear elastic force of a spring, $F(\dot{x})$ – the nonlinear force of resistance, $M(\dot{\varphi})$ – a difference of the rotating moment of an energy source and the moment of forces of resistance to rotation, $\dot{\varphi}$ – the speed of rotation of the engine.

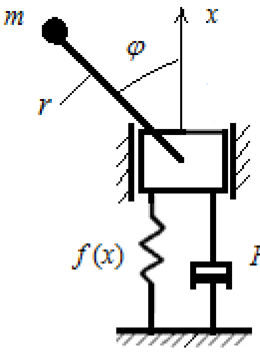

Fig. 1

System presented in fig.1 in case of $f(x) = cx + \gamma x^3$ and $F(\dot{x}) = k\dot{x}$ is considered in work [1] by means of an asymptotic method of averaging.

The nonlinear equations (10) get on the basis of (2) and (4) following linear forms:

$$m\ddot{x} + k_F(\upsilon)\dot{x} + \omega^2(a)x = mr\dot{\varphi}^2\cos\varphi + mr\ddot{\varphi}\sin\varphi, \quad (11)$$

$$J\ddot{\varphi} = B_M(\Omega) + k_M(\Omega)\dot{\varphi} + m\ddot{x}r\sin\varphi + mgr\sin\varphi,$$

where $\omega^2(a)$ and $k_F(\upsilon)$ – coefficients of linearization of nonlinear elastic forces and friction taking into account that at their linearization $\max|x| = a$, $\max|\dot{x}| = \upsilon$.

By a method of replacement of variables with averaging of the solution of the equations (11) it is possible to construct, using ratios of type (7), i.e.

$$y = \upsilon\theta^{-1}\cos\psi, \ \dot{y} = -\upsilon\sin\psi, \quad \psi = \theta t + \xi.$$

On the basis of these ratios we have from (11) equations

$$\frac{d\upsilon}{dt} = \frac{\upsilon}{2m}k_T(\upsilon,\Omega), \ \frac{d\xi}{dt} = \frac{\omega^2 - \Omega^2}{2\Omega}. \quad (12)$$

The second equation (11) according to (8) has an appearance

$$\frac{d\Omega}{dt} = \frac{1}{J}\left[B_{TM}(\upsilon,\Omega) + \Omega\, k_M(\Omega)\right]. \quad (13)$$

From (12) and (13) follow at $\dot{\upsilon} = 0$, $\dot{\xi} = 0$, $\dot{\Omega} = 0$ equations

$$k_T(\upsilon,\Omega) = 0, \ B_{TM}(\upsilon,\Omega) + \Omega k_M(\Omega) = 0, \quad (14)$$

the parameters of stationary movements allowing to define.

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