Proceedings of the International Symposium of Mechanism and Machine Science, 2017
AzC IFToMM – Azerbaijan Technical University
11-14 September 2017, Baku, Azerbaijan

Dual-scheme profiling technique for the liquid rocket engine

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Abstract

The paper studies profiling method of supersonic nozzle based on the velocity distribution of the 2D gas flow along the nozzle axis. Wherein, uniform flow on the exit section of the nozzle is required. The equations of gas dynamics are solved with a new equation for velocity distribution along the nozzle axis by applying the method of characteristics, which stipulates the development of dual-scheme profiling technique for the liquid rocket engine (LRE) nozzle.

Keywords: liquid rocket engine, optimal nozzle, supersonic flow, perfect gas, Prandtl Meyer function, method of characteristics, relative error

1. Introduction

The expansion of the space exploration area demands future enlargement in launching rockets of different classes with payloads from hundreds of kilograms to tens and more tons.

The optimization of the nozzle construction is carried out, in order to ensure the maximum possible payload. The main goal of the optimization process is to select the area expansion ratio of the nozzle ($\bar{A} = A_e / A_*$) which will be the most advantageous combination from the position of specific impulse I_{sp} and the weight of the engine structure. Particular attention is paid to the choice of the optimal expansion ratio of the LRE. Thus, for example, gas pressure falling at the nozzle exit section leads to the growth of I_{sp} and increment of the geometric dimensions of supersonic part of the nozzle. Increasing of the mass and dimensions of the nozzle results in a reduction while the maximum possible payload taking out by the rocket. In addition, the nozzle dimensions may enable LRE to be satisfactorily assembled in the tail part of the rocket. The rate of the specific impulse in the vacuum mostly depends on the losses in the nozzle, which is increased by the expansion ratio of nozzles. In this case, the expansion ratio is determined both as the geometry of the nozzle contour (throat and exit areas), and as the ratio of the pressures at the inlet and outlet. Therefore, one of the main research fields is the retrieval of an optimal contour of the subsonic and supersonic parts of the LRE

2.The aim of the paper

The purpose of the paper is to form methodology for the preliminary optimal geometry design of the nozzle contour based on the analysis of the theoretical and experimental research results of rocket engines

3.Brief information on the theoretical foundations of gas flows modeling in nozzles

The flow of gas in the LRE chamber is researched for the purpose of selecting the geometric parameters of jet nozzles, determining their thrust and energy characteristics. By the development of rocket technology, the price of a unit of the engine specific impulse is increasing. Thus, the growth of a specific impulse by only 0.3% may lead to an increase in the payload weight up to 1.5%. Calculations on one-dimensional theory enable the determination of the nozzle impulse accurately to several percent. In this case, calculation inaccuracy of integral characteristics of gas flows in jet nozzles by numerical methods should not exceed 0.1%. Methods which do not meet this accuracy are suitable only for a qualitative description of the flow [1,2].

In addition, defining the target task and the specific flight trajectory of the rocket, taking into account the limitations on the dimensions and mass of the nozzle is very necessary in selecting the optimal nozzle. These constraints lead to a very complicated task of profiling and optimization of the nozzle.

Despite various limitations, the main difficulties in creating the definite methodology for calculating the LRE nozzle are related to:

- changing properties of real gas
- inconstant gas flow in different sections,
- non-stationary nature of the stream of gas flow,
- heterogeneity of gas,
- variability of gas composition,
- friction between gas and nozzle walls,
- heat transfer through the nozzle walls,

- viscosity and friction between gas layers,
- different directions of vectors and values of gas velocities at different points of the considered cross-section of the nozzle, etc.

Therefore, the LRE nozzles contour formation is led by using the solution of gas dynamic equations system - the momentum equation (Euler equation), energy, and continuity. In the absence of irreversible processes, this system for a stationary non-vortex axisymmetric (two-dimensional) flow of an in viscid and non-heat-conducting gas, by a constant level of the heat capacity ratio γ, can be indicated as below [1,2]:

$$(u^2-a^2)\frac{\partial u}{\partial x}+uv\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+(v^2-a^2)\frac{\partial v}{\partial y}=a^2\frac{v}{y}, \qquad (1)$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=0, \qquad (2)$$

where u, v - components of flow velocity W on the coordinate axis x, y ; x directed along the nozzle axis, the axis y - perpendicular to it.

The system of differential equations (1-2), depending on the flow velocity, can be of various types: elliptical ($M<1$), parabolic ($M=1$), and hyperbolic ($M>1$). Properly, the numerical methods for this system solving are different. Consequently, the questions of profiling the subsonic and supersonic nozzle parts are usually considered separately, which is another drawback of all modern calculation methods.

The profile of the subsonic part of the nozzle can be calculated by solving the equations system (1-2) within specific boundary conditions. However, the solution to this system is extremely difficult. Therefore, a wide application for profiling the subsonic part of the nozzle obtained empirical equations [1, 2, 12, 13].

Differential equations solution (1-2) by the method of characteristics [1, 2, 14, 15] is the base for creating the theoretical profile of the supersonic nozzle part. In addition, other methods of creating optimal nozzles for rocket engines are quite integrated and implemented in practice [14, 15].

In general, the nozzles obtained on the basis of solution of differential equations system (1-2) are characterized by two parameters: the isentropic expansion rate γ and the number M , i.e. Mach number. Herein the specification of the length or radius of the nozzle outlet section within M and γ explicitly defines contour coordinates from the critical (throat) to the exit section of the nozzle.

4. Research analysis in the field of profiling supersonic nozzles of LRE

Generally, development LRE covers two kinds of nozzles: nozzles with uniform characteristics and nozzles with extreme characteristics [8]. The contour of the supersonic part of the extreme nozzle within the same nozzle area expansion ratio is shorter than the nozzles with uniform characteristics. However, the contour of the supersonic part of the nozzle with a uniform characteristic has the advantage that at the same length it has minimum losses for exhaust gases scattering.

Theoretical basis for profiling the supersonic nozzles of LRE in order to achieve the best combination "specific impulse-structural weight" developed in the middle of the last century [3-5]. It has theoretically been proved that the search of an optimal nozzle contour is a variational problem that reduces to numerical integration of the system of ordinary first-order differential equations. Calculations show that the nozzle of the smallest length is not the best by weight characteristics [5]. In other researches presented formulation of the problem and definition of an extreme nozzle as gaining the greatest thrust at given diameters and length [6]. Theoretical-calculation work has been carried out in the field of influence degree of the basic assumptions used in profiling, and proposed a generalized theory of the design of extreme nozzles [7,8]. The rest research works inform about the comparative theoretical studies of the thrust characteristics of rocket nozzles constructed by various methods. It has been shown that the extremely shortest nozzles with a uniform characteristic hold the smallest losses [9].

However, all the mentioned researches do not allow the formation of the initial shape of the nozzle, which should be optimized from the point of view of LRE energy-mass characteristics ($m_{LRE} \to m_{LRE.\min}$ and $I_{s.i} \to I_{s.\max}$). Therefore, the research of gas-dynamic and energy characteristics of nozzles from the area expansion ratio is of great importance, which is closely related to the modeling problem of gas flows in nozzles.

5. Problem formulation and solution in the first approximation

The analysis of the existing researches results shows that the design of the nozzle contour in various formulations does not imply any preliminary correct distribution of the parameters γ and M along the length of the nozzle (on the axis or the contour curve). The solution of equations (1-2) within such problem formulation on nozzle profiling requires large mathematical resources. So, according to the logic and sequence of the performed operations by the thermogasdynamic calculation of the LRE, the values of the gas flow parameters are found step by step. Each subsequent value of any thermogasdynamic parameter is calculated on the its previous value.

Alongside with this, if the required distribution of flow parameters is known at the outlet section of the nozzle under given constraints, the contour providing this distribution of the nozzle is defined by calculation. In this case, the solution of the inverse problem of gas dynamics arises. If in this case one of the parameters defining the task must minimized ($I_{sp.i} \to I_{sp.\max}$ and $m_{LRE} \to m_{LRE.\min}$), then the problem refers to the class of extremes. Since the contour of the nozzle is specified before solving the problem by some unknown function, then the problem becomes variational.

Thus, the question is: Is it possible to preset the preliminarily known distribution of some gas flow parameters along the length of the nozzle (on contour curve or on the axis) in the first approximation? Such kind of opportunity would allow the more rational use of computational resources for the purpose of creating an optimal or extreme LRE nozzle.

In order to answer the current question theoretical, computational and experimental research analysis were led in the field of designing the LRE chamber. Review of researches results shows that on average distribution of thermogasdynamic parameters (p, T, W etc.) along the length of the rocket engine chamber (combustion chamber, subsonic and supersonic part of the nozzle) is of a certain character. This circumstance is related with the basic principles of the nozzle theory. Naturally, the distribution of parameters is determined by the complex nature of the entire system of thermogasdynamic processes (see p. 3), proceeding in a rocket engine.

A preliminary analysis of these mentioned researches shows an approximate asymmetric sigmoid velocity distribution of the gas flow along the length of the chamber (along the nozzle axis) of the LRE. This velocity distribution (combustion chamber + subsonic and supersonic part of the nozzle) can be indicated as following functions:

$$y(x) = \frac{1+\alpha+bx}{1+\alpha e^{-kx}}, \tag{3}$$

$$\text{or} \quad y(x) = d + \frac{a-d}{1+(x/c)^m}, \tag{4}$$

where α, a, d, c, k, m - some unknown coefficients, which can be specified on the basis of numerical calculations or experimental data, x-the coordinate of the point under consideration on the nozzle axis. It should be noted that an attempt to describe the velocity distribution of the free gas stream over the sigmoid was considered in [10,11]. Formulas (1) and (2) for the gas velocity can also be represented for relative lengths $\bar{x} = x/L$.

$$w(x) = \frac{1+\alpha+b\bar{x}}{1+\alpha e^{-k\bar{x}}}, \tag{5}$$

$$\text{or} \quad w(x) = d + \frac{a-d}{1+(\bar{x}/c)^m}. \tag{6}$$

The contour of the LRE nozzle is created thereby. Firstly, for each value x or $\bar{x}$ the velocity values $w(x)$ on the nozzle axis are determined starting from the critical (throat) section (or from the outlet section of the nozzle):

$$w(x) = w_e - \frac{w_e - w_0}{1+(x/c)^m},$$

$$\text{or} \quad w(\bar{x}) = w_e - \frac{w_e - w_0}{1+(\bar{x}/c)^m}. \tag{7}$$

where w_0 - the velocity of the gas flow in the minimum (throat) section of the nozzle, w_e - the velocity of the gas flow at the outlet from the nozzle. It should be noted that the choice of speed w_0 depends on the shape of the transition surface through the sound speed, which can be flat or curved. An analysis of the results of the researches shows that in order to design the optimal contours of LRE nozzles, the most suitable preliminary velocity distribution along the axis is (7). Thus, system (1-2) can be supplemented by one more equation:

$$\frac{du}{dx} = \frac{u_e - u_0}{\left(1+\left(\frac{x}{c}\right)^m\right)^2} \cdot \left(\frac{x}{c}\right)^m \cdot \frac{m}{x}, \quad u = w_x$$

By taking into account the above mentioned, let us consider a dual-scheme calculation technique for determining the main parameters of a supersonic 2D flow and nozzle geometry.

First calculation scheme (C1). As it has already been noted, the method of characteristics (MC) is often used to calculate the parameters of the expanding part of the nozzle. For a supersonic irrotational perfect gas, the application of the MC starts on sonic line and specified by the following equations [14,15]:

1) first characteristics lines group (C^+)

$$d(\nu - \theta) = 0, \; dy/dx = tg(\theta + \mu). \tag{8}$$

2) second characteristics lines group (C^-)

$$d(\nu + \theta) = 0, \; dy/dx = tg(\theta - \mu). \tag{9}$$

where ν - the value of the Prandtl-Mayer function obtained from a given M at the considered point, θ - the local flow angle (the angle of inclination of the velocity $\vec{w}$ with respect to the nozzle axis, μ - the angle between the velocity vectors $\vec{w}$ and the tangent to the characteristic line at the point under consideration. In the real conditions the characteristics are curved. The grid formed by the characteristic curves C^+ and C^- will be accurate when they are very close (Fig.1).

In equations (8) and (9), the Prandtl-Mayer function for the considered points will be determined by the formula [14, 15]:

$$\nu = \nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \cdot \arctan\left(\sqrt{(M^2-1)\frac{\gamma-1}{\gamma+1}}\right) - \arctan(\sqrt{(M^2-1)}) \qquad (10)$$

where γ - the heat capacity ratio, M - the Mach number of the gas flow at mentioned the nozzle point.

Thus, on the characteristic line C^+ (points 1 and 3)

$$\theta_1 - \nu(M_1) = \theta_3 - \nu(M_3), \qquad (11)$$

on the characteristic line C^- (points A and 3)

$$\theta_3 + \nu(M_3) = \theta_A + \nu(M_A). \qquad (12)$$

Then for node 3 with coordinate x_3 we can write (Fig. 2)

$$\theta_3 = \frac{1}{2}\left(\tilde{N}^- + \tilde{N}^+\right), \quad \nu(M_3) = \frac{1}{2}\left(C^- - C^+\right). \qquad (13)$$

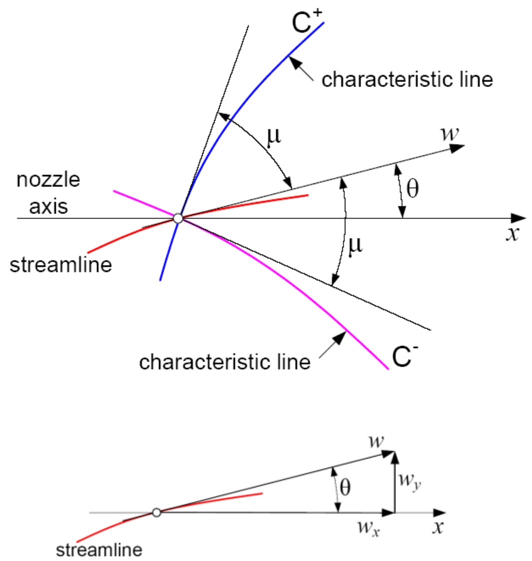

Fig. 1. Geometric representation of the Method of Characteristics

Second calculation scheme (C2). An analysis of the researches shows that the nozzle axis can also be taken as an initial line, by taking into account the equation (7). So the application of the method of characteristics is based on the data obtained from the thermodynamic calculation of the LRE by using a formula (7). Then second calculation scheme C2 is applied in parallel to the C1 calculation scheme.

Thus, on the characteristic line C^+ (points 0 and 2)

$$\theta_0 - \nu(M_0) = \theta_2 - \nu(M_2), \qquad (14)$$

on the characteristic line C^- (points 6 and 2)

$$\theta_6 + \nu(M_6) = \theta_2 + \nu(M_2). \qquad (15)$$

Then for **node 2** with coordinate x_2 we can write (Fig. 2)

$$\theta_2 = \frac{1}{2}\left(C^- + C^+\right), \; \nu(M_2) = \frac{1}{2}\left(C^- - C^+\right), \qquad (16)$$

where

$$M_0 = w_0 / a_{x.0}, \; w_{x.0} = w_0 \cos(\theta_0),$$

$$w_{x.0} = w_e - \frac{w_e - w_0}{1 + \left(x_0 / c\right)^m}.$$

It should be noted that the gas velocity in the critical (throat) section of the nozzle (on the intersection of sonic line and nozzle axis) can be taken as an initial. The **calculation scheme C2** is executed both in x and y direction (x axis is the initial line). Determination of a coordinates of any considered point given in [14,15]. The Mach numbers $M(x)$ or $M(\bar{x})$ at the considered initial points (0, 6, 9, etc.) are known from the thermodynamic calculation of the LRE (Fig. 2).

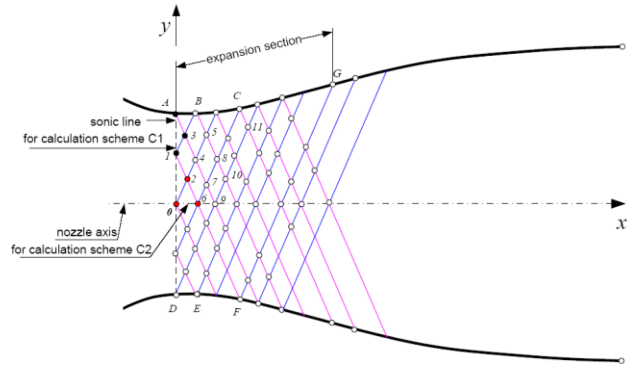

Fig. 2. The calculation scheme (C1+C2) using MC (contour of the nozzle is shown conditionally)

A dual-scheme calculation technique (C1+C2) for determining the main parameters of a supersonic 2D flow and nozzle geometry can be carried out with an unknown or known initial nozzle length L . In the second case, the initial nozzle length after each complete calculation cycle will be verified. The calculation will be repeated with a new value L_i . Accordingly, at the beginning of each calculation cycle, the shape of the distribution $w(x)$ along the nozzle axis will be changed. Therefore, for each new calculation cycle i , the distribution of the parameters values $[M(x)]_i$ and $[a(x)]_i$ given nozzle area expansion ratio $(A_e/A^*)_i = const$ and pressure expansion ratio $\varepsilon_i = (p_c/p_e)_i = const$ will be updated

$$(A_e/A^*)_i = \frac{1}{M_e}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

or $$(A_e/A^*)_i = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\sqrt{\frac{\gamma-1}{\gamma+1}}}{(1/\varepsilon_i)^{\frac{1}{\gamma}}\sqrt{1-(1/\varepsilon_i)^{\frac{\gamma-1}{\gamma}}}},$$

where p_c - pressure in the combustion chamber, p_e - pressure on the nozzle exit section. Thus, refinement of the optimal nozzle length L_{opt} and optimal shape of the nozzle contour curve $(NC)_{opt}$ have been carried out. In the other case, the calculation is performed with the condition

$$[\,\varepsilon = \varepsilon_{opt}\,,\; L_{opt} = const\,,$$

$$(A_e/A^* = \text{var}) \rightarrow (A_e/A^*)_{opt}\,] \Rightarrow (NC)_{opt}$$

Calculation ends when the condition at fulfillment of conditions

$$F((NC)_{opt}) = \min_{L}\ \max_{I}\ \Phi(I,L),$$

$$\lim\left|(NC)_{opt} - (NC)_{C1+C2}\right| \rightarrow \omega\,,$$

where L - the length of the nozzle supersonic part, I - the specific impulse of the engine, ω - the required value of the error.

6. Conclusion

In the current paper we have proposed a dual-scheme calculation technique for determining an optimum contour of the nozzle using the results of the thermodynamic calculation of the parameter values of the supersonic 2D flow, which is covered the internal curved profile of the LRE nozzle. The method is based on semi empirical formula, which was received based on results analysis of the numerical and experimental researches.

The proposed technique allows performing the thermogasdynamic and geometric calculations of the LRE nozzle with optimum contour by considering the required error.

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